

Adaptive Control for Gravitational Wave Detection Formation Considering Time-Varying Communication Delays

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Abstract: A distributed six-degree-of-freedom (6-DOF) cooperative control for multiple spacecraft formation is investigated considering parametric uncertainties, external disturbances, and time-varying communication delays. Unit dual quaternions are used to describe the kinematics and dynamics models of the 6-DOF relative motion of the spacecraft. A distributed coordinated controller based on dual quaternions with time-varying communication delays is proposed. The unknown mass and inertia, as well as unknown disturbances, are then taken into account. An adaptive coordinated control law is developed by combining the coordinated control algorithm with an adaptive algorithm to compensate for parametric uncertainties and external disturbances. The Lyapunov method is used to prove that the tracking errors converge globally asymptotically. Numerical simulations show that the proposed method can realize cooperative control of attitude and orbit for the multi-spacecraft formation.

Keywords: distributed coordinated control; communication delays; adaptive control; dual quaternions



Citation: Zhang, Y.; Liu, Y.; Zhang, J.; Lu, Z.; Yang, J. Adaptive Control for Gravitational Wave Detection Formation Considering Time-Varying Communication Delays. *Sensors* **2023**, *23*, 3003. <https://doi.org/10.3390/s23063003>

Academic Editor: Carlos Silvestre

Received: 2 February 2023

Revised: 23 February 2023

Accepted: 6 March 2023

Published: 10 March 2023



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1. Introduction

Einstein's theory of general relativity was further proven when gravitational waves were first directly detected in LIGO observatories on 14 September 2015 [1]. Space gravitational wave detection has attracted more attention in detecting low-frequency gravitational wave signals. The United States has proposed the laser interferometric space antenna (LISA) [2], and China proposed the "Tianqin Project" [3] and the "Taiji Project" [4].

The space laser interferometer gravitational wave detector consists of three spacecraft, forming an equilateral triangle configuration with a scale of 100,000 km to 1 million km. By adjusting the attitude of the spacecraft, a laser link is established between the two spacecraft to detect gravitational waves. The space gravitational wave detection program currently has a relatively high requirement for configuration stability. However, the orbit injection error and perturbation will lead to the deviation between the actual orbit and the nominal orbit of the spacecraft, which will lead to the destruction of the equilateral triangle configuration and the destruction of the laser link, seriously affecting the implementation of scientific missions. At this time, the scientific observation of gravitational waves needs to be suspended for spacecraft formation configuration reconstruction and attitude adjustment. As is known, actuator configuration leads to dynamic coupling between rotation and translation [5]. In order to achieve high control accuracy of the system, the translation and the rotation of the spacecraft should be simultaneously taken into account. In recent years, Lie group $SE(3)$ [6–8] and dual quaternions [9–11] have been the most popular methods to describe the coupling motion of rigid bodies. A 4×4 homogeneous transformation matrix is utilized when modeling rigid bodies on $SE(3)$, while the model is described more compactly by dual quaternions, which have only eight parameters, and the dual quaternions' multiplications have lower computational cost than homogeneous transformation matrix multiplications [12]. Therefore, this paper uses dual quaternion as a tool

to design an attitude-orbit coupling coordination controller for space gravitational wave detection formation.

There are roughly five methods of multi-spacecraft coordination reported in the literature: the leader-following method [13–15], the behavior-based method [16], the virtual structure method [17], the artificial potential function method [18], and algebraic graph method [19]. The control laws in the previous works require precise inertial parameters of the spacecraft. However, obtaining accurate inertial parameters on fuel consumption during several years of gravitational wave detection missions is challenging. In addition, spacecraft working in the deep space environment are subject to unknown disturbances, including environmental and non-environmental forces and torques. Consequently, it is essential to design a cooperative control algorithm subject to parametric uncertainties and external disturbances for gravitational wave detection missions.

Adaptive control technology as an effective method to deal with parametric uncertainties and external disturbances has been widely used [20–23]. Wang et al. [24] proposed a robust tracking control of unknown models to deal with the problem of model uncertainties. In Ref. [25], a new adaptive nonsingular fast terminal sliding mode surface was developed for the attitude synchronization and tracking control of multiple spacecraft formation systems. Xing et al. [26] used a fuzzy logic system (FLS) to approximate the disturbance. Lin et al. [27] designed an adaptive fast integrating terminal sliding mode control law, which was robust to parameter uncertainties and external disturbances. The literature mentioned above mainly investigates the problem of attitude synchronization control for multiple spacecraft. Some scholars described spacecraft attitude and orbit motion in the dual quaternion framework and combined adaptive control and sliding mode control to deal with parameter uncertainties and external disturbances in spacecraft tracking control [9,28–30]. An adaptive tracking controller was designed in Ref. [10] for satellite proximity operations, which needs no information about the mass and inertia of the chaser spacecraft. On this basis, Gui et al. [11] improved the adaptive control law to reduce the control energy consumption. In Ref. [31], the problem of distributed finite time 6-DOF synchronization control for multiple spacecraft in the presence of external disturbances and parameter uncertainty was investigated, and 6-DOF coupled motion model was the Euler–Lagrange form. Nevertheless, to the best of our knowledge, there is little research on the dual quaternions-based adaptive coordinated controller design for multiple spacecraft.

The communication delay caused by the distance between neighboring spacecraft is another issue that deserves special attention. The current research on spacecraft formation control with communication delay mainly focuses on attitude coordination control [32–34]. In [7], the decentralized leaderless spacecraft consensus was studied considering a constant time delay between two spacecraft. Zhang et al. [8] proposed a nonsingular fast terminal sliding mode scheme to solve the consensus control problem of spacecraft formation flying in the presence of parametric uncertainties, external disturbances, and communication delays. Note that the relative position and attitude in the above works are represented on the Lie group $SE(3)$. However, few studies in the literature discuss the attitude and orbit coupling coordinated control of multiple spacecraft considering parametric uncertainties, external disturbances, and time-varying communication delays in the framework of dual quaternions.

Inspired by this motivation, this paper mainly focuses on discussing the 6-DOF coordinated control problem for multiple spacecraft based on dual quaternions with consideration of parametric uncertainties, external disturbance, and time-varying communication delays. The main contributions in this paper can be summarized as follows:

- (1) Dual quaternion is employed to describe the 6-DOF relative motion of the spacecraft. The gravitational force and torque, the perturbations due to the Earth's J_2 oblateness, the solar pressure perturbation, and the constant external disturbances are considered;
- (2) In the absence of modeling uncertainties and external disturbances, time-delay terms are added to the controller, which guarantees that the controller is effective to solve the cooperative control problem with communication delays;

- (3) In the presence of modeling uncertainties and external disturbances, the cooperation controller with communication delays is developed into an adaptive controller, which can estimate the unknown parameters and external disturbances.

The rest of the paper is organized as follows: In Section 2, quaternions and dual quaternions are introduced. Then, a dual quaternion-based 6-DOF relative motion model is derived. Section 3 presents the proposed control laws and stability analysis. Finally, the simulation results verify the effectiveness of the proposed method in Section 4 followed by conclusions in Section 5.

2. Material Background and Relative Coupled Dynamics

2.1. Quaternions and Dual Quaternions

A quaternion is defined as $q = [\xi, \bar{\eta}]$, where $\xi \in \mathbb{R}$ and $\bar{\eta} \in \mathbb{R}^3$ are the scalar and vector part of the quaternion, respectively. The set of quaternions is defined as $\mathbb{H} = \{q : q = (\xi, \bar{\eta})\}$. Let $\mathbb{H}_v = \{q \in \mathbb{H} : \xi = 0\}$ and $\mathbb{H}_s = \{q \in \mathbb{H} : \bar{\eta} = 0\}$ denote the set of vector quaternions and scalar quaternions, respectively. Given two quaternions $q_1 = (\xi_1, \bar{q}_1)$ and $q_2 = (\xi_2, \bar{q}_2)$ in $\mathbb{H}$. The addition, multiplication, conjugation, dot product, and cross product are defined, respectively, by

$$q_1 + q_2 = (\xi_1 + \xi_2, \bar{q}_1 + \bar{q}_2) \in \mathbb{H} \quad (1)$$

$$q_1 \otimes q_2 = (\xi_1 \xi_2 - \bar{q}_1 \cdot \bar{q}_2, \xi_1 \bar{q}_2 + \xi_2 \bar{q}_1 + \bar{q}_1 \times \bar{q}_2) \in \mathbb{H} \quad (2)$$

$$q^* = (\xi, -\bar{q}) \in \mathbb{H} \quad (3)$$

$$q_1 \cdot q_2 = (\xi_1 \xi_2 + \bar{q}_1 \cdot \bar{q}_2, 0) \in \mathbb{H}_s \quad (4)$$

$$q_1 \times q_2 = (0, \xi_1 \bar{q}_2 - \xi_2 \bar{q}_1 + \bar{q}_1 \times \bar{q}_2) \in \mathbb{H}_v \quad (5)$$

A dual quaternion is defined as $\hat{q} = q_r + \varepsilon q_d$, where $q_r \in \mathbb{H}$ and $q_d \in \mathbb{H}$ are the real and dual parts, respectively. ε is the dual unit that satisfies the property $\varepsilon^2 = 0$ but $\varepsilon \neq 0$. The set of dual quaternions, dual vectors, and dual scalar quaternions are defined as $\mathbb{DQ} = \{\hat{q} : \hat{q} = q_r + \varepsilon q_d : q_r, q_d \in \mathbb{H}\}$, $\mathbb{DQ}_v = \{\hat{q} : \hat{q} = q_r + \varepsilon q_d : q_r, q_d \in \mathbb{H}_v\}$, $\mathbb{DQ}_s = \{\hat{q} : \hat{q} = q_r + \varepsilon q_d : q_r, q_d \in \mathbb{H}_s\}$, respectively. The set of dual scalar quaternions with zero dual part is denoted by $\mathbb{DQ}_r = \{\hat{q} : \hat{q} = q + \varepsilon 0 : q \in \mathbb{H}_s\}$.

Given two dual quaternions $\hat{q}_1 = q_{1r} + \varepsilon q_{1d}$ and $\hat{q}_2 = q_{2r} + \varepsilon q_{2d}$ in $\mathbb{DQ}$ with q_{1r}, q_{1d}, q_{2r} , and q_{2d} in $\mathbb{H}$. The addition, multiplication, conjugation, dot product, and cross product are defined, respectively, by

$$\hat{q}_1 + \hat{q}_2 = (q_{1r} + q_{2r}) + \varepsilon(q_{1d} + q_{2d}) \in \mathbb{DQ} \quad (6)$$

$$\hat{q}_1 \otimes \hat{q}_2 = (q_{1r} \otimes q_{2r}) + \varepsilon(q_{1r} \otimes q_{2d} + q_{1d} \otimes q_{2r}) \in \mathbb{DQ} \quad (7)$$

$$\hat{q}^* = q_r^* + \varepsilon q_d^* \in \mathbb{DQ} \quad (8)$$

$$\hat{q}_1 \cdot \hat{q}_2 = q_{1r} \cdot q_{2r} + \varepsilon(q_{1d} \cdot q_{2r} + q_{1r} \cdot q_{2d}) \in \mathbb{DQ}_s \quad (9)$$

$$\hat{q}_1 \times \hat{q}_2 = q_{1r} \times q_{2r} + \varepsilon(q_{1d} \times q_{2r} + q_{1r} \times q_{2d}) \in \mathbb{DQ}_v \quad (10)$$

The swap product of a dual quaternion is $\hat{q}^s = q_d + \varepsilon q_r \in \mathbb{DQ}$. The $\odot$ product of a dual quaternion is $\hat{c} \odot \hat{q} = (c_r + \varepsilon c_d) \odot (q_r + \varepsilon q_d) = c_r q_r + \varepsilon c_d q_d, \hat{q} \in \mathbb{DQ}$. The circle product of two dual quaternions is $\hat{q}_1 \circ \hat{q}_2 = q_{1r} \cdot q_{2r} + q_{1d} \cdot q_{2d}, \hat{q}_1, \hat{q}_2 \in \mathbb{DQ}_v$.

