Article

Capital Allocation in Decentralized Businesses

Stuart M. Turnbull

Bauer College of Business, University of Houston, TX 77004, USA; stuartmturnbull@gmail.com

Abstract: This paper described a theory of capital allocation for decentralized businesses, taking into account the costs associated with risk capital. We derive an adjusted present value expression for making investment decisions, that incorporates the time varying profile of risk capital. We discuss the implications for business performance measurement.

1. Introduction

The credit worthiness of a bank is a major concern to its management, trading partners, creditors and bank regulators. The lower the credit worthiness, the greater will be the agency and monitoring costs, resulting in increased credit spreads and lower credit ratings. It will also increase the need to provide larger collateral. Steps to increase the credit worthiness will lower these costs while increasing the opportunity cost of holding a protective buffer.

Senior management must decide on the optimal size of a buffer to hold and the allocation to individual businesses within the bank.[1] Erel et al. (2015) (EMR) take a top-down approach to asset and risk capital allocation subject to constraints on the default put to liability ratio and the total amount of risk capital. First, they consider the bank as a whole and then as a multi business. A centralized approach is assumed, with senior management determining the optimal asset allocation for each business. They show that the sum of the product of each asset value of a business and its marginal default put value equals the default value of the bank as a whole. They derive the optimal asset and risk capital allocation to the individual businesses. The model is a single period and debt used to fund investments is assumed to be default free.

For many large multi-business banks, senior management delegate the responsibility of running the businesses to the business managers, subject to various constraints. Individual businesses are held accountable for the risk capital allocated to the business and managers are judged on how well they run the individual businesses. Consequently, managers would prefer performance metrics to be based on factors for which they have direct control. For example, the determination of risk capital for a project should depend on the characteristics of the project and not the characteristics of the bank, as in Erel et al. (2015). The maturity of the debt used to finance a project should be similar to the expected maturity of the project and not the average maturity of the bank's debt.[2] The question of how to allocate capital internally is a question faced by all banks. In this paper, we assume decentralized management, unlike Erel et al. (2015), who assume centralized management.

A manager of a business decides on the asset allocation to maximize the present value of investments, subject to constraints set by senior management. First, there is a limit to the total

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[1] For reviews of extant work on capital allocation, see Bhatia (2009), Matten (2000) and Crouhy et al. (1999). Bajaj et al. (2018) provide a review of the types of methodologies employed by different financial institutions.
[2] This issue does not arise in Erel et al. (2015), as it is a single period model and bonds are assumed to be pari passu if default occurs.

amount of risk capital that a business can employ. Second, there is a limit on the credit risk for each business. The determination of the default put option depends on the characteristics of the business and not the rest of the bank. This implies that we no longer have the aggregation result given in EMR; the sum of the value of the default puts for all the businesses will be greater than the default put for the bank, giving rise to what is termed "the portfolio effect". This is not surprising, given the work of Merton (1973) and Merton and Perold (1993). We show how the credit risk limit assigned to individual businesses by senior management can be set such that the portfolio effect is zero.

With decentralization come issues of setting bonuses for business managers, senior management must determine the relative performance of the different businesses. We show how to determine the adjusted present value of each business. Simple performance measures such as the risk adjusted rate of return on capital (RAROC) do not correctly adjust for credit and market risk (see Wilson (1992), Froot and Stein (1998), Crouhy et al. (1999) and Erel et al. (2015) and the errors can be large, see Demine (1998)). We discuss alternative measures.

Section 2 of the paper provides some basic definitions, drawing on the work of Erel et al. (2015). Section 3 extends the Erel et al. (2015) model to decentralized management. Section 4 examines business performance and conclusions are given in Section 5.

