Supplementary Materials: Classification of Sonar Targets in Air – A Neural Network Approach

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1. Target Strength Calculations

1.1. Total Target Strength

Mathematical derivations are shown here for the target strength estimates in the manuscript.

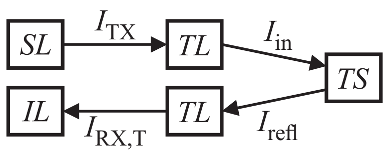

Figure S1. Acoustic intensities along transmission path; SL source level, I_{TX} transmitter intensity, TL transmission loss, I_{in} target input intensity, TS target strength, I_{refl} reflected target intensity, $I_{\mathrm{RX,T}}$ input intensity from target and IL input level.

In analogy to the sonar equation as well as from Figure S1, sonar signal levels are given as

$$IL = SL + TS - 2TL \quad \text{in dB} \quad , \tag{1}$$

$$TS = 10\,\log_{10}\left(\frac{I_{\mathrm{refl}}}{I_{\mathrm{in}}}\right) \quad , \tag{2}$$

$$IL = 10\,\log_{10}\left(\frac{I_{\mathrm{RX,T}}}{I_0}\right) \quad , \tag{3}$$

$$SL = 10\,\log_{10}\left(\frac{I_{\mathrm{TX}}}{I_0}\right) \tag{4}$$

$$\text{and} \quad TL = 10\,\log_{10}\left(\frac{I_{\mathrm{in}}}{I_{\mathrm{TX}}}\right) = 10\,\log_{10}\left(\frac{I_{\mathrm{RX}}}{I_{\mathrm{refl}}}\right) \quad , \tag{5}$$

with
IL: input level at receiver,
SL: source level at 1 m from transmitter,
TS: target strength,
TL: transmission loss,
I_{refl}: acoustic intensity after reflection from target at 1 m distance from target, $[I_{\mathrm{refl}}] = 1\,\mathrm{W\,m^{-2}}$,
I_{in}: acoustic intensity at target position, $[I_{\mathrm{in}}] = 1\,\mathrm{W\,m^{-2}}$,
$I_{\mathrm{RX,T}}$: acoustic intensity at receiver due to target echo, $[I_{\mathrm{RX}}] = 1\,\mathrm{W\,m^{-2}}$,
I_0: reference acoustic intensity, $I_0 = 1 \times 10^{-12}\,\mathrm{W\,m^{-2}}$,
I_{TX}: acoustic intensity at 1 m from transmitter, $[I_{\mathrm{TX}}] = 1\,\mathrm{W\,m^{-2}}$,
All acoustic intensities are RMS values.

Since only geometric spreading is considered for TL, there is no dependency of frequency

$$TL = 20\,\log_{10}\left(\frac{R}{1\,\mathrm{m}}\right) = 20\,\log_{10}\left(\frac{t_{\mathrm{T}}\,c}{2\cdot 1\,\mathrm{m}}\right) \quad , \tag{6}$$

with
R: sound propagation distance (between speaker/microphone and target),
t_{T}: time delay for echo from target in s,

c: speed of sound, $343\,\mathrm{m\,s^{-1}}$ in air.

The relation between acoustic intensity and a pressure signal's auto-correlation function is based on a plane wave assumption (due to small echo wave surface curvature across microphone area) and is given as

$$I = \frac{1}{Z_0}\, r^{\mathrm{T}}_{\mathrm{aa}}(0) \quad , \tag{7}$$

with
I: acoustic intensity RMS in $\mathrm{W\,m^{-2}}$,
Z_0: characteristic specific acoustic impedance in $\mathrm{Pa\,s\,m^{-1}}$,
$r^{\mathrm{T}}_{\mathrm{aa}}(0)$: central peak of short time ACF of pressure signal in Pa^2.

We used the following definition for a short time XCF $r_{\mathrm{ab}}(\tau)$ of two arbitrary pressure signals p_{a} and p_{b}

$$r^{\mathrm{T}}_{\mathrm{ab}}(\tau) = \frac{1}{2\,T_{\mathrm{ref}}} \int_{-\infty}^{+\infty} p_{\mathrm{a}}(t+\tau)\, p_{\mathrm{b}}(t)\, \mathrm{d}t \quad , \tag{8}$$

with
T_{ref}: reference time, e.g., excitation signal duration, in s,
t, τ: time variables in s.