The following properties with the above definitions can be shown:

$$\hat{a} \circ (\hat{b} \otimes \hat{c}) = \hat{b}^s \circ (\hat{a}^s \otimes \hat{c}^*) = \hat{c}^s \circ (\hat{b}^* \otimes \hat{a}^s) \in \mathbb{R}, \hat{a}, \hat{b}, \hat{c} \in \mathbb{DQ} \quad (11)$$

$$\hat{a} \circ (\hat{b} \times \hat{c}) = \hat{b}^s \circ (\hat{c} \times \hat{a}^s) = \hat{c}^s \circ (\hat{a}^s \times \hat{b}) \in \mathbb{R}, \hat{a}, \hat{b}, \hat{c} \in \mathbb{DQ}_v \quad (12)$$

$$\hat{a}^s \circ \hat{b}^s = \hat{a} \circ \hat{b}, \hat{a}, \hat{b} \in \mathbb{DQ} \quad (13)$$

$$||\hat{a}||^2 = \hat{a} \circ \hat{a}, \hat{a} \in \mathbb{DQ}_r \quad (14)$$

2.2. Equations of 6-DOF Relative Motion Based on Dual Quaternions

The 6-DOF relative motion model of spacecraft based on dual quaternions is established in this subsection. Let $\mathcal{F}_I$ represent the Earth-centered-inertial frame with the origin at the center of the Earth. The body-fixed coordinate system $\mathcal{F}_i$ (i means the i -th spacecraft) is defined with the origin at the center of mass.

The kinematics equation of the i -th spacecraft is given by [35]

$$\dot{\hat{q}}_i = \frac{1}{2} \hat{q}_i \otimes \hat{\omega}_i^i \quad (15)$$

where $\hat{q}_i = q_i + \varepsilon \frac{1}{2} q_i \otimes r_i^i$, $r_i^i = [0, \bar{r}_i^i]$, $\bar{r}_i^i \in \mathbb{R}^3$ is the translation vector from the origin of the frame $\mathcal{F}_I$ to the origin of the frame $\mathcal{F}_i$ expressed in the frame $\mathcal{F}_i$. q_i denotes the orientation of the frame $\mathcal{F}_i$ relative to the frame $\mathcal{F}_I$ in terms of unit quaternion. $\hat{\omega}_i^i$ denotes the dual velocity of the i -th spacecraft, given in the body-fixed frame $\mathcal{F}_i$, which is defined as

$$\hat{\omega}_i^i = \omega_i^i + \varepsilon(\dot{r}_i^i + \omega_i^i \times r_i^i) \quad (16)$$

where $\omega_i^i = [0, \bar{\omega}_i^i]$, $\bar{\omega}_i^i \in \mathbb{R}^3$ is the angular velocity of the i -th spacecraft expressed in the frame $\mathcal{F}_i$. $\hat{M}_i$ is the dual inertia matrix, which is defined as [11]

$$\begin{aligned} \hat{M}_i &= m_i \frac{d}{d\varepsilon} I_3 + \varepsilon J_i \\ &= \begin{bmatrix} m_i \frac{d}{d\varepsilon} + \varepsilon J_{i11} & \varepsilon J_{i12} & \varepsilon J_{i13} \\ \varepsilon J_{i21} & m_i \frac{d}{d\varepsilon} + \varepsilon J_{i22} & \varepsilon J_{i23} \\ \varepsilon J_{i31} & \varepsilon J_{i32} & m_i \frac{d}{d\varepsilon} + \varepsilon J_{i33} \end{bmatrix} \end{aligned} \quad (17)$$

where m_i and J_i are the mass and inertia matrix of the i -th spacecraft. The operator $\frac{d}{d\varepsilon}$ is defined by $\frac{d}{d\varepsilon} \hat{a} = \frac{d}{d\varepsilon} (a_r + \varepsilon a_d) = a_d$ and $(\frac{d}{d\varepsilon})^2 = 0$. I_3 is the identity in dimension 3. The inverse of $\hat{M}_i$ is defined as $\hat{M}_i^{-1} = J_i^{-1} \frac{d}{d\varepsilon} + \varepsilon \frac{1}{m_i} I_3$ [36].

The dual quaternion representation of the i -th spacecraft dynamics equation is given by [35]

$$\hat{M}_i \dot{\hat{\omega}}_i^i = \hat{F}_i^i - \hat{\omega}_i^i \times \hat{M}_i \hat{\omega}_i^i \quad (18)$$

For the case of gravitational wave detection in Earth orbit, the total dual force acting on the spacecraft will be decomposed as follows:

$$\hat{F}_i^i = \hat{f}_{gi}^i + \hat{f}_{srpi}^i + \hat{f}_{j2i}^i + \hat{f}_{\nabla gi}^i + \hat{f}_{di}^i + \hat{f}_{ui}^i \quad (19)$$

where $\hat{f}_{gi}^i = \hat{M}_i \hat{a}_{gi}^i$, $\hat{a}_{gi}^i = \mathbf{0} + \varepsilon[0, \bar{a}_{gi}^i]$, $\bar{a}_{gi}^i$ is the gravitational acceleration, including the Earth, Moon, and Sun, given by

$$\bar{a}_{gi}^i = -\frac{\mu_e \bar{r}_i^i}{\|\bar{r}_i^i\|^3} - \mu_m \left(\frac{\bar{r}_i^i - \bar{r}_m^i}{\|\bar{r}_i^i - \bar{r}_m^i\|^3} + \frac{\bar{r}_m^i}{\|\bar{r}_m^i\|^3} \right) - \mu_s \left(\frac{\bar{r}_i^i - \bar{r}_s^i}{\|\bar{r}_i^i - \bar{r}_s^i\|^3} + \frac{\bar{r}_s^i}{\|\bar{r}_s^i\|^3} \right) \quad (20)$$

where $\mu_e = 398,600.44190 \text{ km}^3/\text{s}^2$, $\mu_m = 4902.800076 \text{ km}^3/\text{s}^2$ and $\mu_s = 132,712,440,040.94400 \text{ km}^3/\text{s}^2$ are the gravitational parameter of the Earth, Moon, and Sun, respectively. $\bar{r}_m^i$ and $\bar{r}_s^i$ denote the position vector of the Moon and Sun relative to the Earth expressed in $\mathcal{F}_i$. $\hat{f}_{srpi}^i = \hat{M}_i \hat{a}_{srpi}^i$, $\hat{a}_{srpi}^i = \mathbf{0} + \varepsilon[0, \bar{a}_{srpi}^i]$, the acceleration $\bar{a}_{srpi}^i$ caused by solar radiation pressure can be approximately expressed as

$$\bar{a}_{srpi}^i = -P_{\odot} \frac{A}{m_i} \frac{r_{\odot}}{r_{\odot}^3} AU^2 (1 + \epsilon) \quad (21)$$

where $P_{\odot} = 4.56 \times 10^{-6} \text{ Nm}^2$ is the solar radiation pressure at 1AU (Astronomical Unit), A the occulted segment of the Sun, $\mathbf{r}_{\odot}$ the position vector from the Sun to the spacecraft, ϵ the reflectivity of the surface. $\hat{\mathbf{f}}_{J_2i}^i = \hat{\mathbf{M}}_i \hat{\mathbf{a}}_{J_2i}^i$, $\hat{\mathbf{a}}_{J_2i}^i = \mathbf{0} + \epsilon \mathbf{a}_{J_2i}^i$, $\mathbf{a}_{J_2i}^i = [0, \bar{\mathbf{a}}_{J_2i}^i] = \mathbf{q}_i^* \otimes \mathbf{a}_{J_2i}^I \otimes \mathbf{q}_i$, and $\mathbf{a}_{J_2i}^I = [0, \bar{\mathbf{a}}_{J_2i}^I]$, $\bar{\mathbf{a}}_{J_2i}^I$ is the perturbing acceleration due to Earth's oblateness given by

$$\bar{\mathbf{a}}_{J_2i}^I = -\frac{3}{2} \frac{\mu_e J_2 R_e^2}{\|\bar{\mathbf{r}}_i^I\|^5} \left(D - 5 \left(\frac{r_i^z}{\|\bar{\mathbf{r}}_i^I\|} \right)^2 \mathbf{I}_3 \right) \bar{\mathbf{r}}_i^I \quad (22)$$

where $R_e = 6378.137 \text{ km}$ is the Earth's mean equatorial radius, $J_2 = 0.0010826267$, $D = \text{diag}\{1, 1, 3\}$; $\bar{\mathbf{r}}_i^I = [r_i^x, r_i^y, r_i^z]^T$ represents the coordinates of $\bar{\mathbf{r}}_i$ expressed in the inertial coordinate system. The $\hat{\mathbf{f}}_{\nabla gi}^i$ is the dual force due to the gravity-gradient torque, defined as

$$\hat{\mathbf{f}}_{\nabla gi}^i = 3\mu_e \frac{\hat{\mathbf{r}}_i^i \times \hat{\mathbf{M}}_i \hat{\mathbf{r}}_i^i}{\|\mathbf{r}_i^i\|^5} \quad (23)$$

where $\hat{\mathbf{r}}_i^i = \mathbf{r}_i^i + \epsilon \mathbf{0}$, $\hat{\mathbf{f}}_{di}^i = \mathbf{f}_{di}^i + \epsilon \boldsymbol{\tau}_{di}^i$ and $\hat{\mathbf{f}}_{ui}^i = \mathbf{f}_{ui}^i + \epsilon \boldsymbol{\tau}_{ui}^i$ are the dual disturbance force and the dual control force, with the disturbance force $\mathbf{f}_{di}^i$, the disturbance torque $\boldsymbol{\tau}_{di}^i$, the control force $\mathbf{f}_{ui}^i$ and the control torque $\boldsymbol{\tau}_{ui}^i$ given in the body-fixed frame $\mathcal{F}_i$, respectively.