2. Top-Down Planning

In this section, we briefly describe the Erel et al. (2015) model. Risk capital has been defined as a buffer to absorb unexpected losses and provide confidence to investors and depositors—see Bhatia (2009). It is the amount needed to cover the potential diminution in the value of assets and other exposures over a given time period using some metric as a risk measure. The expected risk capital requirements will vary over the life of a project. For example, the risk capital for a foreign currency swap will increase as its maturity declines. The amount of risk capital is determined internally by the company, using a risk measure such as the value-at-risk (VaR) or expected shortfall.[3] This definition updates the definition given in (Matten 2000, p. 33). Economic capital is defined as

$$\text{Economic Capital} = \text{Risk Capital} + \text{Goodwill}.$$

Instead of using VaR or expected shortfall, Merton and Perold (1993) (MP) introduce a different definition of risk. Risk capital is defined as the cost of buying insurance (default put), so the debt of the firm is default free. If default occurs, the default put pays bondholders. It is assumed that there is no counterparty risk associated with the seller of the insurance. This definition is used by Erel et al. (2015), who consider the issue of capital allocation for a firm with different businesses in a single period model. They show how tax and other costs of risk capital should be allocated to the individual businesses.

The market value of debt, D, can be written as

$$D = L - P, \tag{1}$$

where L denotes the default free value of liabilities and P the value of the put option with strike price equal to the face value plus coupon payment. The term P is a measure of the dollar cost arising from the risk of default and financial distress. We assume that goodwill is zero, a similar assumption is made in Erel et al. (2015). Merton and Perold (1993) define the risk capital C as

$$C = A - L, \tag{2}$$

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[3] Artzner et al. (1999) show that expected short fall is a coherent risk measure.

where A represents the market value of the assets. The risk capital is a measure of the cushion between the assets of the firm and amount of liabilities arising from issuing debt. The larger the risk capital, the greater is the credit worthiness. The risk capital ratio is defined as $c = C/A$, where

$$c = 1 - \frac{L}{A}. \tag{3}$$

The value of the put option is given by

$$P = \int_{z \in Z} [LR_L - AR_A]\pi(z)dz = \int_{z \in Z} A[(1-c)R_L - R_A]\pi(z)dz,$$

where Z is the set of states for which the revenues are insufficient to meet obligations—more formally defined as $Z = \{z; (1-c)R_L - R_A(z) > 0\}$ and $\pi(z)$ is the pricing kernel. If $R_A(z)$ is assumed to have a mean μ_A and standard deviation σ_A, then we can write $R_A(z) = \mu_A + \sigma_A z$, where z has zero mean and unit variance. This does not imply that z is normally distributed. It is assumed that μ_A and σ_A do not depend on A, and a similar assumption is implicitly made in EMR. Note

$$A(1-c)R_L - AR_A(z) > 0 \quad \Leftrightarrow \quad \frac{(1-c)R_L - \mu_A}{\sigma_A} > z.$$

Let $U \equiv \frac{(1-c)R_L - \mu_A}{\sigma_A}$; then,

$$P = A \int_{-\infty}^{U} [(1-c)R_L - (\mu_A + \sigma_A z)]\pi(z)dz. \tag{4}$$

The default put option is a function of the level of assets, the risk capital ratio c and U. The marginal put value is

$$p = \frac{\partial P_j}{\partial A} = \int_{-\infty}^{U} [(1-c)R_L - (\mu_A + \sigma_A z)]\pi(z)dz \tag{5}$$

assuming that μ_A and σ_A are not functions of the asset level A. The above expression implies $P = Ap$. Note that the marginal put value depends on the risk capital ratio, c, given U, implying $p = p(c; U)$.

The term P/L can be interpreted as a measure of the credit risk of the business. For a given level of liabilities, the lower the value of the put option, the lower is the credit risk. We assume the bank sets a limit, α, on the default put to liability ratio

$$q = \frac{P}{L} \leq \alpha, \tag{6}$$

where q is a measure of the credit risk of the bank. The lower the level of α, then the lower is the credit risk of the bank and the greater is the credit worthiness. Most banks are prepared to accept a positive amount of default risk; no large American banks have a triple A credit rating.

