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# Intelligent Decision Support in Proportional–Stop-Loss Reinsurance Using Multiple Attribute Decision-Making (MADM)

Shirley Jie Xuan Wang \* and Kim Leng Poh 

Department of Industrial Systems Engineering and Management, Faculty of Engineering,  
National University of Singapore, Singapore 117576, Singapore; pohkimleng@nus.edu.sg

\* Correspondence: jiexuan.wang@u.nus.edu

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**Abstract:** This article addresses the possibility of incorporating intelligent decision support systems into reinsurance decision-making. This involves the insurance company and the reinsurance company, and is negotiated through reinsurance intermediaries. The article proposes a decision flow to model the reinsurance design and selection process. This article focuses on adopting more than one optimality criteria under a more generic combinational design of commonly used reinsurance products, i.e., proportional reinsurance and stop-loss reinsurance. In terms of methodology, the significant contribution of the study is the incorporation of the well-established decision analysis tool multiple-attribute decision-making (MADM) into the modelling of reinsurance selection. To illustrate the feasibility of incorporating intelligent decision supporting systems in the reinsurance market, the study includes a numerical case study using the simulation software @Risk in modeling insurance claims, as well as programming in MATLAB to realize MADM. A list of managerial implications could be drawn from the case study results. Most importantly, when choosing the most appropriate type of reinsurance, insurance companies should base their decisions on multiple measurements instead of single-criteria decision-making models so that their decisions may be more robust.

**Keywords:** multi-attribute decision-making; reinsurance; proportional reinsurance; non-proportional reinsurance; TOPSIS

## 1. Introduction

### 1.1. Background

Reinsurance is generally known as “insurance for insurance”. Following similar concepts and principles as insurance, it provides financial compensation to insurance companies with respect to the risk of large losses. The reinsured party (or “insurance company”) buys reinsurance from the reinsurer (or “reinsurance company”) in exchange for loss limitation, revenue protection, and freeing up of capital. In recent years, reinsurance has grown both in market value and diversity due to worldwide trends such as global climate change, increases in insurance mega losses, volatility in equity markets, and emerging risks such as terrorism. Regardless of financial size, an insurance company rarely retains all of their risk. Thus, it is of interest to understand the decision-making process with respect to the reinsurance contract, which in reality is usually done using negotiation intermediaries, i.e., reinsurance brokers. Typically, there are two categories of reinsurance decisions, both of which will be addressed in this study:

- The optimal reinsurance form, under given criteria;
- Given the reinsurance form, the choice of reinsurance parameters. (e.g., optimal retention portion for proportional reinsurance, optimal retention limit for stop-loss reinsurance, etc.)

This study focuses on treaty reinsurance, which covers an entire portfolio with multiple single risks. Both facultative and treaty reinsurance could be further broken down into proportional reinsurance and non-proportional reinsurance (Carter 1979). Early works have shown that under variance risk measurement with fixed premium, the stop-loss contract is the optimal reinsurance form for reinsurance buyers (Borck 1960; Hürlimann 2011), whereas the quota-share best addresses the interests of the reinsurer (Vajda 1962). Clearly, there would be conflicts of choice between two parties. Thus, this study addresses a combinational form of proportional and stop-loss treaty reinsurance (see Section 3.2), following the definition by Samson and Thomas (1985). Quota-share reinsurance and stop-loss reinsurance could be considered as special cases of proportional–stop-loss reinsurance. In deciding the optimal reinsurance design parameters, this study attempts to utilize multiple-attribute decision-making (MADM) improving on previous works in the literature which use a single criterion.

## 1.2. Paper Development

The paper is organized as follows. Section 2 reviews the recent research that this study is built upon. Section 3 develops the decision flow based on the form of proportional–stop-loss reinsurance and determines the optimal reinsurance parameters using multi-objective decision-making (MODM). Section 4 includes a numerical case study, which models claims using @Risk and implements MADM for buyer's selection using MATLAB. Section 5 discusses contributions, limitations, and further directions, and concludes the study.

## 2. Literature Review

Decision analysis models using limited criterion have been extensively discussed both for the reinsurer in structuring reinsurance and for the reinsured party for evaluating and selecting the most appropriate reinsurance product (Samson and Thomas 1985). Only recently have researchers begun to look into the cooperative behavior of both parties to reach a joint-party optimality. In addition, the recent growth of promising decision analyses based on multiple criteria has ignited sparks in the field of reinsurance research. In particular, this study is developed upon three recent works (Bazaz and Najafabadi 2015; Karageyik and Şahin 2017; Payandeh-Najafabadi and Panahi-Bazaz 2017) which focus on multi-attribute decision-making (MADM) and proportional–stop-loss reinsurance.

Karageyik and Dickson (2016) first proposed using MADM with respect to the problem of selecting optimal reinsurance levels under competing criteria. On choosing the input alternatives, they use ruin probability as a constraint, i.e., the insurance company should not have a probability of ruin greater than 1%. Loss distribution was modeled as the translated gamma process and the forms considered were pure proportional and pure stop-loss reinsurance. The study also includes comparison with single-criterion decision-making and concludes that MADM is extremely insightful for selecting optimal reinsurance.

Later, Karageyik and Şahin (2017) improved on this research by taking value-at-risk (VaR) measurement into consideration, specifically targeting at optimal retention level in excess-of-loss reinsurance design. Key measurement criteria are expected profit, expected shortfall, finite time ruin probability, and variance of risk. By comparing and contrasting different MADM techniques, the authors safely concluded that under the case of reinsurance where correlation between measurements are low enough, different MADM techniques will generate similar optimal retention level.

However, both studies focused either on pure proportional reinsurance or pure non-proportional reinsurance, with neither considering the combination of both. The most contemporary discussion of general reinsurance model took a viewpoint from both the insurer and reinsurer, and based on TVaR measurement, suggesting that a pareto-optimal solution always exists (Cai et al. 2017). The pareto-optimality studies have been successful in addressing multiple decision objectives, especially in trading cases where both seller and buyer parties could simultaneously make decisions on contracts (Trade procedure P1 in Section 3.1). Our approach of using MADM is different from a nest

of Multiple Objective Decision Models (MODM) in that MADM is assigning rankings to a cluster of available alternatives. This would be especially useful in settings (Trade procedure P2 in Section 3.2) where buyers may have limited, discrete alternatives to choose from. In such settings, deriving an optimal contract would be less meaningful as such contract may not be in the feasible set faced by the buyer party (the reinsured, in our story).

To the best of my knowledge, there is no previous research conducting ranking of proportional-stop-loss reinsurance alternatives based on multiple measurement schemes while considering decisions from both parties. Thus, this study serves the purpose of filling this gap. Section 3 incorporates two-sided deal procedures into the reinsurance optimality study and describes the procedure of applying MADM to reinsurance selection. Following this, Section 4 includes a case study to model the reinsured party's choice of reinsurance in real time.

### 3. Methodology

#### 3.1. Decision Flow

As reinsurance decision-making involves the selling party (the reinsurance company, or the reinsurer) and the buying party (the insurance company, or the reinsured party), it could be safely viewed as a two-sided trade process, which involves negotiation between the selling and buying parties. Furthermore, reinsurance deals could be viewed as the established two-sided trade matching models P1 (Figure 1) or P2 (Figure 2) with the existence of a broker (Liang 2014).

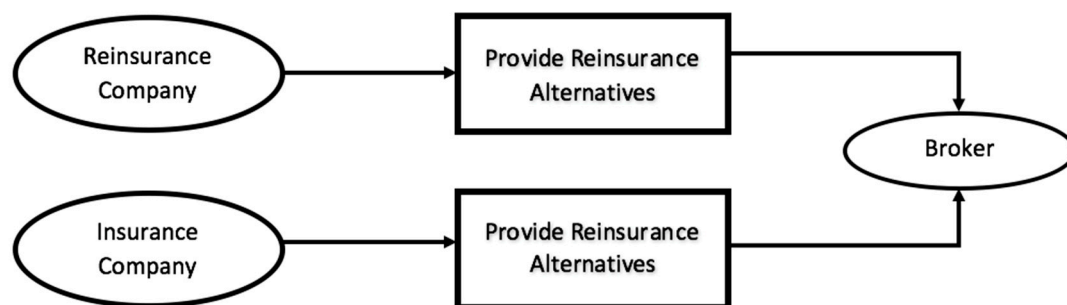


Figure 1. Trade procedure P1 (Liang 2014).