Under the dual quaternion algebra, the motion between the body-fixed frame and its desired frame can be expressed in the $\mathcal{F}_i$ as the relative dual quaternion described by

$$\hat{\mathbf{q}}_{ei} = \hat{\mathbf{q}}_{di}^* \otimes \hat{\mathbf{q}}_i = \mathbf{q}_{ei} + \epsilon \frac{1}{2} \mathbf{q}_{ei} \otimes \mathbf{r}_{ei}^i \quad (24)$$

where $\hat{\mathbf{q}}_{di}^*$ is the conjugate of $\hat{\mathbf{q}}_{di}$. $\hat{\mathbf{q}}_{di}$ denotes the dual quaternion of the frame $\mathcal{F}_i$ relative to the frame $\mathcal{F}_I$. $\mathbf{q}_{ei}$ denotes the orientation of the frame $\mathcal{F}_i$ relative to the frame $\mathcal{F}_{di}$ in terms of unit quaternion. $\mathbf{r}_{ei}^i$ is the relative position between the i -th spacecraft and its desired position, given in the $\mathcal{F}_i$. The relative kinematic and dynamic equations are given by

$$\dot{\hat{\mathbf{q}}}_{ei} = \frac{1}{2} \hat{\mathbf{q}}_{ei} \otimes \hat{\boldsymbol{\omega}}_{ei}^i \quad (25)$$

$$\hat{\mathbf{M}}_i \dot{\hat{\boldsymbol{\omega}}}_{ei}^i = \hat{\mathbf{F}}_i^i - \hat{\boldsymbol{\omega}}_i^i \times \hat{\mathbf{M}}_i \hat{\boldsymbol{\omega}}_i^i + \hat{\mathbf{M}}_i (\hat{\boldsymbol{\omega}}_{ei}^i \times \hat{\boldsymbol{\omega}}_{di}^i) - \hat{\mathbf{M}}_i (\hat{\mathbf{q}}_{ei}^* \otimes \dot{\hat{\mathbf{q}}}_{di}^i \otimes \hat{\mathbf{q}}_{ei}) \quad (26)$$

where $\hat{\boldsymbol{\omega}}_{ei}^i$ is the dual velocity between the $\mathcal{F}_i$ and $\mathcal{F}_{di}$, expressed in the $\mathcal{F}_i$. The kinematics and dynamic models of the desired i -th spacecraft are similar to the i -th spacecraft, which corresponds to (15), (16), and (18), where the notations ' $\bullet_i$ ' and ' $\bullet_i^i$ ' are replaced by ' $\bullet_{di}$ ' and ' $\bullet_{di}^{di}$ '. The total dual force applied to the desired i -th spacecraft is independent of the dual disturbance and dual control force, i.e., $\hat{\mathbf{F}}_{di}^{di} = \hat{\mathbf{f}}_{gdi}^{di} + \hat{\mathbf{f}}_{J_2di}^{di} + \hat{\mathbf{f}}_{\nabla gdi}^{di}$.

2.3. Control Objective

In this paper, $(\hat{\mathbf{q}}_{di}(t), \hat{\boldsymbol{\omega}}_{di}^{di}(t))$ denotes the desired state information of the i -th spacecraft to meet the requirements of gravitational wave detection. $(\hat{\mathbf{q}}_i(t), \hat{\boldsymbol{\omega}}_i^i(t))$ denotes the actual state information of the i -th spacecraft. The objective of this paper is to design an adaptive cooperation control scheme based on dual quaternions such that the state $(\hat{\mathbf{q}}_i(t), \hat{\boldsymbol{\omega}}_i^i(t))$ can track its desired state $(\hat{\mathbf{q}}_{di}(t), \hat{\boldsymbol{\omega}}_{di}^{di}(t))$ in the presence of parametric uncertainties, external disturbances, and time-varying communication delays. In other words, the error state $(\hat{\mathbf{q}}_{ei}(t), \hat{\boldsymbol{\omega}}_{ei}^i(t))$ can converge to an arbitrarily small neighborhood of the origin. That is, when $t \rightarrow \infty$,

$$\begin{aligned} \hat{\mathbf{q}}_{ei}(t) &\rightarrow \pm \hat{\mathbf{1}} \\ \hat{\boldsymbol{\omega}}_{ei}^i(t) &\rightarrow \hat{\mathbf{0}} \end{aligned} \quad (27)$$

where $\hat{\mathbf{1}} = \mathbf{1} + \epsilon \mathbf{0} \in \mathbb{DQ}$, $\hat{\mathbf{0}} = \mathbf{0} + \epsilon \mathbf{0} \in \mathbb{DQ}$, $\mathbf{1} = (1, \bar{\mathbf{0}}) \in \mathbb{H}$ and $\mathbf{0} = (0, \bar{\mathbf{0}}) \in \mathbb{H}$, respectively.

3. Control Law Design

In this section, a distributed coordinated formation control law is designed to solve the 6-DOF coordination control problem with the time-varying communication delays. Then, an adaptive controller is developed to provide the estimations of the parametric uncertainties and external disturbances.

3.1. 6-DOF Coordinated Control Law with Communication Delays

This subsection considers the time-varying communication delays between spacecraft, regardless of parametric uncertainties and external disturbances. An auxiliary state $\hat{s}_i$ is first defined as

$$\hat{s}_i = \hat{\omega}_{ei}^i + \hat{c} \odot \hat{p}_{ei}^i \quad (28)$$

where $\hat{c} = c_r + \varepsilon c_d$ with c_r and c_d are all positive constants. $\hat{p}_{ei}^i$ is defined as $\hat{p}_{ei}^i = \text{vec}(\mathbf{q}_{ei}) + \varepsilon \frac{1}{2} \mathbf{r}_{ei}^i$, $\text{vec}(\mathbf{q}_{ei})$ is the vector part of $\mathbf{q}_{ei}$. When the system dynamics are exactly known, a distributed coordinated controller with communication delays is proposed in the following form:

$$\hat{f}_{ui}^i = -\hat{k}_1 \odot (\hat{s}_i)^s - \hat{k}_2 \odot \sum_{j=1}^n a_{ij} (\hat{s}_i - \hat{s}_j(t - T_{ij}))^s + \hat{\mathbf{M}}_i \mathbf{Y}_i - \hat{f}_{di}^i \quad (29)$$

$$\begin{aligned} \mathbf{Y}_i = & \hat{\mathbf{M}}_i^{-1} (\hat{\omega}_i^i \times \hat{\mathbf{M}}_i \hat{\omega}_i^i) - \hat{\omega}_{ei}^i \times \hat{\omega}_{di}^i + \hat{\mathbf{q}}_{ei}^* \otimes \hat{\omega}_{di}^{di} \otimes \hat{\mathbf{q}}_{ei} - \hat{\mathbf{a}}_{gi}^i \\ & - \hat{\mathbf{a}}_{srpi}^i - \hat{\mathbf{a}}_{2i}^i - \hat{\mathbf{a}}_{\nabla gi}^i - \hat{c} \odot \hat{p}_{ei}^i \end{aligned} \quad (30)$$

where $\hat{k}_1 = k_{1r} + \varepsilon k_{1d}$ and $\hat{k}_2 = k_{2r} + \varepsilon k_{2d}$, with $k_{1r}, k_{1d}, k_{2r}, k_{2d}$ all being positive constants. It assumed that the communication topology between the i -th and the j -th spacecraft is undirected. Therefore, $a_{ij} = 1, i \neq j$. Otherwise, $a_{ij} = 0$. T_{ij} is the time-varying communication delay from the j -th to i -th spacecraft.

Theorem 1. Consider the relative kinematic and dynamic equations given by Equations (25) and (26), and the undirected communication graph is connected. If the time derivative of T_{ij} satisfies $\dot{T}_{ij} \leq 0$, the distributed coordinated formation control law in Equation (29) can ensure $\lim_{t \rightarrow \infty} (\hat{\mathbf{q}}_{ei}, \hat{\omega}_{ei}^i)(t) = (\pm \hat{\mathbf{1}}, \hat{\mathbf{0}})$ for all initial conditions.

Proof of Theorem 1. Consider the Lyapunov function candidate $V_1 = V_{1a} + V_{1b}$, where

$$V_{1a} = \frac{1}{2} \sum_{i=1}^n (\hat{s}_i)^s \odot (\hat{\mathbf{M}}_i \hat{s}_i) \quad (31)$$

$$V_{1b} = \frac{1}{2} \hat{k}_2 \odot \sum_{i=1}^n \sum_{j=1}^n a_{ij} \int_{t-T_{ij}}^t \hat{s}_j(\tau) \odot \hat{s}_j(\tau) d\tau \quad (32)$$

It can be verified that $V_1 \geq 0$ for all $\hat{s}_i$ and $V_1 = 0$ if and only if $\hat{s}_i = \hat{\mathbf{0}}$.

Taking the time derivative of V_{1a} and V_{1b} along the trajectories of the formation system (25) and (26), we can obtain

$$\dot{V}_{1a} = \sum_{i=1}^n (\hat{s}_i)^s \odot (\hat{\mathbf{M}}_i \dot{\hat{s}}_i) \quad (33)$$

$$\dot{V}_{1b} = \frac{1}{2} \hat{k}_2 \odot \sum_{i=1}^n \sum_{j=1}^n a_{ij} (\hat{s}_j(t) \odot \hat{s}_j(t) - (1 - \dot{T}_{ij}) \hat{s}_j(t - T_{ij}) \odot \hat{s}_j(t - T_{ij})) \quad (34)$$

By taking a derivative of (28), substituting it into (33) yields

$$\begin{aligned}\dot{V}_{1a} &= \sum_{i=1}^n (\hat{\mathbf{s}}_i)^s \circ \hat{\mathbf{M}}_i (\dot{\hat{\omega}}_{ei}^i + \hat{\mathbf{c}} \odot \dot{\hat{\mathbf{p}}}_{ei}^i) \\ &= \sum_{i=1}^n (\hat{\mathbf{s}}_i)^s \circ \left(-\hat{\mathbf{k}}_1 \odot (\hat{\mathbf{s}}_i)^s - \hat{\mathbf{k}}_2 \odot \sum_{j=1}^n a_{ij} (\hat{\mathbf{s}}_i - \hat{\mathbf{s}}_j(t - T_{ij}))^s \right)\end{aligned}\quad (35)$$

Then,

$$\begin{aligned}\dot{V}_1 &= \dot{V}_{1a} + \dot{V}_{1b} \\ &= \sum_{i=1}^n (\hat{\mathbf{s}}_i)^s \circ \left(-\hat{\mathbf{k}}_1 \odot (\hat{\mathbf{s}}_i)^s - \hat{\mathbf{k}}_2 \odot \sum_{j=1}^n a_{ij} (\hat{\mathbf{s}}_i - \hat{\mathbf{s}}_j(t - T_{ij}))^s \right) \\ &\quad + \frac{1}{2} \hat{\mathbf{k}}_2 \odot \sum_{i=1}^n \sum_{j=1}^n a_{ij} ((\hat{\mathbf{s}}_j)^s \circ (\hat{\mathbf{s}}_j)^s) \\ &\quad - \frac{(1 - \hat{T}_{ij})}{2} \hat{\mathbf{k}}_2 \odot \sum_{i=1}^n \sum_{j=1}^n a_{ij} ((\hat{\mathbf{s}}_j(t - T_{ij}))^s \circ (\hat{\mathbf{s}}_j(t - T_{ij}))^s)\end{aligned}\quad (36)$$