2.1. Top Down Planning

The net present value of the bank's assets

$$NPV(A, q) = \int_A npv(y, q)dy,$$

where $npv(y, q)$ denotes the marginal net present value. It is assumed to be a function of the credit risk, q. There are additional costs imposed on the bank as it alters its asset mix—first are the costs associated with the risky debt used to finance investments due to default and financial distress, as measured by the value of the default put P. The value of the put option will depend on the amount of the assets and the risk capital. Second, there are costs from holding risk capital, τC. Holding risk capital imposes

an implicit opportunity cost, as the capital could be employed to generate additional income. Here, we assume that τ is positive. It is not uncommon for τ to be set equal to the required rate of return on equity. The bank is assumed to maximize the net present value of allocation of assets subject to the constraint of maintaining a given level of credit quality. The bank places a restraint on the level of credit risk it is prepared to accept. Expression (6) can be written in the form

$$P \leq \alpha A(1 - c).$$

2.2. Allocation to Individual Businesses

We assume the bank has N internal businesses. The assets in business j are denoted by A_j and the face value of debt liabilities attributed to the business by L_j. The total return at the end of the period is given by $R_{A,j}A_j$ and total debt payments $R_{L,j}L_j$.[4] For the bank,

$$R_A A \equiv \sum_{j=1}^{N} R_{A,j}A_j \quad \text{and} \quad R_L L \equiv \sum_{j=1}^{N} R_{L,j}L_j,$$

where

$$A = \sum_{j=1}^{N} A_j \quad \text{and} \quad L = \sum_{j=1}^{N} L_j. \tag{7}$$

The value of the default put for the bank can be expressed in the form

$$P = \int_{-\infty}^{U} \sum_{j=1}^{N} A_j[(1 - c_j)R_{L,j} - R_{A,j}]\pi(z)dz, \tag{8}$$

where $c_j = C_j/A_j, = (1 - L_j/A_j)$, $j = 1, \ldots, N$. The contribution by the j business to the default put is described by:

$$p_j = \frac{\partial P_j}{\partial A_j} = \int_{-\infty}^{U} [(1 - c_j)R_{L,j} - R_{A,j}]\pi(z)dz, \tag{9}$$

implying that the marginal contribution to the put option is a function of U and c_j: $p_j = p_j(U, c_j)$. Given that the range of integration U does not explicitly depend on the individual business, then we have the additivity result first derived by Erel et al. (2015) (EMR):

$$P = \sum_{j=1}^{N} A_j p_j. \tag{10}$$

EMR assumes that senior management directly determines the size of the individual businesses, subject to constraints on the aggregate credit risk

$$\sum_{j=1}^{N} A_j p_j \leq \alpha \sum_{j=1}^{N} A_j(1 - c_j)$$

and the amount of risk capital

$$\sum_{j=1}^{N} c_j A_j < \bar{C}. \tag{11}$$

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[4] It should be remembered that while specifying possibly different funding rates for each business, given the assumptions in the Erel et al. (2015) model - single period and pari passu if default risk, all the rates will be equal.

In the capital allocation program, different amounts of risk capital will be assigned to the different businesses, given the constraint that the total risk capital for the bank is $\bar{C}$. The capital allocation program facing management can be expressed in the form

$$
\begin{aligned}
V(A,c) \quad = \quad & \max_{\{A_k,c_k\}}\{\textstyle\sum_{k=1}^{N}\int_{A_k} npv_k(y,q)dy - \tau c_k A_k - w A_k p_k \\
& + \lambda[\alpha(1-c_k)A_k - p_k A_k]\} + \kappa[\bar{C} - \textstyle\sum_{k=1}^{N} c_k A_k],
\end{aligned}
\tag{12}
$$

where the decision variables are the assets allocated to each business $\{A_k\}$, and the risk capital $\{C_k\}$; λ and κ are Lagrangian coefficients.

3. Decentralized Management

At the senior management level, the central planning system helps to determine the amount of risk capital to allocate to the individual businesses within the bank. However, to encourage entrepreneurship at the business level, operating decisions are left to the business managers, subject to various constraints. Each business is treated as having its own balance sheet. Usually, business managers try to match borrowing requirements to the average duration of the business assets. The business borrows an amount L_j from the bank and the cost of borrowing is determined by the current yield on the bank's debt for a specified maturity. Let $R_{L,j}$ denote the borrowing rate for the j th business. By definition, the risk capital of the business is $C_j = A_j - L_j$. The cash flow at the end of the period is $A_j R_{A,j} - L_j R_{L,j}$.