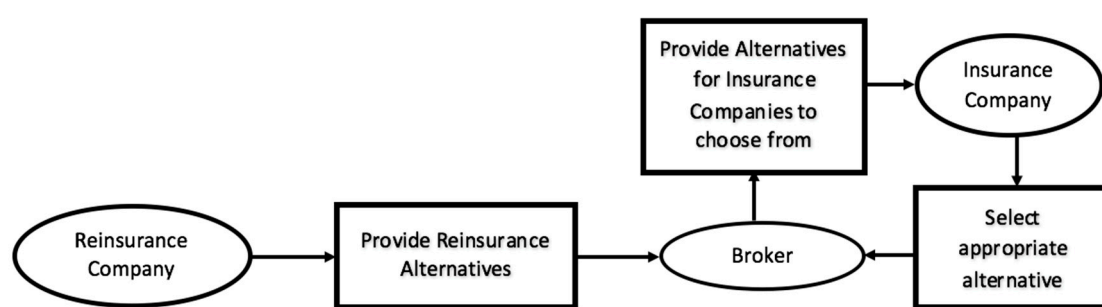


Figure 2. Trade procedure P2 (Liang 2014).

For the first trade procedure P1, both the reinsurer and the reinsured party exchange information through the broker. Previous research on two-sided optimality could largely be viewed as P1 when both firms make decisions simultaneously. In this case, an equilibrium strategy either optimizing one side or both side's benefits could be reached (see Borch 1960; Wang 2003; Cai et al. 2017), while few works in literature have discussed reinsurance deals settled under procedure P2. Under P2, the seller (the reinsurer) will provide several plans for the buyer to choose from. Noting the prevalence of procedure P2 in reinsurance industry practice, this study attempts to model reinsurance scenarios

under P2 by using classic MODM in providing reinsurance alternatives and by using MADM in selecting an appropriate reinsurance design for the reinsured party. The rest of the section follows this decision flow. Section 3.2 defines the proportional-stop-loss reinsurance form; Section 3.4 constructs one simplified set of reinsurer's offerings using MODM. Section 3.5 defines each of the key criteria the reinsured may consider. Finally, Section 3.6 presents the application of MADM in aiding the decision of ranking the reinsurance alternatives.

### 3.2. The Proportional–Stop-Loss Reinsurance Model

The paper discusses proportional–stop-loss reinsurance, adopting definitions from Samson and Thomas (1985), Hürlimann (2011) and Payandeh-Najafabadi and Panahi-Bazaz (2017), under which, given a single loss of  $X$  and a reinsurance arrangement with parameters  $(a, M)$ , the reinsurer is bonded to pay a claim amount of:

$$X_r = a(X - M)_+ \Leftrightarrow X_r = \begin{cases} 0 & \text{if } X \leq M \\ a(X - M) & \text{if } X > M \end{cases} \quad (1)$$

where  $a$  is the fraction ceded to the reinsurer and  $M$  is the retention limit. The reinsured party will pay the rest of the claim  $X_i = X - X_r$ , where  $X_r$  represents the amount of claim paid by the reinsurer. When  $M = 0$ , the proportional–stop-loss model becomes the classical quota-share reinsurance model, and when  $a = 1$ , it becomes the classical stop-loss reinsurance model.

### 3.3. Variable Definition

To ensure the consistency of notations in this paper, we define the key variables as shown in Table 1. Almost all definitions follow previous literature, and necessary elaborations will be given in later sections.

**Table 1.** Key variable definitions.

Variable	Variable Explanation
$t$	the time period of one contract, in our case study $t = 1$ ;
$N$	the number of claims incurred in period $t$ (during one contract);
$W(t)$	the wealth held by insurance company at time $t$ ;
$\zeta$	the loading factor of the reinsurance premium paid to the reinsurer;
$\theta$	the loading factor of the premium paid to the reinsured party;
$X$	the claim amount of one single loss;
$X_i$	the claim amount payable by the insurance company (the reinsured party);
$X_r$	the claim amount payable by the reinsurance company (the reinsurer);
$S(t)$	the aggregate loss of an insurance portfolio;
$S_i(t)$	the aggregate claim (loss) incurred to insurance company (the reinsured party);
$S_r(t)$	the aggregate claim (loss) incurred to the reinsurance company (reinsurer);
$F_S(X)$	the cumulative distribution function of $S$ ;
$\overline{F_S} = 1 - F_S(X)$	the survival distribution function of $S$ ;
$(a, M)$	the proportional–stop-loss reinsurance parameter, $X_r = a(X - M)_+$ ;
$c$	the total premium per unit time;
$c_i$	the premium gained by the insurance company;
$c_r$	the premium payable to the reinsurer;
$ES_\alpha$	the expected shortfall with a confidence level of $\alpha$ ;
$PROFIT_i$	the expected profit gained by insurance company;
$\psi(i)$	the ruin probability of insurance company's wealth $U(t)$ ;
$U_i(t)$	the utility of insurance company at the end of period $t$ ;

### 3.4. Simulating Alternatives Using MODM

Considering the reinsurance practices and following previous research on the joint-party reinsurance problem, we first attempt to model the reinsurer pricing objectives. As such, we provide



a list of alternatives for the reinsured party to choose from. We attempt to formulate a model maximizing the reinsurer's expected profit while minimizing the variance of profit. In deciding a reinsurance design, the reinsurer needs to specify the premium and the arrangement of the reinsurance claim amount, in other words, the reinsurance premium loading factor  $\zeta$  and the reinsurance design parameter  $(a, M)$ . Under the expected value premium principle, the insurance premium must be at least greater than the expected individual loss (Karageyik and Şahin 2017). Thus, the bi-objective model is formulated as:

$$\begin{aligned} & \max_{a \in [0,1], M \geq 0, \zeta \in [1,0]} E[c_r \cdot t - S_r] \\ & \min_{a \in [0,1], M \geq 0, \zeta \in [1,0]} \text{Var}[c_r \cdot t - S_r] \\ & \text{subject to.} \quad M \geq \ln(\zeta/\theta) \\ & \quad \quad \quad \zeta \geq \theta \end{aligned} \quad (2)$$

where  $S_r$  is defined as the aggregated claim of loss (compounded from individual loss  $X_r$ ), and  $c_r$  is the premium paid to the reinsurance company per unit of time, defined according to the expected value premium principle (formulas are in Section 3.5.1).

Clearly, there is conflict between two objectives and there is no single design of  $(\zeta, a, M)$  that can achieve all objectives. The closed-form derivations (Hürlimann 2011) are omitted and the optimal solution set would be an efficient frontier analyzed in closed form. The optimal pairs will satisfy:

$$\zeta = e^{M/\lambda} \cdot \theta \text{ subject to : } \zeta \geq \theta \quad (3)$$

Note that for an increasing ceding level  $a$ , the reinsurer risk and expected profit will both increase proportionally; thus, the reinsurer preference will be ambiguous for different ceding portion  $a$  while fixing the pair of  $(\zeta, M)$ . This is in line with the work of Payandeh-Najafabadi and Panahi-Bazaz (2017) where it is suggested that optimal design  $(a, M, \zeta)$  depends on the loss distribution (in our case,  $\lambda$ ) but not on the market premium ( $\theta$ ), and does not depend on the portion retained ( $a$ ). Thus, it would be flexible for the reinsurance company to select an appropriate ceding portion  $a$  given their risk appetite and their financial capability (which is often not necessarily known by the broker). In Section 5, we will briefly discuss the resulting effects of choosing different ceding portions of  $a$ , based on a numerical case study.

Thus, the alternatives provided by the reinsurance firm will be in the form of  $(a, M, \zeta)$ . These are inputting alternatives we will use to apply MADM.