Note that the undirected topology is balanced, meaning that $\sum_{j=1}^n a_{ij} = \sum_{j=1}^n a_{ji}$ for $i = 1, \dots, n$; then, it follows that

$$\sum_{i=1}^n \sum_{j=1}^n a_{ij} \hat{\mathbf{s}}_i = \sum_{j=1}^n \sum_{i=1}^n a_{ji} \hat{\mathbf{s}}_i = \sum_{i=1}^n \sum_{j=1}^n a_{ij} \hat{\mathbf{s}}_j \quad (37)$$

Then,

$$\begin{aligned}\dot{V}_1 &= -\hat{\mathbf{k}}_1 \odot \sum_{i=1}^n (\hat{\mathbf{s}}_i)^s \circ (\hat{\mathbf{s}}_i)^s + \hat{T}_{ij} \hat{\mathbf{k}}_2 \odot \sum_{i=1}^n \sum_{j=1}^n a_{ij} (\hat{\mathbf{s}}_j(t - T_{ij}))^s \circ (\hat{\mathbf{s}}_j(t - T_{ij}))^s \\ &\quad - \hat{\mathbf{k}}_2 \odot \sum_{i=1}^n \sum_{j=1}^n a_{ij} (\hat{\mathbf{s}}_i - \hat{\mathbf{s}}_j(t - T_{ij}))^s \circ (\hat{\mathbf{s}}_i - \hat{\mathbf{s}}_j(t - T_{ij}))^s\end{aligned}\quad (38)$$

Therefore, if $\hat{T}_{ij} < 0$, it follows that $\dot{V}_1(t) \leq 0$. Since $V_1(t) \geq 0$ and $\dot{V}_1(t) \leq 0$, $\hat{\mathbf{s}}_i$ is bounded. According to Equation (28), the boundedness of $\hat{\mathbf{s}}_i$ means that $\dot{\hat{\mathbf{p}}}_{ei}^i$ and $\dot{\hat{\omega}}_{ei}^i$ are bounded. In addition, the boundedness of $\dot{\hat{\mathbf{p}}}_{ei}^i$ and $\dot{\hat{\omega}}_{ei}^i$ means that $\hat{\mathbf{f}}_{ui}^i$ and thus $\dot{\hat{\omega}}_{ei}^i$ are bounded. Hence, $\dot{\hat{\omega}}_{ei}^i$ and $\dot{V}_1$ are uniformly continuous. It follows from Barbalat's lemma that $\lim_{t \rightarrow \infty} \dot{V}_1(t) = 0$ and thus $\lim_{t \rightarrow \infty} \dot{\hat{\omega}}_{ei}^i(t) = \hat{\mathbf{0}}$. Furthermore, since $\ddot{\hat{\omega}}_{ei}^i$ is bounded and thus $\dot{\hat{\omega}}_{ei}^i$ is uniformly continuous, it follows from Barbalat's lemma that $\lim_{t \rightarrow \infty} \dot{\hat{\omega}}_{ei}^i(t) = \hat{\mathbf{0}}$ and thus $\lim_{t \rightarrow \infty} \dot{\hat{\mathbf{p}}}_{ei}^i(t) = \hat{\mathbf{0}}$, which implies $\lim_{t \rightarrow \infty} (\hat{\mathbf{q}}_{ei}, \dot{\hat{\omega}}_{ei}^i)(t) = (\pm \hat{\mathbf{1}}, \hat{\mathbf{0}})$. (Note that $\hat{\mathbf{q}}_{ei} = \hat{\mathbf{1}}$ and $\hat{\mathbf{q}}_{ei} = -\hat{\mathbf{1}}$ are the same pose.) Thus, the control objective is achieved. $\square$

3.2. Adaptive 6-DOF Coordinated Control Law with Communication Delays

In this subsection, let us consider the delayed 6-DOF coordination control problem with model and disturbances uncertainties and propose an adaptive coordinated controller in the following form:

$$\hat{\mathbf{f}}_{ui}^i = -\hat{\mathbf{k}}_1 \odot (\hat{\mathbf{s}}_i)^s - \hat{\mathbf{k}}_2 \odot \sum_{j=1}^n a_{ij} (\hat{\mathbf{s}}_i - \hat{\mathbf{s}}_j(t - T_{ij}))^s + \widetilde{\mathbf{M}}_i \mathbf{Y}_i - \widetilde{\mathbf{f}}_{di}^i \quad (39)$$

$$\begin{aligned}\mathbf{Y}_i &= \widetilde{\mathbf{M}}_i^{-1} (\dot{\hat{\omega}}_i^i \times \widetilde{\mathbf{M}}_i \dot{\hat{\omega}}_i^i) - \dot{\hat{\omega}}_{ei}^i \times \dot{\hat{\omega}}_{di}^i + \hat{\mathbf{q}}_{ei}^* \otimes \dot{\hat{\omega}}_{di}^{di} \otimes \hat{\mathbf{q}}_{ei} - \hat{\mathbf{a}}_{gi}^i \\ &\quad - \hat{\mathbf{a}}_{srpi}^i - \hat{\mathbf{a}}_{j2i}^i - \hat{\mathbf{a}}_{\nabla gi}^i - \hat{\mathbf{c}} \odot \dot{\hat{\mathbf{p}}}_{ei}^i\end{aligned}\quad (40)$$

where $\tilde{M}_i$ and $\tilde{f}_{di}^i$ are the estimation of $\hat{M}_i$ and $\hat{f}_{di}^i$, respectively. All gains are the same as the ones in the last subsection. To simplify notation, the following is introduced as

$$\hat{a} \circ (\hat{M}_i \hat{b}) = h^T(\hat{a}, \hat{b}) \Gamma(\hat{M}_i) \quad (41)$$

where $\hat{a} = a_r + \varepsilon a_d$, $\hat{b} = b_r + \varepsilon b_d$ are dual quaternions, with $a_r = [a_{r0}, a_{r1}, a_{r2}, a_{r3}]$, $a_d = [a_{d0}, a_{d1}, a_{d2}, a_{d3}]$, $b_r = [b_{r0}, b_{r1}, b_{r2}, b_{r3}]$, $b_d = [b_{d0}, b_{d1}, b_{d2}, b_{d3}]$. The function h is defined as $h(\hat{a}, \hat{b}) = [a_{d1}b_{r1}, a_{d2}b_{r1} + a_{d1}b_{r2}, a_{d3}b_{r1} + a_{d1}b_{r3}, a_{d2}b_{r2} + a_{d1}b_{r3}, a_{d3}b_{r2} + a_{d2}b_{r3}, a_{d3}b_{r3}, a_{r1}b_{d1} + a_{r2}b_{d2} + a_{r3}b_{d3}]^T$, and $\Gamma(\hat{M}_i) = [J_{i11}, J_{i12}, J_{i13}, J_{i22}, J_{i23}, J_{i33}, m_i]^T$. The updating law for $\tilde{M}_i$ and $\tilde{f}_{di}^i$ can be designed as

$$\begin{aligned} \frac{d}{dt} \Gamma(\tilde{M}_i) = & W_M [h((\hat{s}_i)^s, \hat{\omega}_{ei}^i \times \hat{\omega}_{di}^i - \hat{q}_{ei}^* \otimes \hat{\omega}_{di}^{di} \otimes \hat{q}_{ei} + \hat{a}_{gi}^i + \hat{a}_{srpi}^i + \hat{a}_{2i}^i + \hat{c} \odot \hat{p}_{ei}^i) \\ & - h((\hat{s}_i \times \hat{\omega}_i^i)^s, \hat{\omega}_i^i) + h((\hat{s}_i \times \frac{3\mu_e \hat{r}_i^i}{\|\hat{r}_i^i\|^5})^s, \hat{r}_i^i)] \end{aligned} \quad (42)$$

$$\frac{d}{dt} \tilde{f}_{di}^i = \hat{W}_d (\hat{s}_i)^s \quad (43)$$

where $W_M \in \mathbb{R}^{7 \times 7}$ is a positive definite matrix. $\hat{W}_d = W_{d2} \frac{d}{d\varepsilon} + \varepsilon W_{d1}$ with W_{d1} and $W_{d2} \in \mathbb{R}^{3 \times 3}$ being positive definite matrices.

Theorem 2. Consider the relative kinematic and dynamic equations given by Equations (25) and (26), and the undirected communication graph is connected. If the time derivative of T_{ij} satisfies $\dot{T}_{ij} \leq 0$, the distributed coordinated formation control law in Equations (39) and (40), with the adaptive law (42) and (43) can ensure $\lim_{t \rightarrow \infty} (\hat{q}_{ei}, \hat{\omega}_{ei}^i)(t) = (\pm \hat{1}, \hat{0})$ for all initial conditions.

Proof of Theorem 2. The dual inertia matrix and dual disturbance force estimation errors are defined as $\Delta \hat{M}_i = \tilde{M}_i - \hat{M}_i$ and $\Delta \hat{f}_{di}^i = \tilde{f}_{di}^i - \hat{f}_{di}^i$, respectively. Consider a Lyapunov function candidate as $V_2 = V_{2a} + V_{2b} + V_{2c}$, where

$$\begin{aligned} V_{2a} &= \frac{1}{2} \sum_{i=1}^n (\hat{s}_i)^s \circ (\hat{M}_i \hat{s}_i) \\ V_{2b} &= \frac{1}{2} \hat{k}_2 \odot \sum_{i=1}^n \sum_{j=1}^n a_{ij} \int_{t-T_{ij}}^t (\hat{s}_j(\tau))^s \circ (\hat{s}_j(\tau))^s d\tau \\ V_{2c} &= \frac{1}{2} \sum_{i=1}^n \Gamma^T(\Delta \hat{M}_i) W_M^{-1} \Gamma(\Delta \hat{M}_i) + \frac{1}{2} \sum_{i=1}^n \Delta \hat{f}_{di}^i \circ (\hat{W}_d^{-1} \Delta \hat{f}_{di}^i) \end{aligned} \quad (44)$$

V_2 is a valid candidate Lyapunov function since $V_2 \geq 0$ for all $\hat{s}_i$, $\Gamma^T(\Delta \hat{M}_i)$ and $\Delta \hat{f}_{di}^i$; $V_2 = 0$ if and only if $\hat{s}_i = \hat{0}$, $\Gamma^T(\Delta \hat{M}_i) = 0_{7 \times 1}$ and $\Delta \hat{f}_{di}^i = \hat{0}$.