The value of the default put option to the business is

$$
P_j = \int_{z \in Z_j} A_j[(1-c_j)R_{L,j} - R_{A,j}]\pi(z)dz,
$$

where $c_j = C_j/A_j$, the capital ratio for the business, Z_j is the set of states for which the revenues are insufficient to meet obligations—more formally defined as $Z_j = \{z; (1-c_j)R_{L,j} - R_{A,j}(z) > 0\}$ and $\pi(z)$ is the pricing kernel. Note that the definition for Z_j is business specific, unlike the definition for Z that referenced the conditions for the whole bank. This difference implies that we will no longer have the additivity result (10). The business wants its risk capital to depend only on the operations of the business. It does not want is risk capital being directly influenced by other businesses within the bank. The strike price of the put option depends on the duration of the business's liabilities, $R_{L,j}$, and the magnitude of its debt, L_j.

If $R_{A,j}(z)$ is assumed to have a mean $\mu_{A,j}$ and standard deviation $\sigma_{A,j}$, then we can write $R_{A,j}(z) = \mu_{A,j} + \sigma_{A,j}z$, where z has zero mean and unit variance. It is assumed that $\mu_{A,j}$ and $\sigma_{A,j}$ do not depend on A_j. Let $U_j \equiv \frac{(1-c_j)R_{L,j} - \mu_{A,j}}{\sigma_{A,j}}$, then

$$
P_j = A_j \int_{-\infty}^{U_j}[(1-c_j)R_{L,j} - R_{A,j}]\pi(z)dz.
\tag{13}
$$

The upper limit of integration depends on the characteristics of the business, unlike the upper limit of integration in expression (4) that depends on the cash flows of the whole bank.

The marginal contribution to the default put is

$$
p_j = \frac{\partial P_j}{\partial A_j} = \int_{-\infty}^{U_j}[(1-c_j)R_{L,j} - R_{A,j}]\pi(z)dz,
\tag{14}
$$

implying that the marginal contribution to the put option is a function of c_j and U_j: $p_j = p_j(c_j, U_j)$. The greater the capital ratio c_j, the lower is the value of the default put option.

Senior management is assumed to impose a common limit, α_B, for the credit risk applied to all businesses:

$$q_j = \frac{P_j}{L_j} \leq \alpha_B. \tag{15}$$

In general, this rate may differ from the rate used by the bank for central planning.

The business wants to pick an asset level A_j to maximize the net present value. There are also constraints—first, the constraint on credit risk, written in the form

$$p_j A_j \leq \alpha_B (1 - c_j) A_j, \tag{16}$$

second the risk capital for the business can not exceed the limit imposed by the senior management

$$c_j A_j \leq \bar{C}_j,$$

where $\bar{C}_j$ denotes the amount of risk capital assigned to the business.

The optimization facing the business is described by

$$
\begin{aligned}
V_j(A,c) \;=\; & \max_{A_j, c_j} \int npv_j(A_j, q_j) dA_j - \tau c_j A_j - w p_j A_j \\
& + \lambda_j [\alpha_B (1 - c_j) A_j - p_j A_j] + \kappa_j (\bar{C}_j - c_j A_j),
\end{aligned} \tag{17}
$$

where $npv_j(A_j, q_j)$ denotes the marginal net present value, $and \lambda_j$ and κ_j are the Lagrangians arising from the two constraints. One way to interpret this specification is the business picks an asset class(es) with credit risk, q_j. Different businesses will have different inherent credit risks and different returns. The greater the credit risk, the larger are the costs associated with risky debt. Ex ante, the business hopes to offset these costs with higher returns. The business must decide on the appropriate asset class and level of investment. It is instructive to compare the above objective function with the top-down objective represented by expression (12). The obvious difference is that the top-down approach considers the asset allocation for all businesses and the constraints refer to the bank as a whole, while (17) is at the business level. The Lagrangian coefficients λ_j and κ_j and the value of the default put P_j are business specific.