### 3.5. Calculating Decision Criteria

Now, we need to define the selection criteria for reinsurance design. In this study, we are concerned with expected profit, expected shortfall, ruin probability, and expected utility as selection criteria. All of these factors are calculated taking the viewpoints of reinsurance buyers (the reinsured parties).

#### 3.5.1. Expected Profit of the Insurance Company (Reinsured Party): $PROFIT_i$

In general, the expected profit of the reinsured party is calculated as the difference between the insurer's income and the claims paid to the policyholders. Net premium gained by the insurance company is calculated under the expected value premium principle, defined as:

$$c^* = \text{Total Premium Income} - \text{Reinsurance Premium} = (1 + \theta)E[S] - (1 + \zeta)E[S_r] \quad (4)$$

The net insurance profit after considering the reinsurance arrangement is:

$$Profit_i = c^* - E[S_i] \quad (5)$$

Our objective is to maximize the expected profit of the insurance company.

### 3.5.2. Expected Shortfall $ES_\alpha$

Expected shortfall is calculated under value at risk (VaR) measurement. VaR, given a confidence level of  $\alpha \in (0, 1)$ , is defined as the smallest  $l$ , such that the probability of loss  $L < l^*$  is at least  $\alpha$  (Bazaz and Najafabadi 2015), i.e.,

$$VaR_\alpha(L) = \min(l^* \in \mathbb{R} : Pr(L \leq l^*) \geq \alpha) \quad (6)$$

Expected shortfall is the financial risk measurement to investigate market risk of the portfolio. It is calculated as the expected value of tail distribution of  $VaR_\alpha$  as follows:

$$ES_\alpha(L) = \frac{1}{1 - \alpha} \int_\alpha^1 VaR_u(L) du \quad (7)$$

An increase in retention level  $M$  will cause the insurer's liability to insurance policyholders to increase, and thus  $ES$  will increase accordingly. In contrast, a larger ceding portion will release the insurer from burden and thus will decrease the amount of liability held by the reinsured party. Our objective is to find the optimal  $(a, M)$  pair that could minimize the expected shortfall of the insurance company.

### 3.5.3. Ruin Probability

The ruin probability criterion is based on definitions of finite time ruin probability measurement. The insurer's asset is represented as  $W(t)$  and is defined by:

$$W_i(t) = w_i(0) + c^* \cdot t - S_i(t) \quad (8)$$

In Equation (8)  $c^*$  is the net premium income per unit time gained by the insurance company, and  $S(t)$  is the aggregate claim amount up to time  $t$ , which is calculated by:

$$S_i(t) = \sum_{i=1}^{N(t)} X_i \quad (9)$$

The finite time ruin probability,  $\psi(w_0, t)$ , is given as:

$$\psi(w_0, t) = Pr(W(s) < 0) \text{ for some } s, 0 < s \leq t \quad (10)$$

In our study, the ruin probability is approximated through a simulation study, as the closed form ruin probability for compounding exponential loss distribution under proportional-stop-loss reinsurance design is hard to obtain. Our objective is to minimize the ruin probability of  $\psi(w, t)$  such that the insurance company would be less likely to go bankrupt if there is a large loss incurred.

### 3.5.4. Expected Utility

To address the utility theory used in vast literature on reinsurance optimization (Samson and Thomas 1983), the utility function of the reinsured is defined as exponential utility function, which assumes constant absolute risk aversion:

$$U_i^t(W_i(t)) = -e^{-kW_i(t)} \quad (11)$$

In reality, utility function may have much more complexity and may be different for different insurance companies. However, as long as the value of utility could be obtained in numeric value, decisions could be made through MADM. In deciding the ranking of reinsurance alternatives, one of our objectives is to maximize the expected utility of the insurance company.

### 3.6. Selecting the Best Alternative Using MADM

In Section 2, we reviewed decision analysis techniques on reinsurance decisions under single measurement. In order to model the decision of the reinsurance purchasing party (the insurance company or the reinsured) under multiple measurement criteria, this study adopts multi-criteria decision-making techniques. In particular, the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), reviewed in prior works (Bazaz and Najafabadi 2015) is the most popular MADM technique and is the most suitable for pure numerical criteria. TOPSIS was originally developed by Hwang and Yoon (1981), and was later developed by Yoon (1987) and Hwang et al. (1993).

TOPSIS is thus applied to the reinsurance selection problem. Furthermore, as suggested by Karageyik and Şahin (2017), the correlation amongst criteria in reinsurance problem is small enough to return similar results from different TOPSIS methodologies. Thus, in this study we choose the classical TOPSIS method to support our analysis.

Following similar definitions of TOPSIS in a previous study (Bazaz and Najafabadi 2015; Ameri Sianaki 2015; Karageyik and Şahin 2017), we briefly describe the steps of applying the method as follows. This study attempts to implement the TOPSIS decision supporting system by storing reinsurance alternatives in Excel and processing the input matrices with a MATLAB code. Part of the MATLAB code was developed with reference to previous efforts by Amari (Ameri Sianaki 2015), and was revised accordingly to serve the needs of this study. Below is the complete procedure of conducting TOPSIS.

1. Formulate decision matrix  $D$  with  $m$  alternatives  $A_1, A_2, \dots, A_m$  and  $n$  decision criteria  $C_1, C_2, \dots, C_n$ . The attribute value of  $A_i$  on  $C_j$  for  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$  is represented as  $d_{ij}$ .
2. Calculate weight of the criteria using entropy technique as follows:

$$\begin{aligned} q_{ig} &= \frac{d_{ig}}{d_{1g} + d_{2g} + \dots + d_{mg}}; \forall g \in \{1, 2, \dots, c\} \\ \Delta_g &= -k \sum q_{ig} \cdot \log_2(q_{ig}); \forall g \in \{1, 2, \dots, c\} \\ d_g &= 1 - \Delta_g, w_g = \frac{d_g}{(d_1 + \dots + d_g)} \\ w_g' &= \frac{\lambda_g \cdot w_g}{\lambda_1 \cdot w_1 + \lambda_2 \cdot w_2 + \dots + \lambda_c \cdot w_c} \end{aligned} \quad (12)$$

3. Normalize the decision matrix using the following formula:

$$r_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^m d_{ij}^2}} \quad (13)$$

One may notice that by scaling the criteria (multiplying a constant to  $d_{ij}$ ), the decision will not change. However, it will not necessarily return same decision for different utility functions that generate the same decision under expected utility measurement, as adding a constant to  $d_{ij}$  in the  $r_{ij}$  formula will change the resulting  $r_{ij}$ .

4. Calculate the weighted normalized decision matrix by using the normalized decision matrix parameter  $r_{ij}$  and weight vector  $\omega = (\omega_1, \omega_2, \dots, \omega_n)$  to return the weighted normalized decision matrix parameter  $V_{ij} = \omega_j \cdot r_{ij}$ . If criteria are given same weight,  $\omega_1 = \omega_2 = \dots = \omega_n = \frac{1}{n}$ .
5. Compute the vectors of positive ideal solutions and the negative ideal solutions, denoted by:

$$\begin{aligned} S^+ &= (S_1^+, S_2^+, S_3^+, \dots, S_n^+) \\ S^- &= (S_1^-, S_2^-, S_3^-, \dots, S_n^-) \end{aligned} \quad (14)$$

6. Calculate the distance between each alternative and the positive and negative ideal points. The distance between alternative  $A_i$  and the positive ideal points is:

$$D_i^+ = \sqrt{\sum_{j=1}^n (V_{ij} - S_j^+)^2}, \text{ for } i = 1, 2, \dots, m; \quad (15)$$

The distance between alternative and the negative ideal solutions are:

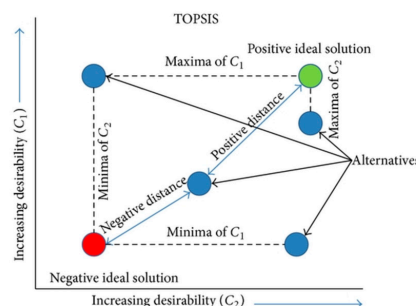
$$D_i^- = \sqrt{\sum_{j=1}^n (V_{ij} - S_j^-)^2}, \text{ for } i = 1, 2, \dots, m; \quad (16)$$

7. Calculate the relative closeness coefficient of each alternative represented as:

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-}, C_i \in [0, 1] \quad (17)$$

8. Rank the alternatives according to  $C_i$ . The alternative with higher  $C_i$  value is preferred over lower  $C_i$  alternatives.