By taking the derivative of Formula (44), we can arrive at

$$\begin{aligned} \dot{V}_{2a} &= \sum_{i=1}^n (\hat{s}_i)^s \circ (\dot{\hat{M}}_i \hat{s}_i) \\ &= \sum_{i=1}^n (\hat{s}_i)^s \circ (-\hat{k}_1 \odot (\hat{s}_i)^s - \hat{k}_2 \odot \sum_{j=1}^n a_{ij} (\hat{s}_i - \hat{s}_j(t - T_{ij}))^s) \\ &\quad - \sum_{i=1}^n \Gamma^T(\Delta \hat{M}_i) W_M^{-1} \dot{\Gamma}(\Delta \hat{M}_i) - \sum_{i=1}^n \Delta \hat{f}_{di}^i \circ (\hat{W}_d^{-1} \dot{\Delta \hat{f}}_{di}^i) \end{aligned} \quad (45)$$

$$\begin{aligned} \dot{V}_{2b} &= \frac{1}{2} \hat{k}_2 \odot \sum_{i=1}^n \sum_{j=1}^n a_{ij} ((\hat{s}_j)^s \circ (\hat{s}_j)^s \\ &\quad - (1 - \dot{T}_{ij}) (\hat{s}_j(t - T_{ij}))^s \circ (\hat{s}_j(t - T_{ij}))^s) \end{aligned} \quad (46)$$

$$\dot{V}_{2c} = \sum_{i=1}^n \mathbf{r}^T (\Delta \hat{\mathbf{M}}_i) \mathbf{W}_M^{-1} \dot{\mathbf{r}} (\Delta \hat{\mathbf{M}}_i) + \sum_{i=1}^n \Delta \hat{\mathbf{f}}_{di}^i \circ (\hat{\mathbf{W}}_d^{-1} \Delta \hat{\mathbf{f}}_{di}^i) \quad (47)$$

Then,

$$\begin{aligned} \dot{V}_2 &= \dot{V}_{a2} + \dot{V}_{b2} + \dot{V}_{c2} \\ &= \sum_{i=1}^n (\hat{\mathbf{s}}_i)^s \circ (-\hat{\mathbf{k}}_1 \odot (\hat{\mathbf{s}}_i)^s - \hat{\mathbf{k}}_2 \odot \sum_{j=1}^n a_{ij} (\hat{\mathbf{s}}_i - \hat{\mathbf{s}}_j(t - T_{ij}))^s) \\ &\quad + \frac{1}{2} \hat{\mathbf{k}}_2 \odot \sum_{i=1}^n \sum_{j=1}^n a_{ij} ((\hat{\mathbf{s}}_j)^s \circ (\hat{\mathbf{s}}_i)^s) \\ &\quad - \frac{(1 - \dot{T}_{ij})}{2} \hat{\mathbf{k}}_2 \odot \sum_{i=1}^n \sum_{j=1}^n a_{ij} ((\hat{\mathbf{s}}_j(t - T_{ij}))^s \circ (\hat{\mathbf{s}}_j(t - T_{ij}))^s) \\ &\leq -\hat{\mathbf{k}}_1 \odot \sum_{i=1}^n (\hat{\mathbf{s}}_i)^s \circ (\hat{\mathbf{s}}_i)^s + \dot{T}_{ij} \hat{\mathbf{k}}_2 \odot \sum_{i=1}^n \sum_{j=1}^n a_{ij} ((\hat{\mathbf{s}}_j(t - T_{ij}))^s \circ (\hat{\mathbf{s}}_j(t - T_{ij}))^s) \end{aligned} \quad (48)$$

Therefore, if $\dot{T}_{ij} < 0$, it follows that $\dot{V}_2(t) \leq 0$. Since $V_2(t) \geq 0$ and $\dot{V}_2(t) \leq 0$, $\hat{\mathbf{s}}_i$, $\mathbf{r}(\hat{\mathbf{M}}_i)$, and $\hat{\mathbf{f}}_{di}^i$ are bounded. Then, based on the similar analysis and proof in the previous section, it can be concluded that $\lim_{t \rightarrow \infty} (\hat{\mathbf{q}}_{ei}, \hat{\boldsymbol{\omega}}_{ei}^i)(t) = (\pm \hat{\mathbf{1}}, \hat{\mathbf{0}})$. We complete the proof. $\square$

Remark 1. The proposed adaptive law (42) and (43) can only guarantee that $\Delta \hat{\mathbf{M}}_i$ and $\Delta \hat{\mathbf{f}}_{di}^i$ are bounded. The estimates of the dual inertia matrix and the external disturbance will not be guaranteed to converge to their actual values.

Remark 2. It is worth noting that the dynamic model and controller analysis in this paper is based on continuous time. It is necessary to discretize the controller in the process of practical engineering implementation.

4. Numerical Simulations

This section applies the proposed controller to the earth-centered orbital space gravitational wave detection system. It requires that the variation of the formation arm length (the side length of the triangle) is less than 1%, the relative speed is less than 5 m/s, and the breathing angle (the inner angle of the triangle) is less than 0.1° [37]. When the position error, velocity error, and attitude error between the actual state and the desired state of the spacecraft should be less than 5 m, 2 mm/s, and 1 mrad, respectively, it can meet the requirements of the gravitational wave detection mission.

The inertia matrix and masses of the spacecraft are assumed to be

$$J = \begin{bmatrix} 162.5 & 3 & 2 \\ 3 & 162.5 & 2.5 \\ 2 & 2.5 & 325 \end{bmatrix} \text{ kg} \cdot \text{m}^2 \quad (49)$$

and $m = 650$ kg, respectively. The desired orbit parameters of SC1~3 are shown in Table 1, assuming that the three spacecraft form an equilateral triangle. The desired attitude and the desired angular velocity are ground orientation and orbital angular velocity, respectively. The initial position errors $\mathbf{r}_{ei} = [r_{eix}, r_{eiy}, r_{eiz}]^T$, velocity errors $\dot{\mathbf{r}}_{ei} = [\dot{r}_{eix}, \dot{r}_{eiy}, \dot{r}_{eiz}]^T$, angular velocity errors $\boldsymbol{\omega}_{ei} = [\omega_{eix}, \omega_{eiy}, \omega_{eiz}]^T$, and attitude errors $\boldsymbol{\theta}_{ei} = [\theta_{eix}, \theta_{eiy}, \theta_{eiz}]^T$ ($i = 1, 2, 3$) for each spacecraft are presented in Table 2. Note that we use the Euler angle rather than quaternion to describe the attitude in the simulation, which is easier to understand. The communication time delay between the neighboring spacecraft is supposed to be $T_{ij} = 0.6 - 0.1 \times |\sin(0.01t)|$ s. In this paper, the maximum control forces and torques are set to 10^{-3} N and 10^{-4} N · m in each axis, respectively. The minimum impulse bit is set to 10^{-7} N. The position and linear velocity measurement accuracy are assumed to be

0.1 m and 10^{-6} m/s, respectively. The attitude and angular velocity measurement accuracy are assumed to be 10^{-6} rad and 10^{-7} rad/s, respectively. Those measurement errors are assumed to be normally distributed.

Table 1. Desired orbital parameters.

Parameter	Value	Unit
Perigee altitude	9.999×10^7	m
Eccentricity	0.00043	-
Inclination	74.5362	deg
Argument of perigee	346.5528	deg
RAAN	211.6003	deg
True anomaly (SC1)	61.3296	deg
True anomaly (SC2)	181.3296	deg
True anomaly (SC3)	301.3296	deg

Table 2. Initial conditions.

	Initial Position Error (m)	Initial Velocity Error (m · s ⁻¹)	Initial Angular Velocity Error (rad · s ⁻¹)	Initial Attitude Error (rad)
SC1	$[-60 \ 80 \ -100]^T$	$[1.2 \ -0.22 \ 0.57]^T \times 10^{-3}$	$[0.8 \ -2 \ 1]^T \times 10^{-5}$	$[0.8727 \ -0.5236 \ 0.3491]$
SC2	$[160 \ 100 \ -40]^T$	$[1.2 \ -3.5 \ -3.9]^T \times 10^{-3}$	$[0.7 \ -2 \ 2]^T \times 10^{-5}$	$[-0.3491 \ 0.8727 \ 1.0472]$
SC3	$[-80 \ 120 \ 100]^T$	$[2.2 \ 1.7 \ -0.29]^T \times 10^{-3}$	$[0.9 \ -1 \ 1]^T \times 10^{-5}$	$[0.5236 \ -0.8727 \ 0.8727]$

4.1. 6-DOF Coordinated Control Law with Communication Delays

Using a trial and error procedure, we select the gains for the controller (29) as $k_{1d} = 0.05$, $k_{1r} = 0.06$, $k_{2d} = 0.001$, $k_{2r} = 0.001$, $c_r = 0.01$, and $c_d = 0.035$. Under the designed 6-DOF coordination control law (29), simulation results are presented in Figures 1–3, which validate the stability analysis of the proposed control schemes.

Figure 1 shows the time histories of the position errors and linear velocity errors of each spacecraft with communication delays, respectively. It can be seen that the position errors and linear velocity errors converge to the region $|r_{eiw}| < 2.5$ m and $|\dot{r}_{eiw}| < 2 \times 10^{-4}$ m/s, ($w = x, y, z$). The performance of position tracking and linear velocity tracking during the transient phase and the final accuracy is acceptable. Figure 2 shows the time histories of attitude errors and angular velocity errors of each spacecraft with communication delays, respectively. It can be observed that attitude errors and angular velocity errors converge to the region $|\theta_{eiw}| < 5 \times 10^{-4}$ rad and $|\omega_{eiw}| < 5 \times 10^{-7}$ rad/s. Figure 3 shows the control forces and control torques of each spacecraft.

As shown in Figures 1 and 2, it can be seen that the convergence time is about 18 h and 12 h for the translation and rotation, respectively. These two figures indicate that the errors of the relative position and attitude could rapidly converge and satisfy the accuracy requirements when the time-varying communication delay is considered.

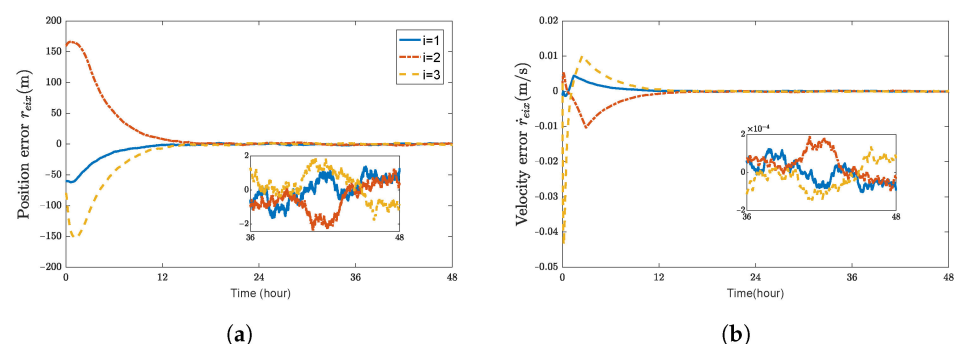


Figure 1. Cont.