The first order condition for the asset level is given by

$$
\begin{aligned}
\frac{\partial V_j}{\partial A_j} \;=\; & npv_j(A_j, q_j) + \frac{\partial q_j}{\partial A_j} A_j \frac{\partial}{\partial q_j} npv_j(A_j, q_j) \\
& - (\tau + \kappa_j) c_j - w p_j + \lambda_j [\alpha_B (1 - c_j) - p_j] \\
=\; & 0,
\end{aligned}
$$

where, following Erel et al. (2015), we have used the assumption $\frac{\partial q_j}{\partial A_j} \frac{\partial}{\partial q_j} \int (npv_j(A_j, q_j) dA_j = \frac{\partial q_j}{\partial A_j} A_j \frac{\partial}{\partial q_j} npv_j(A_j, q_j)$. Now $\frac{\partial q_j}{\partial A_j} = \frac{\partial q_j}{\partial c_j} \frac{\partial c_j}{\partial A_j}$ and at the optimum, changes in the risk capital ratio do not affect the credit quality, $\frac{\partial q_j}{\partial c_j} = 0$. Therefore, we have

$$\frac{\partial V_j}{\partial A_j} = npv_j(A_j, q_j) - (\tau + \kappa_j) c_j - w p_j + \lambda_j [\alpha_B (1 - c_j) - p_j] = 0. \tag{18}$$

For the amount of risk capital, we can write, after simplification,

$$\frac{\partial V_j}{\partial c_j} = -[(\tau + \kappa_j) + w \frac{\partial p_j}{\partial c_j} + \lambda_j (\alpha_B + \frac{\partial p_j}{\partial c_j})] A_j = 0. \tag{19}$$

For the constraints, the Kuhn–Tucker conditions are

$$\frac{\partial V_j}{\partial \lambda_j} = A_j[\alpha_B(1 - c_j) - p_j]\lambda_j = 0.$$

If the constraint is binding, $\lambda_j > 0$, then the optimal risk capital ratio for business j is determined by solving the equation

$$\alpha_B(1 - c_j^*) = p_j(U_j, c_j^*) \tag{20}$$

so that $c_j^* = c_j(U_j, \alpha_B)$. For the risk capital constraint,

$$\frac{\partial V_j}{\partial \kappa_j} = (\bar{C}_j - c_j A_j)\kappa_j = 0. \tag{21}$$

We can solve the above four numbered equations to determine the optimum values for A_j^* and c_j^* and the Lagrangians λ_j and κ_j.

Aggregation

Each individual business determines its own level of asset investment $\{A_j\}$ and risk capital $\{C_j\}$. This will differ from the asset allocation determined by the bank in a top-down approach for at least three reasons. First, the bank uses the distribution generated by the aggregated cash flows. Second, in the top-down approach, the strike price of the default put is given by the aggregated level of debt payments. Third, the level of credit risk, q, is determined at the aggregate level.

The sum of the business put options is given

$$\sum_j P_j = \alpha_B \sum (A_j - C_j).$$

At the aggregate level, the total level of debt payments is $A(1 - c)R_L = \sum_j A_j(1 - c_j)R_{L,j}$ and the total cash flow generated by the individual businesses is given by $AR_A = \sum A_j R_{A,j}$, which defines the return R_A with mean μ_{BU} and standard deviation σ_{BU}. We can write $R_A = \mu_{BU} + \sigma_{BU}z$, where z has zero mean and unit variance. Given the individual business decisions, the value of the default put option for the bank is

$$P = \int_{-\infty}^{U_{BU}} [A(1 - c)R_L - AR_A]^+ \pi(z)dz, \tag{22}$$

where $U_{BU} = \frac{(1-c)R_L - \mu_A}{\sigma_A}$. We continue to assume that the only uncertainty is with respect to the aggregate rate of return from the different businesses. Note the upper limit of integration in the above expression is U_{BU}, while, in expression (13), the upper limit of integration is U_j and, consequently, we do not have the top-down aggregation result (10). From Merton and Perold (1993), we know that the sum of the individual business put options is greater than the put option on the bank, $\sum_j P_j \geq P$. The difference is often called the "portfolio effect".