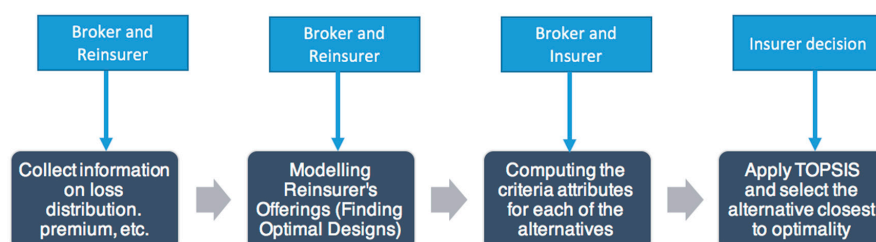
A graphical representation of TOPSIS is shown in Figure 3. Each blue ball represents one available alternative. The red ball represents the negative ideal solution and the green ball represents the positive ideal solution. The blue ball that is relatively near to the green ball and away from the red ball would be the best alternative amongst all. Given at least four selection criteria, it would be hard to visualize the alternatives in 3-dimensional space, thus the calculation of distance is coded using MATLAB.



**Figure 3.** Graphical representation of technique for order of preference by similarity to ideal solution (TOPSIS) (Chauhan and Vaish 2013).

#### 4. Case Study

Following the decision flow in Section 3, this study models the reinsurance deal procedure as illustrated in Figure 4. We will choose a loss distribution model in Section 4.1, generate reinsurance offering alternatives in Section 4.2, tabulate the decision matrix in Section 4.3, and finally apply MADM in selecting from alternatives in Section 4.4.



**Figure 4.** Case study flowchart in modeling real-world reinsurance deals.

#### 4.1. Loss Distribution Modeling

Under treaty reinsurance, which covers the entire line of insurance business handled by the insurance company, the aggregated claim  $S$  is a compounding distribution of  $N$  single risks or claims incurred during time period  $t$ .

Following the majority of research on modeling losses or claims (e.g., Samson and Thomas 1985; Bazaz and Najafabadi 2015; Payandeh-Najafabadi and Panahi-Bazaz 2017; Karageyik and Şahin 2017); our case study chooses to model individual claims as an exponential loss model with the parameter  $\mu = 100$ , i.e.,

$$Pr(X = x^*) = \frac{1}{\mu} e^{-\frac{1}{\mu} x^*} \quad (18)$$

The occurrence of a claim follows Poisson distribution with mean  $\lambda = 10$ , i.e.,

$$Pr(N(t) = n) = \frac{(\lambda^n) e^{-\lambda}}{n!} \quad (19)$$

Thus,  $S$  is the compounding distribution of  $N(t)$  identical, independently distributed risks, each with distribution  $X$ .

In this case study, we set  $t = 1$ ,  $u(0) = 1500$  and original insurance premium parameter  $\theta = 0.1$ .

#### 4.2. Generating Alternatives from the Viewpoints of Reinsurers

Following Section 3.2, we define the case by setting the portion ceded as  $a = 0.6$ ,  $a = 0.75$ ,  $a = 0.9$  and  $a = 1$ , respectively, in assessing the differences in results when the retention level is changed. The optimal pair  $(\zeta, M)$  (Figure 5 with  $\zeta$  on x-axis and  $M$  on y-axis) is sought by a grid search, as solving Formula 3.2 in mathematical form may not be succinct.

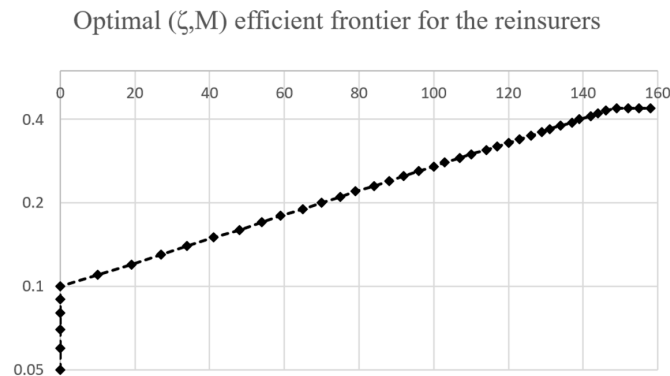


Figure 5. Optimal pairs of reinsurance design given constant ceding portion  $a$ .

When  $\zeta < 0.1$ , constraint  $\zeta < \theta$  will be violated; all points to the right and below of the efficient frontier are deemed as inferior to the points on the efficient frontier. In managerial terms, the reinsurance designs with parameters to the southeast of the efficient frontier will cause the reinsurance company to likely generate less profit while suffering from a larger risk.

#### 4.3. Constructing Decision Matrix

For illustration, Figure 6 shows the optimal pairs of  $(\zeta, M)$  given other parameters in the case study. For the first trial, we fixed the ceding portion at  $a = 0.6$  and select 35 pairs of the optimal  $(\zeta, M)$  as the reinsurance design parameters for TOPSIS alternatives. Profit for the insurance company and expected utility after claims are calculated using theoretical mean of the random distributions. However, given that the claim process compounds exponential loss with Poisson occurrence, ruin probability and expected shortfall at 95% confidence level are hard to obtain in analytic terms. Thus, by using Monte

Carlo simulation with 100,000 iterations, losses and claims are modeled as exponential values with Poisson occurrence, and  $ES_{0.95}$  and ruin probability  $\psi(a, M)$  are calculated in Excel as follows in Figure 6:

A	B	C	D	E	F	G	H	I	J	K
Theo U <sub>i</sub> (t)	1538.37786	1538.97666	1540.0545	1541.43174	1542.92874	1545.26406	1548.4377	1551.4317	1554.06642	1558.43766
theta	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
zeta	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19
u	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
t	1	1	1	1	1	1	1	1	1	1
a	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
M	0	10	19	27	34	41	48	54	59	65
N	10	10	10	10	10	10	10	10	10	10
X	100	100	100	100	100	100	100	100	100	100
Xi	40	46	51.4	56.2	60.4	64.6	68.8	72.4	75.4	79
Xr	60	54	48.6	43.8	39.6	35.4	31.2	27.6	24.6	21
S	996.5980549	996.5980549	996.5980549	996.5980549	996.5980549	996.5980549	996.5980549	996.5980549	996.5980549	996.5980549
[Si]	399.2	459.08	512.972	560.876	602.792	644.708	686.624	722.552	752.492	788.42
Sr	598.8	538.92	485.028	437.124	395.208	353.292	311.376	275.448	245.508	209.58
U(t)	1538.37786	1538.97666	1540.0545	1541.43174	1542.92874	1545.26406	1548.4377	1551.4317	1554.06642	1558.43766
value-at-risk	75.18886993	98.1306547	99.16616383	99.48271089	75.21509366	74.64912875	74.45636394	74.40275484	54.06642042	58.43766042
c*	437.5778604	498.0566604	553.0265004	602.3077404	645.7207404	689.9720604	735.0617004	773.9837004	806.5584204	846.8576604
Var(Si)	14719.36	19466.3536	24304.97522	29056.38462	33561.61274	38391.40274	43545.75462	48222.0953	52301.19794	57414.7036
PROFITr	59.88	59.2812	58.20336	56.82612	55.32912	52.9938	49.82016	46.82616	44.19144	39.8202
Var (Sr)	33118.5600	26826.0336	21729.0872	17648.8806	14426.4447	11528.5707	8955.2586	7007.8873	5567.2299	4057.0236
ES (0.95)	670.1016	705.6377	730.6881	763.1634	783.3937	812.4188	836.4823	855.6909	886.3125	893.6415
Psi (u,t)	0.087312252	0.086452339	0.086887538	0.089444258	0.089566722	0.089115785	0.089900436	0.090491325	0.092109158	0.091589813
PROFITi	38.37786042	38.97666042	40.05450042	41.43174042	42.92874042	45.26406042	48.43770042	51.43170042	54.06642042	58.43766042
Expected Utility	0.142588906	0.142640246	0.142732651	0.142850709	0.142979015	0.143179133	0.143451014	0.143707427	0.143933006	0.144307132

Figure 6. Screenshot of criteria calculation using Monte Carlo simulation with @Risk.