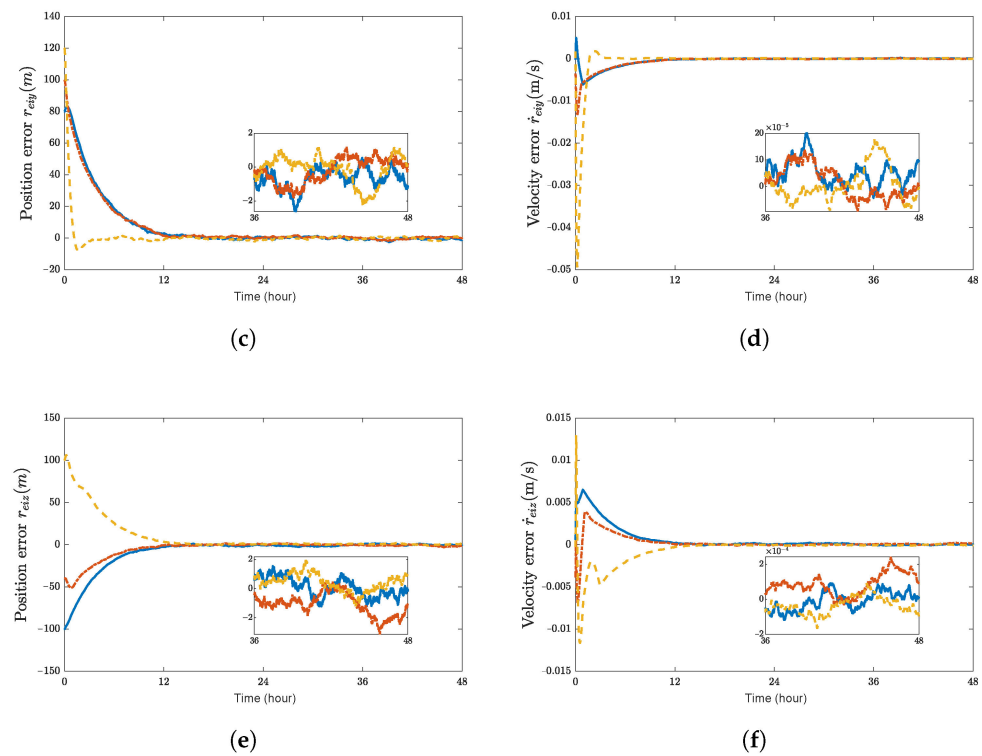


Figure 1. Position errors and velocity errors of spacecraft SC1~3. (a) position error r_{eix} (m); (b) velocity error $\dot{r}_{eix}$ (m/s); (c) position error r_{eiy} (m); (d) velocity error $\dot{r}_{eiy}$ (m/s); (e) position error r_{eiz} (m); (f) velocity error $\dot{r}_{eiz}$ (m/s).

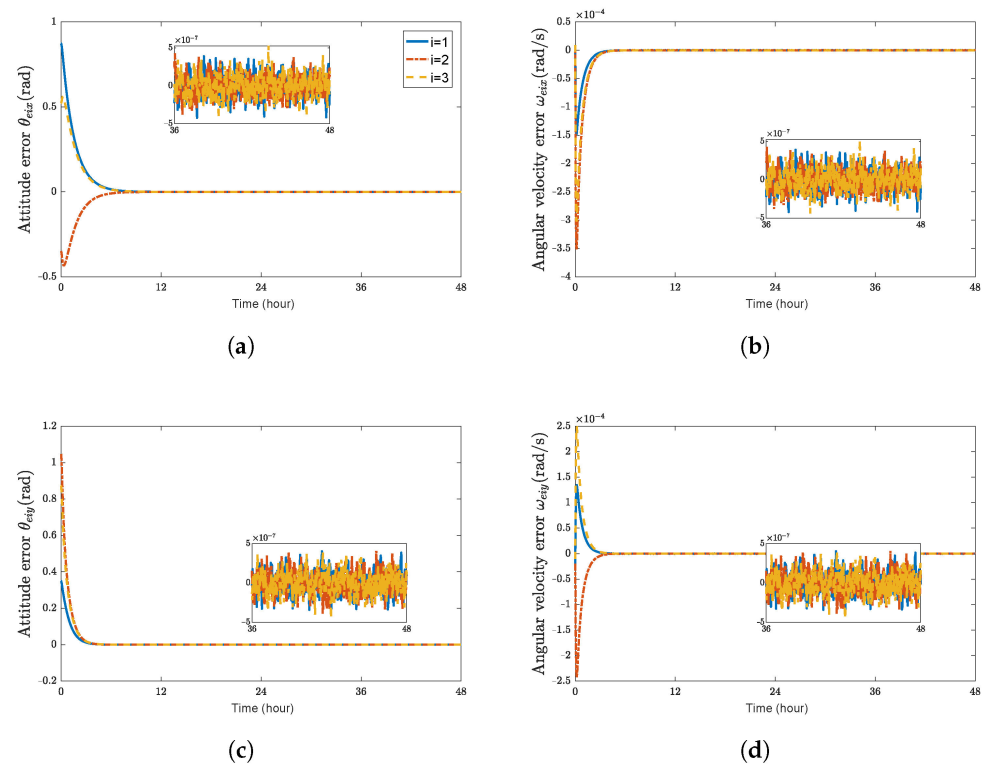


Figure 2. Cont.

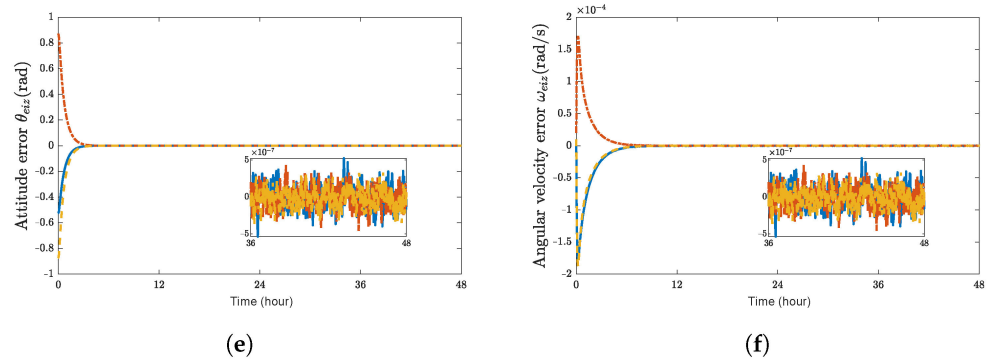


Figure 2. Attitude errors and angular velocity errors of spacecraft SC1~3. (a) attitude error θ_{eix} (rad); (b) angular velocity error ω_{eix} (rad/s); (c) attitude error θ_{eiy} (rad); (d) angular velocity error ω_{eiy} (rad/s); (e) attitude error θ_{eiz} (rad); (f) angular velocity error ω_{eiz} (rad/s).

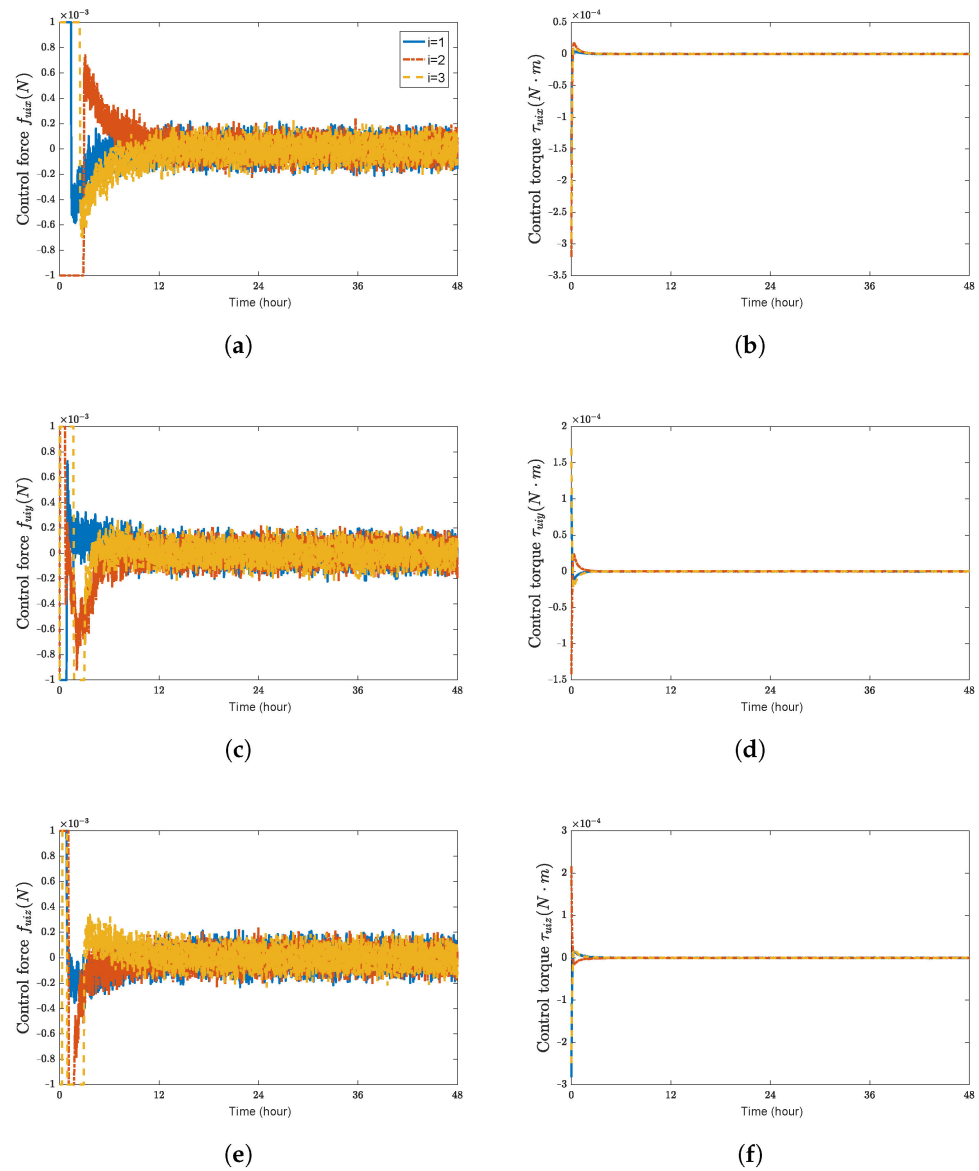


Figure 3. Control forces and control torques of spacecraft SC1~3. (a) control force f_{uix} (N); (b) control torque τ_{uix} (N · m); (c) control force f_{uiy} (N); (d) control torque τ_{uiy} (N · m); (e) control force f_{uiz} (N); (f) control torque τ_{uiz} (N · m).

4.2. Adaptive 6-DOF Coordinated Control Law with Communication Delays

Considering the 6-DOF coordination adaptive control law (39), (40), (42), and (43), selecting the adaptive gain parameter as $W_M = 10 \times \text{diag}\{1, 1, 1, 1, 1, 1\}$, $W_{d1} = 10^{-5} \mathbf{I}_3$, $W_{d2} = 2 \times 10^{-2} \mathbf{I}_3$. The initial values of the estimated variables are set to $\tilde{\mathbf{f}}_{di}^i(0) = \hat{\mathbf{0}}$ and $\Gamma(\hat{\mathbf{M}}_i)(0) = [160, 0, 0, 160, 0, 320, 640]^T$. Other parameters for control law (39) remain the same as those in the previous section. The simulation figures are given in Figures 4–8.