One of the consequences of decentralization is businesses pick, subject to constraints, the asset level $\{A_j\}$, risk capital $\{C_j\}$ and credit risk $\{q_j\}$. The individual businesses can have a lower degree of credit worthiness than the bank. By aggregating across the businesses and assuming that returns are not perfectly correlated, the value of the default put for the bank is lower than the sum for the individual businesses. The actual value of α_B is set by senior management and involves a trade-off between return and the impact on the bank's credit worthiness. What is required is a way to measure the relative performance of the different businesses.

4. Business Performance

The need to employ risk capital affects the determination of the value of investment projects and business performance metrics. Each project a business undertakes affects the credit risk of the business and management of the business wants to determine the trade-off between how the project enhances the value of the business and the associated credit risk. We start the analysis by considering a business undertaking a project and will discuss some of the many practical issues that arise. We do not consider the implications of corporate taxation at the business level. It is quite difficult for businesses to determine the effective tax rate, as senior management at the bank level has flexibility in adjusting when certain cash flows are recorded. To reduce agency problems at the business level, taxation is usually ignored when comparing business performance.

The initial implicit price per share of equity is $\bar{S}_0$ and the number of shares h_0, so that the implicit value of equity in the business is $\bar{S}_0 \times h_0$, reflecting current projects. When a business undertakes a new project, the investment is assumed to be financed with debt. The cost of the debt financing reflects the credit risk of the bank and the maturity of the project. We will denote the interest rate for the project as R_d, where d denotes the expected duration of the project. For each project, the business is required to assign risk capital to the project, so that it does not adversely affect the credit worthiness of the bank. Let $\{C_0, C_1, \ldots, C_{n-1}\}$ denote the required risk capital for the project. Note that the risk capital in general changes over the life of the project. Some projects initially require little risk capital, such as foreign exchange swaps. The business uses a bank approved risk metric to estimate the required risk capital. The implicit assumption is that risk capital is financed by equity. The actual determination of the required risk capital is far from easy. There is an array of different approaches: value at risk, expected shortfall or estimating the value of implicit default put options, each with advantages and disadvantages when used for actual estimation arising from issues of data availability and underlying model assumptions.

In practice, the bank allocates a certain amount of risk capital to a business and charges the business for the use of the capital. The cost is often specified as the required rate of return on equity, denoted by the symbol k. The business to partially off set the cost of risk capital assumes it can invest in default free assets of the life of the project. We denote the rate of interest as r and, for the sake of simplicity, assume it is constant. At time i, the business has risk capital C_{i-1} and the net cost of risk capital is $C_{i-1} \times (k - r)$.

At the end of the i period, the risk capital changes from C_{i-1} to C_i. The risk capital is financed via equity. We employ the following convention by calling the change $(C_i - C_{i-1})$ a cash inflow, recognizing that the change could be negative implying that shares are implicitly repurchased.

When a business undertakes a project, it is possible for the project to be prematurely terminated: the business may decide to cut losses or to take profits. To concentrate the focus on establishing a framework for judging the performance of a business within a bank, we ignore the possibility of premature termination.

At the maturity of the project, the cash flow before tax generated by the project is

$$Z_n \equiv F_n - I_0(1 + R_d) + C_{n-1} \times (-\bar{k}),$$

where the term F_n represents the cash flow when the project is matures, I_0 is the initial outlay to finance the project and $\bar{k} = k - r$. Again, for simplicity, we assume there is only one initial outlay. This assumption can be dropped, though it adds complexity without adding additional insight. The second term, $I_0(1 + R_d)$, represents the repayment of principal and interest from the business to the bank. As the project terminates, $C_n \equiv 0$ and the remaining capital C_{n-1} is used to repurchase equity. Note that the terminal cash flow Z_n may be negative. The business records the loss, with the bank acting as lender of last resort.