By retrieving criteria value and reformatting in the tabulated workbook for processing TOPSIS, the decision matrix is built as shown in Figure 7 and is ready for processing using MATLAB, coded and developed in Section 2.

	A	B	C	D	E	F	G	H	I
1	Alternatives					ES0.95	Ruin Probability	PROFITi	Expected Utility
2	Alternatives	a	M	zeta	riteri	c1	c2	c3	c4
3	A1	0.6	0	0.1		670.1016436	0.087312252	38.37786042	0.142588906
4	A2		10	0.11		705.6377247	0.086452339	38.97666042	0.142640246
5	A3		19	0.12		730.688082	0.086887538	40.05450042	0.142732651
6	A4		27	0.13		763.1634052	0.089444258	41.43174042	0.142850709
7	A5		34	0.14		783.3936523	0.089566722	42.92874042	0.142979015
8	A6		41	0.15		812.4187793	0.089115785	45.26406042	0.143179133
9	A7		48	0.16		836.4822701	0.089900436	48.43770042	0.143451014
10	A8		54	0.17		855.6908783	0.090491325	51.43170042	0.143707427
11	A9		59	0.18		886.3125475	0.092109158	54.06642042	0.143933006
12	A10		65	0.19		893.6414999	0.091589813	58.43766042	0.144307132
13	A11		70	0.2		910.8349333	0.091607518	62.32986042	0.14464012
14	A12		75	0.21		935.2232616	0.092231426	66.82086042	0.145024176
15	A13		79	0.22		936.9221891	0.092097565	70.59330042	0.145346649
16	A14		84	0.23		957.8482411	0.092074192	76.22202042	0.145827574
17	A15		88	0.24		976.5046079	0.092530559	81.01242042	0.146236659
18	A16		92	0.25		992.2091456	0.09388913	86.28186042	0.146686426
19	A17		96	0.26		1007.423898	0.093290762	92.03034042	0.147176811
20	A18		100	0.27		1011.69034	0.09339054	98.25786042	0.147707743
21	A19		103	0.28		1043.980452	0.094080032	98.25786042	0.147707743
22	A20		107	0.29		1097.52394	0.095797522	98.25786042	0.147707743
23	A21		110	0.3		1123.563791	0.094323856	98.25786042	0.147707743
24	A22		114	0.31		1170.495524	0.093745244	98.25786042	0.147707743
25	A23		117	0.32		1205.441455	0.094158638	98.25786042	0.147707743
26	A24		120	0.33		1231.082995	0.094099739	98.25786042	0.147707743
27	A25		123	0.34		1255.11953	0.09403522	98.25786042	0.147707743
28	A26		126	0.35		1279.785066	0.093324499	98.25786042	0.147707743
29	A27		129	0.36		1299.283714	0.092930586	98.25786042	0.147707743
30	A28		131	0.37		1353.999274	0.094426982	98.25786042	0.147707743
31	A29		134	0.38		1372.454608	0.094210057	98.25786042	0.147707743
32	A30		137	0.39		1382.934637	0.094673436	98.25786042	0.147707743
33	A31		139	0.4		1422.224375	0.09415124	98.25786042	0.147707743
34	A32		142	0.41		1444.980799	0.092483131	98.25786042	0.147707743
35	A33		144	0.42		1477.163227	0.095391474	98.25786042	0.147707743
36	A34		146	0.43		1512.923601	0.094778562	98.25786042	0.147707743
37	A35		149	0.44		1531.323057	0.093241787	98.25786042	0.147707743
38	criteria Sign range	1				-1.00	-1	1	1
39	W(Lambda)					1.00	1	1	1

Figure 7. Construction of a decision matrix for preparation of TOPSIS.



#### 4.4. Selecting Alternatives Using TOPSIS

With the above alternatives inputting the decision matrix, we observe that if we only consider minimizing expected shortfall and ruin probability, the profit or expected utility will be exceptionally low. The color scale shows intuitively this conflict with red representing smallest value and green representing largest value. We define the weight vector of criteria as all equals (equal values in blue cells Range F39:I39). Recalling the Matlab function built earlier in Section 3 by executing the following code:

```
topsis (decisionMakingMatrix,lambdaWeight,criteriaSign)
```

we could get normalized weight matrix, the identified ideal solutions and the distance between each alternative and the ideal optimality. All these results are stored in Excel sheet “TOPSIS OUTPUT Variables” with selection ranking results shown in Figure 8.

Alternative	Alt1	Alt2	Alt3	Alt4	Alt5	Alt6	Alt7	Alt8	Alt9	Alt10	Alt11	Alt12
D+	0.0569751	0.056478	0.0555945	0.0545868	0.05343051	0.0517104	0.0492624	0.0469969	0.0455135	0.0419515	0.0393043	0.0367563
D-	0.0693894	0.0665287	0.0645277	0.0619595	0.06041675	0.0582922	0.0567964	0.0558353	0.0540705	0.0548093	0.0549429	0.0551283
C	0.5491211	0.5408545	0.5371836	0.5316299	0.53068253	0.5299168	0.5355181	0.5429752	0.5429638	0.5664413	0.5829654	0.5999735
	18	22	23	25	26	27	24	19	20	15	13	11
Alternative	Alt13	Alt14	Alt15	Alt16	Alt17	Alt18	Alt19	Alt20	Alt21	Alt22	Alt23	Alt24
D+	0.0339859	0.0312588	0.0296431	0.0283441	0.0278169	0.0275222	0.0301239	0.034438	0.036536	0.0403173	0.0431329	0.0451989
D-	0.056861	0.0585793	0.0603648	0.0629628	0.0662408	0.0707038	0.069195	0.0668414	0.0657687	0.0639637	0.0627341	0.0618979
C	0.6258991	0.6520543	0.6706616	0.6895734	0.70425705	0.7198073	0.6966954	0.6599706	0.6428709	0.6133785	0.5925748	0.5779624
Rank	9	7	5	4	2	1	3	6	8	10	12	14
Alternative	Alt25	Alt26	Alt27	Alt28	Alt29	Alt30	Alt31	Alt32	Alt33	Alt34	Alt35	
D+	0.0471355	0.0491228	0.0506938	0.0551023	0.05658928	0.0574337	0.0605993	0.0624327	0.0650258	0.067907	0.0693894	
D-	0.061167	0.0604723	0.0599641	0.0587391	0.05839528	0.058216	0.0576492	0.0573983	0.057142	0.0569944	0.0569751	
C	0.5647793	0.5517793	0.541887	0.5159732	0.50785324	0.5033824	0.487526	0.4789934	0.4677338	0.4563152	0.450879	
Rank	16	17	21	28	29	30	31	32	33	34	35	

Figure 8. Ranking of the alternatives based on TOPSIS.

From the TOPSIS output, we could identify that Alternative 18 (0.27, 100) is the best choice, followed closely by Alternative 17 (0.26, 96). Alternative 19 (0.28, 103) is not far away as the third best alternative identified. Alternative 35 (0.44, 149) is the furthest from ideal solution. Multiple trials were tested fixing  $a$  at  $a = 0.75$  (Trial 2),  $a = 0.9$  (Trial 3) and  $a = 1$  (Trial 4). Trial 4 resembles pure Excess-of-loss Reinsurance to compare and contrast decision differences under different ceding portion in Proportional-Stop-loss Reinsurance design. Results of these trials are included in Appendix A. From the result, we could draw insightful managerial implications.

## 5. Managerial Implications

The result shows several interesting findings:

1. The best alternative suggested by TOPSIS does not necessarily optimize any one single criterion, rather, it has an overall highest ranking due to its relative weighted closeness to all four criteria. In reality, if reinsurance is chosen merely according to expected profit, the insurance company may suffer from a high probability of financial crisis. On the other hand, if the decision merely considers constraining higher shortfalls, the insurance company may appear to have poor performance based on their profit and loss statement due to the low profits obtained.