Based on correlation, factors from the sonar equation can be expressed as

$$TS = 10\,\log_{10}\left(\frac{r^{\mathrm{T}}_{\mathrm{xx,refl}}(0)}{r^{\mathrm{T}}_{\mathrm{xx,in}}(0)}\right) \quad , \tag{9}$$

$$IL = 10\,\log_{10}\left(\frac{r^{\mathrm{T}}_{\mathrm{xx,RX,T}}(0)}{r_0}\right) \tag{10}$$

$$\text{and} \quad SL = 10\,\log_{10}\left(\frac{r^{\mathrm{T}}_{\mathrm{xx,TX}}(0)}{r_0}\right) \quad , \tag{11}$$

with
$r^{\mathrm{T}}_{\mathrm{xx,refl}}(\tau)$: short time ACF of acoustic pressure $p_{\mathrm{refl}}(t)$ at 1 m distance from target, in Pa^2,
$r^{\mathrm{T}}_{\mathrm{xx,in}}(\tau)$: short time ACF of acoustic pressure $p_{\mathrm{in}}(t)$ at target position, in Pa^2,
$r^{\mathrm{T}}_{\mathrm{xx,RX,T}}(\tau)$: short time ACF of acoustic pressure $p_{\mathrm{RX,T}}(t)$ at receiver due to echo from target, in Pa^2,
r_0: reference value for correlation functions of acoustic pressure, $r_0 = I_0 Z_0 = p_0^2$, in Pa^2,
$r^{\mathrm{T}}_{\mathrm{xx,TX}}(\tau)$: short time ACF of acoustic pressure $p_{\mathrm{TX}}(t)$ at 1 m from transmitter, in Pa^2.

The pressure signal at the receiver is given as

$$p_{\mathrm{RX}}(t) = p_{\mathrm{RX,T}}(t) + p_{\mathrm{uncorr}}(t) \quad , \tag{12}$$

with
$p_{\mathrm{RX}}(t)$: acoustic pressure signal at the receiver, in Pa
$p_{\mathrm{RX,T}}(t)$: echo from target at the receiver, in Pa
$p_{\mathrm{uncorr}}(t)$: uncorrelated acoustic pressure, not from target echo, e.g., noise and other ultrasonic sound

sources, in Pa.

Multiple echoes i can occur at a target such as a hollow hemisphere:

$$p_{\mathrm{RX,t}} = \sum_{i=1}^{N} a_i\, p_{\mathrm{TX}}(t - t_i) \tag{13}$$
$$\text{with} \quad t_i = t_\mathrm{T} + \Delta t_i, \tag{14}$$
$$\Delta t_{i+1} > \Delta t_i \tag{15}$$
$$\text{and} \quad \Delta t_1 = 0, \tag{16}$$

with
i: echo number, starts at 1,
N: number of reflections,
a_i: relative magnitude of single echo,
t_i: propagation delay for echo i,
Δt_i: delay difference for echo i.

Cross-correlation between p_{RX} and p_{TX} leads to pulse-compression and thus removal of uncorrelated parts from the echo signal:

$$r^{\mathrm{T}}_{\mathrm{yx}}(\tau) = \sum_{i=1}^{N} a_i\, r^{\mathrm{T}}_{\mathrm{xx,TX}}(\tau - t_i) \tag{17}$$

with
$r^{\mathrm{T}}_{\mathrm{yx}}(t_\mathrm{T})$: pulse-compressed echo's value at t_T
(see Appendix 2.1 for detailed calculations).

A relation between $r^{\mathrm{T}}_{\mathrm{xx,RX,T}}(\tau)$ and $r^{\mathrm{T}}_{\mathrm{yx}}(\tau)$ can be derived, too:

$$r^{\mathrm{T}}_{\mathrm{xx,RX,T}}(\tau) = \sum_{i=1}^{N} a_i\, r^{\mathrm{T}}_{\mathrm{yx}}(\tau + t_i) \tag{18}$$

(see Appendix 2.2 for detailed calculations).

As a consequence, it can be inferred for a reflector with a single dominant echo (e.g., disc and cylinder) that

$$TS_1 = 20\,\log_{10}\left(\frac{r^{\mathrm{T}}_{\mathrm{yx}}(t_T)}{r^{\mathrm{T}}_{\mathrm{xx,TX}}(0)}\right) + 40\,\log_{10}\left(\frac{t_\mathrm{T}\, c}{2 \cdot 1\,\mathrm{m}}\right) \tag{19}$$
$$= 2\,(\Delta TS_1 + TS_{1,\mathrm{const}}) \tag{20}$$

with
TS_1: total target strength estimate
t_T: target's main peak delay in pulse-compressed echo
ΔTS_1: relative total target strength estimate,
$TS_{1,\mathrm{const}}$: constant part of total target strength estimate
with

$$\Delta TS_1 = 10\,\log_{10}\left(\frac{r^{\mathrm{T}}_{\mathrm{yx}}(t_{\mathrm{T}})}{r_0}\right) + 20\,\log_{10}\left(\frac{t_{\mathrm{T}}}{t_0}\right) \tag{21}$$

$$\text{and} \quad TS_{1,\mathrm{const}} = 20\,\log_{10}\left(\frac{c\,t_0}{2\cdot 1\,\mathrm{m}}\right) - 10\,\log_{10}\left(\frac{r^{\mathrm{T}}_{\mathrm{xx,TX}}(0)}{r_0}\right) \tag{22}$$

with
r_0: arbitrary constant in Pa^2,
t_0: arbitrary time constant in s.