At time $n - 1$, the market value of the business's equity project is

$$(h_{n-1} + m_{n-1})S_{n-1} = PV_{n-1}(B_n) + PV_{n-1}(Z_n) - PV_{n-1}(C_n - C_{n-1}|n),$$

where B_n is the cash flow generated by existing projects within the business. The last term on the right side represents the present value at time $t = n - 1$ of the issuance of equity to finance the change in risk capital at $t = n$. Note that $C_n = 0$, as the project has matured. To simplify the analysis, we assume that the risk capital for the existing projects is constant. Financing the change in required risk capital implies issuing or repurchasing equity

$$m_{n-1}S_{n-1} = C_{n-1} - C_{n-2}$$

so that the total value of the equity of the business is

$$
\begin{aligned}
h_{n-1}S_{n-1} = {} & PV_{n-1}(B_n) + PV_{n-1}(F_n) - PV_{n-1}[I_0(1 + R_d)] \\
& + PV_{n-1}(C_{n-1} \times (1 - \bar{k})|n] - C_{n-1} + C_{n-2}.
\end{aligned}
$$

At time $n - 1$, the cash flow for the project is

$$Z_{n-1} \equiv F_{n-1} - I_0 \times R_d + C_{n-2} \times (-\bar{k}).$$

At time $n - 2$, the investment in risk capital is given by

$$m_{n-2}S_{n-2} = C_{n-2} - C_{n-3}.$$

Hence,

$$
\begin{aligned}
h_{n-2}S_{n-2} = {} & PV_{n-2}(B_n) + PV_{n-2}(B_{n-1}) \\
& + PV_{n-2}[F_n - I_0(1 + R_d)|n] + PV_{n-2}[F_{n-1} - I_0 \times R_d|n - 1] \\
& + PV_{n-2}[C_{n-1}(1 - \bar{k})|n] - PV_{n-2}[C_{n-1}|n - 1] \\
& + PV_{n-2}[C_{n-2} \times (1 - \bar{k})|n - 1] - PV_{n-2}[C_{n-2} - C_{n-3}|n - 2].
\end{aligned}
$$

Repeating this analysis and simplifying gives

$$h_0 S_0 = \sum_{j=1}^{n} PV_0(B_j) + \sum_{j=1}^{n} PV_0(F_j - I_j) + \sum_{j=1}^{n} PV_0(C_{j-1} \times (1 - \bar{k})|j) - \sum_{j=0}^{n-1} PV_0(C_j|j),$$

where $C_{-1} \equiv 0$, as $m_0 S_0 = C_0$, $C_n \equiv 0$, and I_j is the debt payment at time j. Before the new project is undertaken, we have

$$h_0 \bar{S}_0 = \sum_{j=1}^{n} PV_0(B_j)$$

so that

$$h_0(S_0 - \bar{S}_0) = \sum_{j=1}^{n} PV_0(F_j) - \sum_{j=1}^{n} PV_0(I_j) + \sum_{j=1}^{n} PV_0(C_{j-1} \times (1 - \bar{k})|j) - \sum_{j=0}^{n-1} PV_0(C_j|j). \tag{23}$$

The above expression represents the adjusted present value of the project. The first two terms on the right side represent the net present value of the project, ignoring the costs arising from risk capital. The first term on the right side is the present value of the project's cash flows. The discount rate used in determining the present value represents the risk of the project's cash flows and in general this differs

from the bank's required rate of return on equity or weighted average cost of capital.[5] The second term is the present value of the debt funding payments. The last set of terms arise because the project affects the credit worthiness of the bank and represents the present value of the costs from risk capital.