By increasing the ceding amount from  $a = 0.6$  to  $a = 0.75, 0.9$ , one result from Trial 1,2,3,4 (Trial 2–4 are reported in Appendix A) suggests that the ranking of alternatives is different when parameters are changed. When the ceding portions are fixed at relatively lower level (such as  $a = 0.6$  to  $a = 0.75$ ) the best alternative to choose will have the retention limit equaling to mean value of loss. Thus, if the given reinsurance parameters (either  $a$ ,  $\theta$  or  $M2$ ) are altered, it is recommended for the insurance company to evaluate once again the reinsurance plans instead of extrapolating conclusions from previous experiences.

- In addition, Trial 4 with  $a = 1$  models an excess-of-loss reinsurance form where  $X_r = \text{MAX}(0, X - M) = 1 * (0, X - M)_+$ . Accordingly, results from Trial 4 are in accordance with previous knowledge on excess-of-loss reinsurance. Under excess-of-loss reinsurance, the best form is given at  $M = M_{max}$ , which is in correspondence with Section 2 in [Payandeh-Najafabadi and Panahi-Bazaz \(2017\)](#).
- In each trial, the Alternative 1  $M = 0$  simulates the scenario of pure proportional reinsurance. Trial 5 attempts to model different retention level under proportional reinsurance ( $M = X$ ) with fixed reinsurance premium loading factor  $\theta$ . The result shows that given same premium loading factor, retention level of 0.6 would be most preferable.
- By setting  $(a, M)$  to  $(0, 0)$ , we could also model the scenario of no reinsurance. The results show that with no reinsurance, the expected shortfall of insurance company will be significantly higher than all other alternatives, and the ruin probability will be higher as well. This suggests that insurance company without reinsurance is more likely to become bankrupt if large losses are incurred. As compensation, the expected profit and utility will increase by a small amount for the insurance company due to high profit from insurance premium and low probability of large losses. However, noting the high ruin probability, which suggests a much higher risk of bankruptcy, the insurance company will often seek for reinsurance to keep ruin probability low.
- Furthermore, through the simulation process, the variance and profitability of the reinsurer are also being observed and calculated (as can be seen from Figure 9). The result was in correspondence with our previous argument that by scaling the ceding portion  $a$  to larger values, both the variance and the profitability of the reinsurer will increase, suggesting that there is a trade-off between high profit and high risk of large losses. Thus, this supports our previous assumption that the reinsurer is ambiguous towards a design that only differs with respect to parameter  $a$ .

	A	B	C	D	E	F	G	H	I	J	K
1	Alternatives					ES0.95	Ruin Probability	PROFITi	Expected Utility	Rank	C
2	Alternatives	a	M	zeta	riteri	c1	c2	c3	c4		
3	A1	0.1	0	0.15		4046.645008	0.079521227	88.27786042	0.146856731	7	0.307501
4	A2	0.2	0	0.15		3537.668564	0.074630029	76.30186042	0.145834394	6	0.287943
5	A3	0.3	0	0.15		3093.458715	0.079440396	62.32986042	0.14464012	5	0.2594
6	A4	0.4	0	0.15		2747.712124	0.08434825	46.36186042	0.14327319	3	0.21561
7	A5	0.5	0	0.15		2206.42891	0.08018247	28.39786042	0.141732783	2	0.142364
8	A6	0.6	0	0.15		1767.175006	0.090638388	8.437860424	0.140017971	1	0.004457
9	A7	0.7	0	0.15		1306.695578	0.090656182	-13.51813958	0.13812772	4	0.219885
10	A8	0.8	0	0.15		877.5800343	0.093619193	-37.47013958	0.136060889	8	0.460288
11	A9	0.9	0	0.15		373.9148882	0.105017214	-63.41813958	0.133816229	9	0.721277
12	A10	1	0	0.19		91.36213958	0.108302642	-91.36213958	0.13139238	10	0.988854
18	criteria Sign range	1				-1.00	-1	1	1		
19	W(Lambda)					1.00	1	1	1		

Figure 9. Trial 5: Pure proportional reinsurance selection.

## 6. Conclusions

### 6.1. Contributions

The research has the following contributions:

- To the best of our knowledge, this is the first theoretical study using MADM to approach proportional-stop-loss reinsurance model, though there are a few recent studies using MADM in designing either pure proportional or pure stop-loss reinsurance contracts;
- This is one of the few studies taking a non-discriminatory position considering both the insurance and the reinsurance company in designing an optimal reinsurance contract, and the study made

significant contribution by incorporating existing MODM models and the promising MADM model into one decision flow process to arrive at a robust decision for reinsurance design;

3. This study demonstrates the feasibility of incorporating intelligent decision supporting systems in reinsurance deal-making. As observed by the author through industry experiences, @Risk has grown its popularity recently for actuarial study in modeling risk and claims. The prototype of TOPSIS implemented through Matlab suggests that a software of multi-criteria decision support would be promising.
4. As previous research suggested (Bazaz and Najafabadi 2015), MADM is not likely to address finding of optimal type of reinsurance. However, with the generic formulation of proportional-stop-loss reinsurance, we would be able to model proportional reinsurance and stop-loss reinsurance as special cases of proportional-stop-loss, thus the choice between proportional and non-proportional reinsurance using MADM could be possible under this formulation of reinsurance.

## 6.2. Limitations

There are still some limitations for this research, specifically in the following aspects:

1. In terms of the scope of study, due to time and resource constraints the study only considers proportional-stop-loss treaty reinsurance, while basing the decision process on ruin probability, CVaR, and expected utility criteria. Other types of reinsurance and decision measurements have not been elaborated and tested.
2. In terms of methodology, this study attempts to utilize the simulation software @Risk to model the loss and claim distribution and to use numerical TOPSIS model in modeling decisions from the insurance company, without reaching to a close-form solution. Thus, the conclusions were drawn based on simulation result rather than robust theoretical derivation.
3. In terms of model implementation, due to resource constraints, this study only includes a numerical made-up case instead of existing cases to conduct archival research in addressing the decision process in the reinsurance purchase decisions.

## 6.3. Future Directions

In reality, trade contracts will usually go through lengthy negotiations with broking firms acting as intermediaries, thus, empirical study with cases from existing broking firms may be more realistic and practical in addressing the usefulness of this decision framework. In addition, behavioral study of both the reinsurer and the reinsured party would be of great importance to suggest whether they are rational players in the reinsurance market.

Furthermore, it would be promising for mathematical and quantitative researchers to look into the closed-form optimization for proportional-stop-loss under each single measurement. As pure proportional or stop-loss reinsurance could be regarded as special cases of proportional-stop-loss reinsurance could, this will reconcile existing mathematical models on either side and help in calculating the precise decision matrix for MADM analysis.