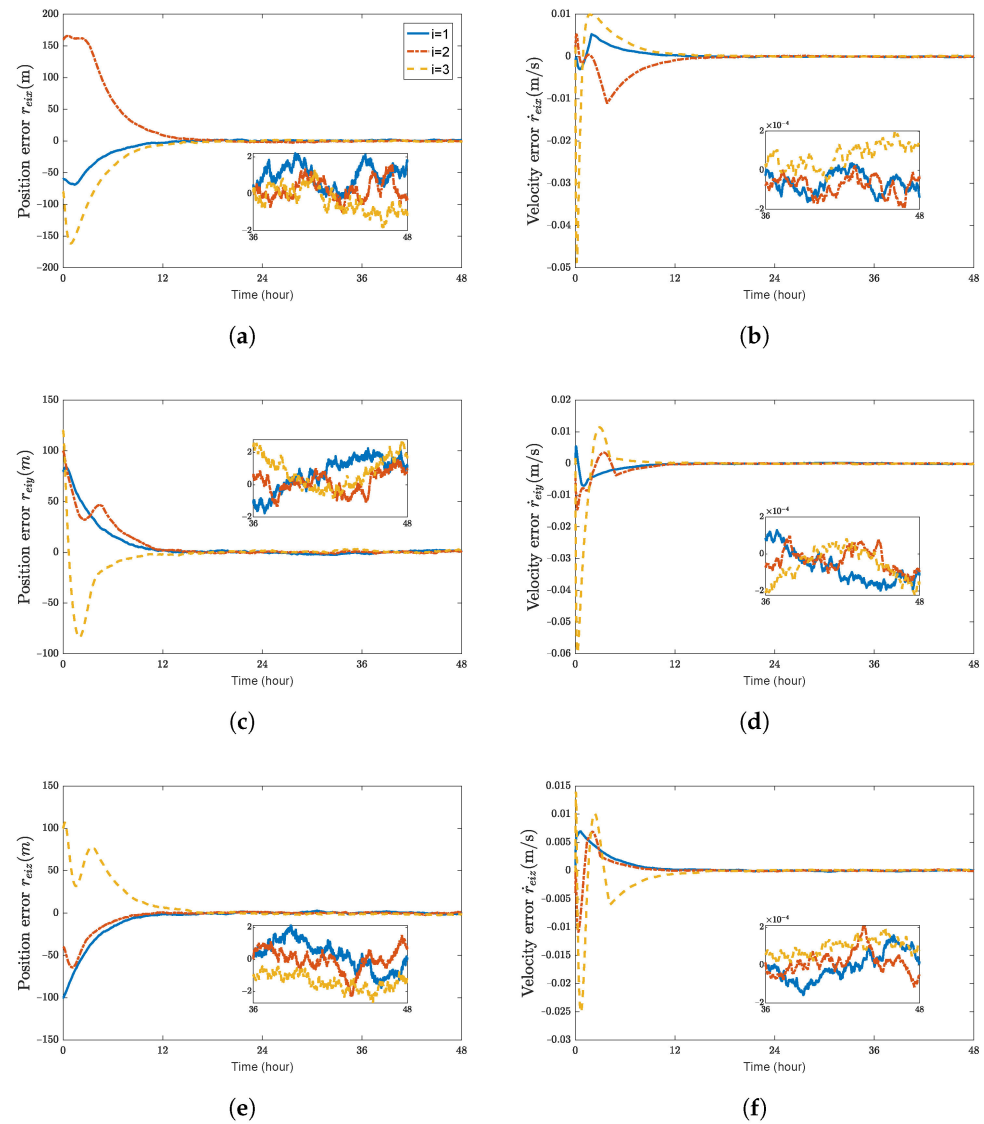


Figure 4. Position errors and velocity errors of spacecraft SC1~3 with adaptive control law. (a) position error r_{eix} (m); (b) velocity error $\dot{r}_{eix}$ (m/s); (c) position error r_{eiy} (m); (d) velocity error $\dot{r}_{eiy}$ (m/s); (e) position error r_{eiz} (m); (f) velocity error $\dot{r}_{eiz}$ (m/s).

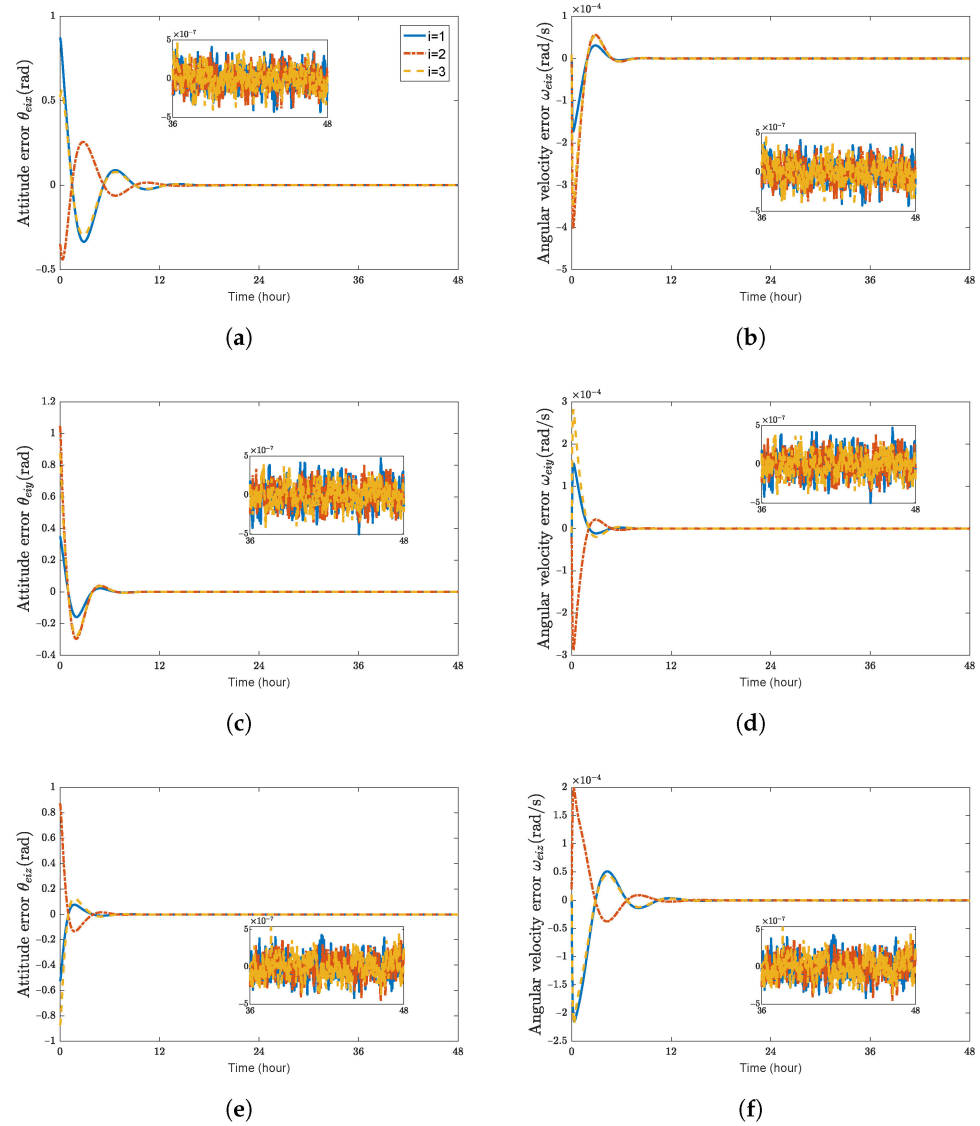


Figure 5. Attitude errors and angular velocity errors of spacecraft SC1~3 with adaptive control law. (a) attitude error θ_{eix} (rad); (b) angular velocity error ω_{eix} (rad/s); (c) attitude error θ_{eiy} (rad); (d) angular velocity error ω_{eiy} (rad/s); (e) attitude error θ_{eiz} (rad); (f) angular velocity error ω_{eiz} (rad/s).

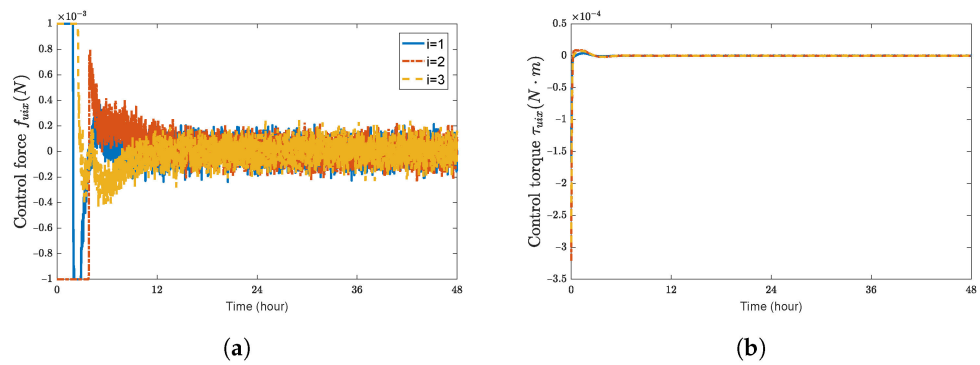


Figure 6. Cont.

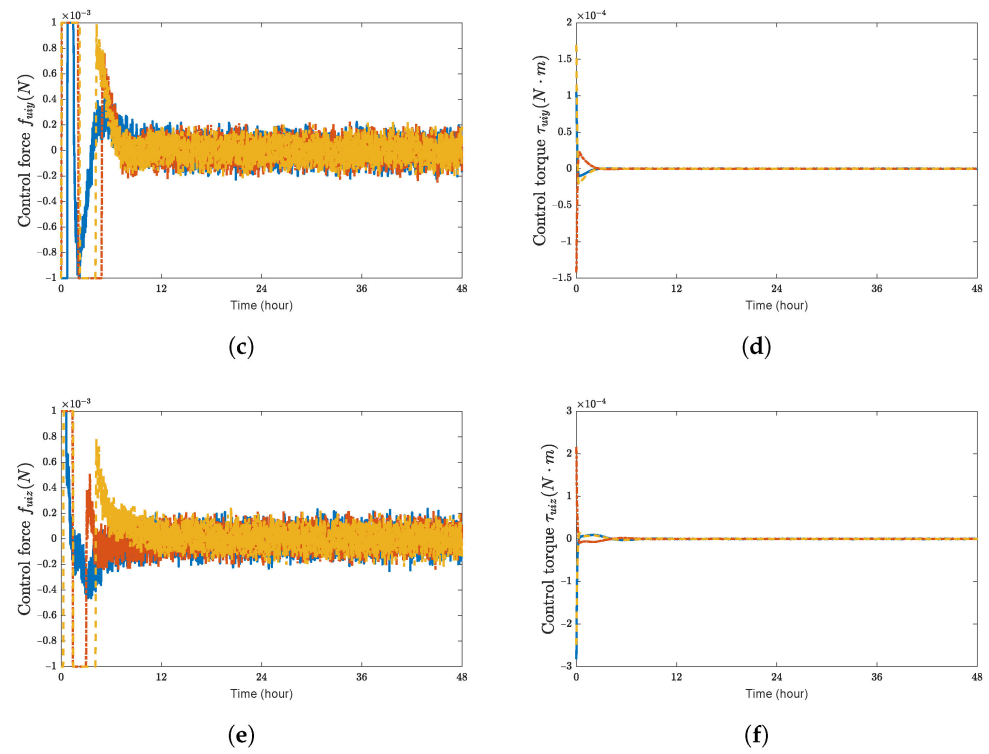


Figure 6. Control forces and control torques of spacecraft SC1~3 with adaptive control law. (a) control force f_{uix} (N); (b) control torque τ_{uix} (N·m); (c) control force f_{uiy} (N); (d) control torque τ_{uiy} (N·m); (e) control force f_{uiz} (N); (f) control torque τ_{uiz} (N·m).