4.1. Return on Capital for a Project

The return on the project over the first period is given by

$$
\begin{aligned}
\{ & F_1 - I_0 \times R_d + C_0 \times (1 - \bar{k}) - C_1 \\
& + \textstyle\sum_{j=2}^{n} PV_1(F_j) - \sum_{j=2}^{n} PV_1(I_j) \\
& + \textstyle\sum_{j=2}^{n} PV_1(C_{j-1} \times (1-\bar{k})|j+1] - \sum_{j=1}^{n-1} PV_1(C_j|j)]\} / m_0 S_0,
\end{aligned}
\tag{24}
$$

where $m_0 S_0 = C_0$, the initial risk capital. This expression with the denominator being the initial risk capital looks similar to the definition of the risk adjusted rate of return on capital (RAROC)—see Chapter 9 in Matten (2000) and Turnbull (2009). The attraction of a measure like RAROC is that it is simple and it is claimed that it can be used to compare the performance of different businesses, on a risk adjusted basis. However, it is well known that this is incorrect as it cannot accommodate projects with different systematic risk. Different projects will have different expected rates of return. There is no single rate of return, such as the required rate of return on equity, to provide a benchmark. Management must use its own judgement when ranking projects.

For many types of derivatives, such as forward contracts and foreign exchange swaps, the initial of risk capital is small. Consequently, a RAROC type of measure, which is myopic in nature, tends to be relatively large. The above expression (24) takes into account the changing risk capital profile and avoids the limitations of the traditional measure.

4.2. Return on Capital for a Business

Expression (24) can be re-interpreted to provide the expected rate of return on a business by first aggregating cash flows in the numerator and second aggregating across the change in the required risk capital for all projects in the business. Management can determine the adjusted present value for projects within a business and rank projects on the basis of the adjusted present value per unit of investment, APV_j / A_j. However, the issue of an appropriate benchmark remains. The required rate of return on equity is a common benchmark. But this is generally inappropriate, as businesses are influenced by different risk factors, implying different systematic risk.[6] The errors introduced by employing a fixed benchmark are well known—see (Brealey et al. 2014, chp. 19). Senior management must judge the relative performance of the different businesses, while recognizing the deficiencies of the performance metrics.

5. Conclusions

When determining bonuses for the managers of the individual businesses within the bank, senior management must attempt to judge the relative performance of the different businesses. Consequently, business managers want performance measures that depend on factors under their control. For example, funding costs should reflect the average duration of the assets of the business and risk capital assigned to projects within the business to depend on the assets within the business ignoring the rest of the bank. This paper addresses some of the issues that arise from decentralized management.

We extend the analysis of EMR by allowing individual businesses to make their own asset allocations subject to constraints on the total amount of risk capital and credit risk of the business. Relaxing the assumptions of EMR implies that we no longer have the aggregation result that the risk

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[5]　See (Brealey et al. 2014, chp. 19).
[6]　The systematic risk of the different businesses could be calculated and the capital market line used as a benchmark.

capital allocations based on marginal default values add up exactly. The break-down of this result is to be expected, given the work of Merton and Perold (1993).

Each project has its own risk capital requirements. Risk capital is not free and consequently it is important to determine how the time profile of required risk capital affects the adjusted present value of a project. We derive an expression for the adjusted present value of a project that considers the time profile of risk capital. This necessitates determining the present value of the risky cash flows, as well as the funding costs and the present value of the risk capital costs. From there, we derive an expression for expected rate of return for a project. However, there is no simple benchmark, such as risk adjusted rate of return on capital (RAROC) with which to compare relative performance.

We derive an expression for the adjusted present value of a business by aggregating across the individual projects within a business. Senior managers of a bank need to rank the relative performance of the bank's businesses. There is no simple way to do this, given the absence of a benchmark. Management can rank the businesses using the ratio of the adjusted present value to asset size $\{APV_j / A_j\}$, as suggested by Erel et al. (2015). While this ratio is a percentage measure and useful in comparing large versus small businesses, it does not address the issue that projects have different risk profiles, even with risk capital. It suffers from similar limitations as RAROC.

Business managers can take steps to alter the risk profile of a project by undertaking hedging and entering into collateral agreements. While these activities alter the risk profile, quantifying the impact for over-the-counter contracts can be challenging, especially in stressed financial conditions.

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