**Author Contributions:** The authors contributed equally to the paper.

**Conflicts of Interest:** The authors declare no conflict of interest.

## Appendix A. TOPSIS Trials #2, #3 and #4

	A	B	C	D	E	F	G	H	I
1	Alternatives					ESO.95	Ruin Probability	PROFITi	Expected Utility
2	Alternatives	a	M	zeta	riteri	c1	c2	c3	c4
3	A1	0.75	0	0.1		834.7716436	0.087312252	23.40786042	0.141304401
4	A2		10	0.11		855.1880247	0.086452339	24.15636042	0.141368672
5	A3		19	0.12		866.495922	0.086887538	25.50366042	0.141484347
6	A4		27	0.13		886.6509352	0.089444258	27.22521042	0.141632132
7	A5		34	0.14		896.0279323	0.089566722	29.09646042	0.141792739
8	A6		41	0.15		913.9902293	0.089115785	32.01561042	0.142043226
9	A7		48	0.16		926.7813101	0.089900436	35.98266042	0.142383515
10	A8		54	0.17		936.2594183	0.090491325	39.72516042	0.142704417
11	A9		59	0.18		958.7374075	0.092109158	43.01856042	0.142986713
12	A10		65	0.19		955.9915499	0.091589813	48.48261042	0.143454861
13	A11		70	0.2		964.7269333	0.091607518	53.34786042	0.14387149
14	A12		75	0.21		980.5075116	0.092231426	58.96161042	0.144351965
15	A13		79	0.22		975.2753291	0.092097565	63.67716042	0.144755355
16	A14		84	0.23		987.3092011	0.092074192	70.71306042	0.145356885
17	A15		88	0.24		998.7799679	0.092530559	76.70106042	0.145868492
18	A16		92	0.25		1007.179146	0.09389913	83.28786042	0.146430906
19	A17		96	0.26		1014.968778	0.093290762	90.47346042	0.147044026
20	A18		100	0.27		1011.69034	0.09339054	98.25786042	0.147707743
21	A19		103	0.28		1043.980452	0.094080032	98.25786042	0.147707743
22	A20		107	0.29		1097.52394	0.095797522	98.25786042	0.147707743
23	A21		110	0.3		1123.563791	0.094323856	98.25786042	0.147707743
24	A22		114	0.31		1170.495524	0.093745244	98.25786042	0.147707743
25	A23		117	0.32		1205.441455	0.094158638	98.25786042	0.147707743
26	A24		120	0.33		1231.082995	0.094099739	98.25786042	0.147707743
27	A25		123	0.34		1255.11953	0.09403522	98.25786042	0.147707743
28	A26		126	0.35		1279.785066	0.093324499	98.25786042	0.147707743
29	A27		129	0.36		1299.283714	0.092930586	98.25786042	0.147707743
30	A28		131	0.37		1353.999274	0.094426982	98.25786042	0.147707743
31	A29		134	0.38		1372.454608	0.094210057	98.25786042	0.147707743
32	A30		137	0.39		1382.934637	0.094673436	98.25786042	0.147707743
33	A31		139	0.4		1422.224375	0.09415124	98.25786042	0.147707743
34	A32		142	0.41		1444.980799	0.092483131	98.25786042	0.147707743
35	A33		144	0.42		1477.163227	0.095391474	98.25786042	0.147707743
36	A34		146	0.43		1512.923601	0.094778562	98.25786042	0.147707743
37	A35		149	0.44		1531.323057	0.093241787	98.25786042	0.147707743
38	criteria Sign range	1				-1.00	-1	1	1
39	W(Lambda)					1.00	1	0.5	0.5

Figure A1. Trial 2: Screen-shot of crucial TOPSIS output parameters.

Alternative	Alt1	Alt2	Alt3	Alt4	Alt5	Alt6	Alt7	Alt8	Alt9	Alt10	Alt11	Alt12
D+	0.1320813	0.1307614	0.1283854	0.1253516	0.12205232	0.116908	0.1099147	0.1033174	0.0975231	0.0878841	0.0793125	0.0694342
D-	0.0170302	0.0165837	0.01667	0.0171408	0.01849387	0.0214132	0.0266617	0.0322606	0.0373297	0.0464293	0.0546185	0.0641677
C	0.1142111	0.1125501	0.1149215	0.120293	0.13158572	0.1548078	0.1952147	0.2379488	0.2768181	0.345679	0.4078107	0.4802904
	34	35	33	32	31	30	29	28	27	26	25	24
Alternative	Alt13	Alt14	Alt15	Alt16	Alt17	Alt18	Alt19	Alt20	Alt21	Alt22	Alt23	Alt24
D+	0.0611182	0.0487488	0.0382502	0.0267505	0.01442573	0.0043257	0.0051152	0.0064243	0.0070609	0.0082083	0.0090627	0.0096896
D-	0.0723485	0.0845284	0.094939	0.1064393	0.1190163	0.1326909	0.1326177	0.1325065	0.132457	0.1323756	0.1323214	0.1322851
C	0.5420717	0.63423	0.712813	0.7991551	0.89189515	0.9684294	0.9628616	0.953759	0.9493909	0.9416129	0.9359003	0.9317513
Rank	23	22	21	20	17	1	2	3	4	5	6	7
Alternative	Alt25	Alt26	Alt27	Alt28	Alt29	Alt30	Alt31	Alt32	Alt33	Alt34	Alt35	
D+	0.0102773	0.0108803	0.011357	0.0126948	0.01314599	0.0134022	0.0143628	0.0149192	0.015706	0.0165803	0.0170302	
D-	0.1322538	0.1322244	0.1322031	0.1321524	0.13213841	0.1321311	0.1321082	0.1320982	0.1320879	0.1320821	0.1320813	
C	0.9278946	0.9239697	0.9208903	0.9123575	0.9095155	0.9079096	0.901941	0.8985211	0.8937302	0.8884699	0.8857891	
Rank	8	9	10	11	12	13	14	15	16	18	19	

Figure A2. Trial 2: Screen-shot of crucial TOPSIS output parameters.



	A	B	C	D	E	F	G	H	I
1	Alternatives					ES0.95	Ruin Probability	PROFITi	Expected Utility
2	Alternatives	a	M	zeta	riteri	c1	c2	c3	c4
3	A1	0.9	0	0.1		988.2699744	0.085088195	8.437860424	0.140017971
4	A2		10	0.11		999.1228241	0.088673751	9.336060424	0.140095211
5	A3		19	0.12		1002.263826	0.088121149	10.95282042	0.140234225
6	A4		27	0.13		992.671412	0.0883292	13.01868042	0.140411823
7	A5		34	0.14		996.8261654	0.089295076	15.26418042	0.140604822
8	A6		41	0.15		969.8431979	0.089787781	18.76716042	0.140905813
9	A7		48	0.16		1018.170562	0.092399405	23.52762042	0.141314684
10	A8		54	0.17		970.0579878	0.089894564	28.01862042	0.141700233
11	A9		59	0.18		1003.176245	0.089720096	31.97070042	0.142039373
12	A10		65	0.19		990.9022899	0.089280019	38.52756042	0.142601742
13	A11		70	0.2		1042.748026	0.089877328	44.36586042	0.14310217
14	A12		75	0.21		947.1232568	0.08922491	51.10236042	0.143679225
15	A13		79	0.22		979.2784188	0.089342529	56.76102042	0.144163651
16	A14		84	0.23		999.4414069	0.093276822	65.20410042	0.144885935
17	A15		88	0.24		1001.186805	0.096410473	72.38970042	0.145500165
18	A16		92	0.25		1006.397288	0.094699571	80.29386042	0.146175309
19	A17		96	0.26		971.5311893	0.090507333	88.91658042	0.146911221
20	A18		100	0.27		1036.796887	0.091251777	98.25786042	0.147707743
21	A19		103	0.28		1025.100632	0.09641785	98.25786042	0.147707743
22	A20		107	0.29		1092.039887	0.096444365	98.25786042	0.147707743
23	A21		110	0.3		1095.142832	0.096424704	98.25786042	0.147707743
24	A22		114	0.31		1173.781477	0.098787514	98.25786042	0.147707743
25	A23		117	0.32		1214.496515	0.098542238	98.25786042	0.147707743
26	A24		120	0.33		1208.626958	0.098701899	98.25786042	0.147707743
27	A25		123	0.34		1215.531859	0.09889659	98.25786042	0.147707743
28	A26		126	0.35		1273.365626	0.094173914	98.25786042	0.147707743
29	A27		129	0.36		1281.506042	0.090339338	98.25786042	0.147707743
30	A28		131	0.37		1327.804543	0.095786712	98.25786042	0.147707743
31	A29		134	0.38		1383.018954	0.099597592	98.25786042	0.147707743
32	A30		137	0.39		1394.675135	0.092995609	98.25786042	0.147707743
33	A31		139	0.4		1362.567173	0.09055014	98.25786042	0.147707743
34	A32		142	0.41		1452.76873	0.094773467	98.25786042	0.147707743
35	A33		144	0.42		1421.889442	0.091291142	98.25786042	0.147707743
36	A34		146	0.43		1431.78405	0.090361098	98.25786042	0.147707743
37	A35		149	0.44		1543.429196	0.09393016	98.25786042	0.147707743
38	criteria Sign range	1				-1.00	-1	1	1
39	W(Lambda)					1.00	1	1	1

Figure A3. Trial 3: Screen-shot of crucial TOPSIS output parameters.