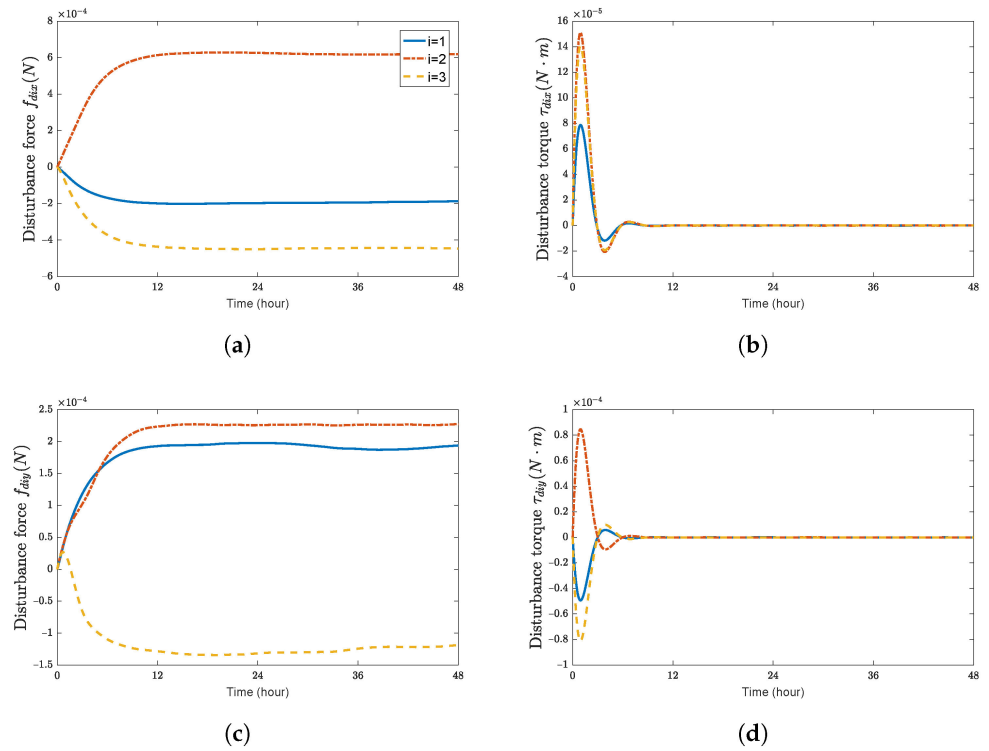


Figure 7. Cont.

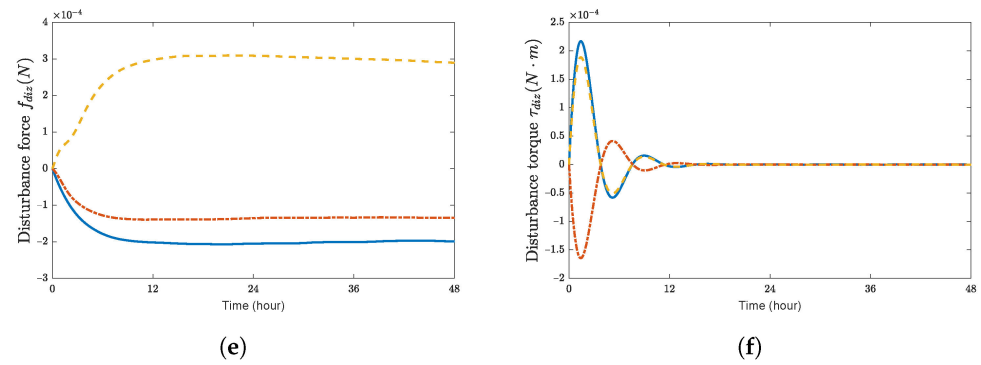


Figure 7. Disturbance forces and disturbance torques of spacecraft SC1~3 with adaptive control law. (a) disturbance force f_{dix} (N); (b) disturbance torque τ_{dix} (N·m); (c) disturbance force f_{diy} (N); (d) disturbance torque τ_{diy} (N·m); (e) disturbance force f_{diz} (N); (f) disturbance torque τ_{diz} (N·m).

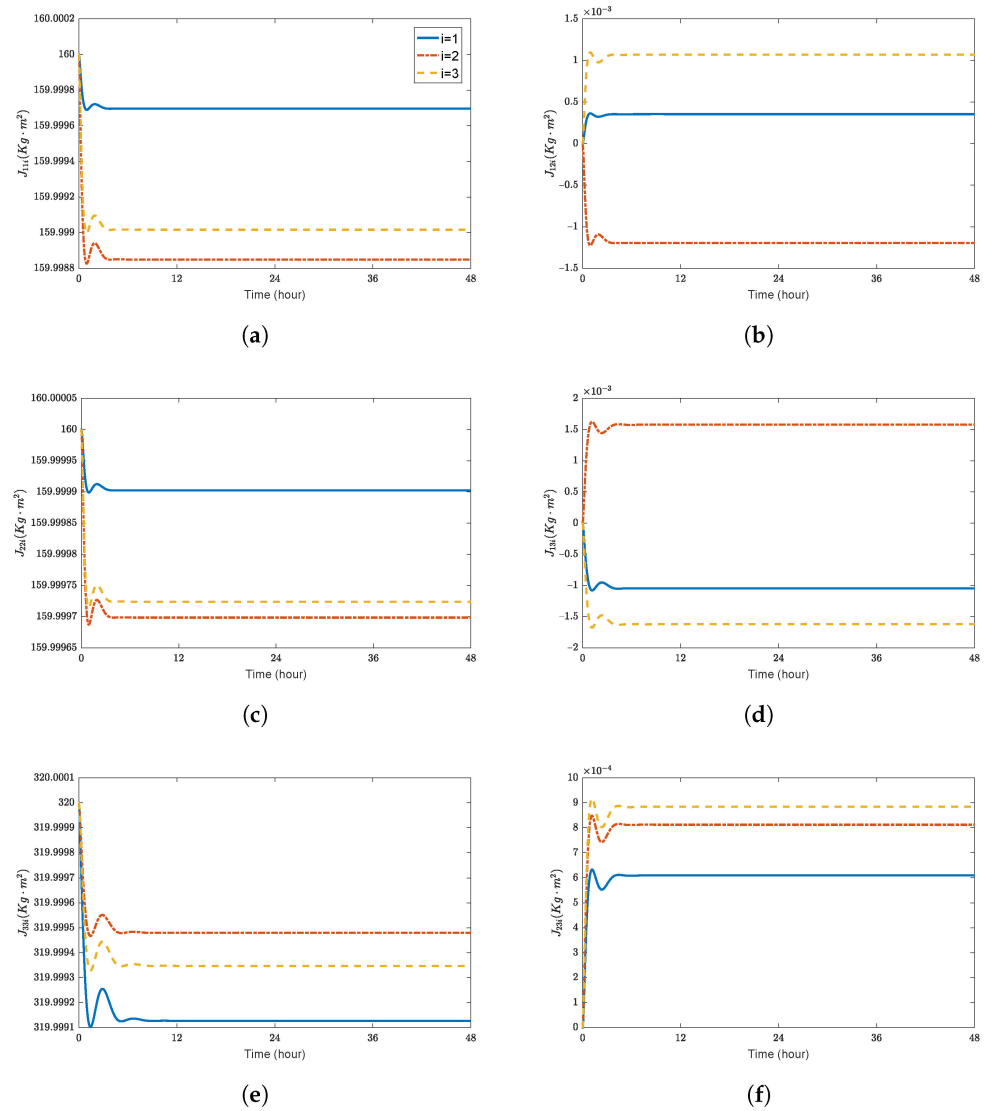


Figure 8. Cont.

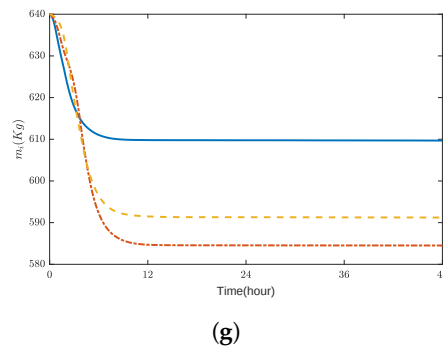


Figure 8. Estimation of the inertia matrix and mass under the proposed adaptive controller. (a) J_{11i} (Kg · m²); (b) J_{12i} (Kg · m²); (c) J_{22i} (Kg · m²); (d) J_{13i} (Kg · m²); (e) J_{33i} (Kg · m²); (f) J_{23i} (Kg · m²); (g) m_i (Kg).

Figures 4 and 5 show the time histories of the position errors, linear velocity errors, attitude errors, and angular velocity errors of each spacecraft with communication delays, model uncertainties, and external disturbances, respectively. It can be seen that the spacecraft can asymptotically track their desired positions, and the tracking error can converge to the region $|r_{eiv}| < 2.5$ m, $|\dot{r}_{eiv}| < 2 \times 10^{-4}$ m/s, $|\theta_{eiv}| < 3 \times 10^{-4}$ rad, and $|\omega_{eiv}| < 5 \times 10^{-7}$ rad/s, ($w = x, y, z$).

Figure 6 shows the control forces and control torques of each spacecraft, respectively. The estimation of the external disturbances, the inertia matrix, and the mass under the proposed adaptive controller are shown in Figures 7 and 8. Although the updating laws given by (42) and (43) do not converge to the actual values of the spacecraft, the asymptotic convergence of the position errors and the attitude errors are still guaranteed.

The transient response of the control law (39) is less smooth than that of the control law (29), which does not consider parametric uncertainties and external interference. This is because it takes time for the updating law (42) and (43) to adjust the estimations of the dual inertia and external disturbances to achieve a fine compensation. However, the settling time for the two controllers is identical, and the accuracy of the relative position errors and attitude errors are the same, which can meet the requirements of gravitational wave detection for the initial pose error.

5. Conclusions

This paper has investigated the attitude and orbit coupled tracking control problem for multiple spacecraft formation. A distributed 6-DOF coordinated control law based on dual quaternions has been designed with time-varying communication delays. Moreover, an adaptive control law has been further developed by consideration of parametric uncertainties and external disturbances, where the asymptotic stability of the closed-loop system is guaranteed. Numerical simulation results show that the controller can realize the coordination of relative orbit and attitude, and make the formation configuration meet the requirements of space gravitational wave detection. In future work, the distributed attitude-orbit coordinated control with velocity-free could be studied.

Author Contributions: Conceptualization, Y.Z., Y.L., J.Z., Z.L. and J.Y.; Methodology, Y.Z.; Software, Y.Z. and J.Z.; Validation, Y.Z., J.Z., Z.L. and J.Y.; Writing—original draft, Y.Z.; Writing—review and editing, Y.Z. and Y.L. All authors have read and agreed to the published version of the manuscript.

Funding: This work is supported partially by the Guangdong Major Project of Basic and Applied Basic Research (Grant No. 2019B030302001) and National Key R&D Program of China (Grant No. 2022YFB3902800). The authors would like to thank the reviewers and the editor for their comments and constructive suggestions that helped to improve the paper significantly.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The authors would like to thank the editors and reviewers for their constructive comments that improve the paper.

Conflicts of Interest: The authors declare no conflict of interest.

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