Alternative	Alt1	Alt2	Alt3	Alt4	Alt5	Alt6	Alt7	Alt8	Alt9	Alt10	Alt11	Alt12
D+	0.1811699	0.1793585	0.1760976	0.1719304	0.16740133	0.1603351	0.1507347	0.1416747	0.1337043	0.1204785	0.108706	0.095114
D-	0.0056236	0.0058032	0.0074685	0.0107931	0.01484024	0.0216293	0.0308979	0.0399196	0.0477808	0.0609492	0.072645	0.0862672
C	0.0301062	0.0313411	0.0406857	0.0590679	0.08143167	0.1188654	0.1701119	0.2198285	0.2632766	0.3359422	0.4005768	0.4756127
Rank	35	34	33	32	31	30	29	28	27	26	25	24
Alternative	Alt13	Alt14	Alt15	Alt16	Alt17	Alt18	Alt19	Alt20	Alt21	Alt22	Alt23	Alt24
D+	0.0837009	0.0666725	0.0521798	0.036239	0.0188433	0.0009097	0.0007959	0.0014709	0.0015023	0.0022985	0.0027103	0.002651
D-	0.0976365	0.1146316	0.1291095	0.1450376	0.16243114	0.1812421	0.1812455	0.1812271	0.1812263	0.1812081	0.1812001	0.1812012
C	0.5384244	0.6322614	0.712174	0.8000902	0.89605096	0.9950056	0.9956281	0.9919487	0.9917786	0.9874744	0.985263	0.985581
Rank	23	22	21	20	19	2	1	3	4	5	7	6
Alternative	Alt25	Alt26	Alt27	Alt28	Alt29	Alt30	Alt31	Alt32	Alt33	Alt34	Alt35	
D+	0.0027209	0.0033049	0.0033867	0.0038564	0.00441623	0.004533	0.0042076	0.0051215	0.0048084	0.0049085	0.0060394	
D-	0.1811999	0.1811901	0.1811889	0.1811826	0.18117673	0.1811757	0.1811787	0.1811718	0.1811736	0.181173	0.1811695	
C	0.9852061	0.9820869	0.9816515	0.9791591	0.97620477	0.9755909	0.9773038	0.9725085	0.9741461	0.9736219	0.9677397	
Rank	8	9	10	11	13	14	12	17	15	16	18	

Figure A4. Trial 3: Screen-shot of crucial TOPSIS output parameters.

	A	B	C	D	E	F	G	H	I
1	Alternatives					ES0.95	Ruin Probability	PROFITi	Expected Utility
2	Alternatives	a	M	zeta	riteri	c1	c2	c3	c4
3	A1	1	0	0.1		1098.049974	0.085088195	-1.542139576	0.13915928
4	A2		10	0.11		1098.823024	0.088673751	-0.544139576	0.139245188
5	A3		19	0.12		1092.802386	0.088121149	1.252260424	0.1393998
6	A4		27	0.13		1074.996432	0.0883292	3.547660424	0.139597319
7	A5		34	0.14		1071.915685	0.089295076	6.042660424	0.139811963
8	A6		41	0.15		1037.557498	0.089787781	9.934860424	0.1401467
9	A7		48	0.16		1078.369922	0.092399405	15.22426042	0.140601391
10	A8		54	0.17		1023.770348	0.089894564	20.21426042	0.141030124
11	A9		59	0.18		1051.459485	0.097200962	24.60546042	0.141407232
12	A10		65	0.19		1032.46899	0.085280019	31.89086042	0.142032523
13	A11		70	0.2		1078.676026	0.089877328	38.37786042	0.142588906
14	A12		75	0.21		977.3127568	0.08722491	45.86286042	0.143230438
15	A13		79	0.22		1004.847179	0.085934253	52.15026042	0.143768954
16	A14		84	0.23		1019.082047	0.093276822	61.53146042	0.144571825
17	A15		88	0.24		1016.037045	0.096410473	69.51546042	0.145254526
18	A16		92	0.25		1016.377288	0.094699571	78.29786042	0.146004869
19	A17		96	0.26		976.5611093	0.088507333	87.87866042	0.146822672
20	A18		100	0.27		1036.796887	0.091251777	98.25786042	0.147707743
21	A19		103	0.28		1025.100632	0.08641785	98.25786042	0.147707743
22	A20		107	0.29		1092.039887	0.096444365	98.25786042	0.147707743
23	A21		110	0.3		1095.142832	0.096424704	98.25786042	0.147707743
24	A22		114	0.31		1173.781477	0.098787514	98.25786042	0.147707743
25	A23		117	0.32		1214.496515	0.098542238	98.25786042	0.147707743
26	A24		120	0.33		1208.626958	0.088701899	98.25786042	0.147707743
27	A25		123	0.34		1215.531859	0.09889659	98.25786042	0.147707743
28	A26		126	0.35		1273.365626	0.094173914	98.25786042	0.147707743
29	A27		129	0.36		1281.506042	0.090339338	98.25786042	0.147707743
30	A28		131	0.37		1327.804543	0.095786712	98.25786042	0.147707743
31	A29		134	0.38		1383.018954	0.099597592	98.25786042	0.147707743
32	A30		137	0.39		1394.675135	0.092995609	98.25786042	0.147707743
33	A31		139	0.4		1362.567173	0.09055014	98.25786042	0.147707743
34	A32		142	0.41		1452.76873	0.094773467	98.25786042	0.147707743
35	A33		144	0.42		1421.889442	0.091291142	98.25786042	0.147707743
36	A34		146	0.43		1431.78405	0.090361098	98.25786042	0.147707743
37	A35		149	0.44		1543.429196	0.09393016	98.25786042	0.147707743
38	criteria Sign range	1				-1.00	-1	1	1
39	W(Lambda)					1.00	1	1	1

Figure A5. Trial 4: Screen-shot of crucial TOPSIS output parameters.

Alternative	Alt1	Alt2	Alt3	Alt4	Alt5	Alt6	Alt7	Alt8	Alt9	Alt10	Alt11	Alt12
D+	0.2122375	0.2101156	0.2062963	0.2014154	0.1961105	0.1878342	0.1765874	0.1659774	0.1566397	0.1411504	0.1273582	0.1114441
D-	0.0021764	0.0004588	0.0038411	0.0087105	0.01401154	0.0222867	0.0335335	0.0441442	0.0534823	0.0689743	0.0827689	0.0986859
C	0.0101505	0.0021788	0.0182791	0.0414537	0.06668285	0.106066	0.1595914	0.210089	0.2545299	0.3282541	0.3938992	0.4696421
	34	35	33	32	31	30	29	28	27	26	25	24
Alternative	Alt13	Alt14	Alt15	Alt16	Alt17	Alt18	Alt19	Alt20	Alt21	Alt22	Alt23	Alt24
D+	0.0980792	0.078138	0.0611706	0.0425172	0.02221186	0.0025705	0.0024108	0.0021459	0.0020171	0.0017849	0.0016121	0.0014852
D-	0.1120561	0.1320054	0.1489836	0.1676596	0.18803338	0.210105	0.2101051	0.2101054	0.2101057	0.2101065	0.2101072	0.2101078
C	0.5332568	0.6281682	0.7089253	0.7977074	0.89435261	0.9879134	0.988656	0.9898898	0.9904909	0.9915761	0.9923858	0.9929807
	23	22	21	20	19	18	17	16	15	14	13	12
Alternative	Alt25	Alt26	Alt27	Alt28	Alt29	Alt30	Alt31	Alt32	Alt33	Alt34	Alt35	
D+	0.0013663	0.0012443	0.0011479	0.0008772	0.0007859	0.0007341	0.0005397	0.0004272	0.0002679	9.105E-05	6.183E-06	
D-	0.2101085	0.2101092	0.2101098	0.2101118	0.21011258	0.210113	0.2101148	0.2101159	0.2101176	0.2101196	0.2101207	
C	0.9935391	0.9941126	0.9945665	0.9958425	0.99627358	0.9965186	0.997438	0.997971	0.9987265	0.9995669	0.9999706	
	11	10	9	8	7	6	5	4	3	2	1	

Figure A6. Trial 4: Screen-shot of crucial TOPSIS output parameters.



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