1.2. Spectral Target Strength

Spectral target strength relations:

$$TS = 10\,\log_{10}\left(\frac{\int_{-\infty}^{+\infty} S^{\mathrm{T}}_{\mathrm{xx,refl}}(f)\,\mathrm{d}f}{\int_{-\infty}^{+\infty} S^{\mathrm{T}}_{\mathrm{xx,in}}(f)\,\mathrm{d}f}\right) = 10\,\log_{10}\left(\frac{\int_{-\infty}^{+\infty} S^{\mathrm{T}}_{\mathrm{xx,in}}(f)\,\widetilde{TS}_{lin}(f)\,\mathrm{d}f}{\int_{-\infty}^{+\infty} S^{\mathrm{T}}_{\mathrm{xx,in}}(f)\,\mathrm{d}f}\right) \quad , \tag{23}$$

$$\widetilde{TS}(f) = 10\,\log_{10}\left(\frac{S^{\mathrm{T}}_{\mathrm{xx,refl}}(f)}{S^{\mathrm{T}}_{\mathrm{xx,in}}(f)}\right) \quad , \tag{24}$$

$$IL = 10\log_{10}\left(\frac{\int_{-\infty}^{+\infty} S^{\mathrm{T}}_{\mathrm{xx,RX,T}}(f)\,\mathrm{d}f}{r_0}\right) \tag{25}$$

$$\text{and} \quad SL = 10\,\log_{10}\left(\frac{\int_{-\infty}^{+\infty} S^{\mathrm{T}}_{\mathrm{xx,TX}}(f)\,\mathrm{d}f}{r_0}\right) \tag{26}$$

with
$S^{\mathrm{T}}_{\mathrm{xx,refl}}(f)$: auto power spectral density for $p_{\mathrm{refl}}(t)$, in $\mathrm{Pa}^2\,\mathrm{Hz}^{-1}$,
$S^{\mathrm{T}}_{\mathrm{xx,in}}(f)$: auto power spectral density for $p_{\mathrm{in}}(t)$, in $\mathrm{Pa}^2\,\mathrm{Hz}^{-1}$,
$\widetilde{TS}(f)$: spectral target strength,
$S^{\mathrm{T}}_{\mathrm{xx,RX,T}}(f)$: auto power spectral density for $p_{\mathrm{RX,T}}(t)$, in $\mathrm{Pa}^2\,\mathrm{Hz}^{-1}$,
$S^{\mathrm{T}}_{\mathrm{xx,TX}}(f)$: auto power spectral density for $p_{\mathrm{TX}}(t)$, in $\mathrm{Pa}^2\,\mathrm{Hz}^{-1}$.

Relations between correlation functions and spectral properties are

$$S^{\mathrm{T}}_{\mathrm{ab}}(f) = R^{\mathrm{T}}_{\mathrm{ab}}(f) = \mathcal{F}\{r^{\mathrm{T}}_{\mathrm{ab}}(t)\} \tag{27}$$

$$\text{and} \quad r^{\mathrm{T}}_{\mathrm{ab}}(t) = \int_{-\infty}^{+\infty} S^{\mathrm{T}}_{\mathrm{ab}}(f)\,\mathrm{e}^{\mathrm{j}2\pi f t}\,\mathrm{d}f \tag{28}$$

with
$S^{\mathrm{T}}_{\mathrm{ab}}(f)$: cross power spectral density for two arbitrary signals $p_{\mathrm{a}}(t)$ and $p_{\mathrm{b}}(t)$, here in $\mathrm{Pa}^2\,\mathrm{Hz}^{-1}$,
$R^{\mathrm{T}}_{\mathrm{ab}}(f)$: fourier transform of cross correlation of $p_{\mathrm{a}}(t)$ and $p_{\mathrm{b}}(t)$, here in $\mathrm{Pa}^2\,\mathrm{s}$,
$\mathcal{F}\{\cdots\}$: fourier transform.

From equations 17 and 18 as well as 28, it follows that

$$S^{\mathrm{T}}_{\mathrm{yx}}(f) = S^{\mathrm{T}}_{\mathrm{xx,TX}}(f) \sum_{i=1}^{N} a_i \, \mathrm{e}^{-\mathrm{j}2\pi f t_i} \tag{29}$$

$$\text{and} \quad S^{\mathrm{T}}_{\mathrm{xx,RX,T}}(f) = S^{\mathrm{T}}_{\mathrm{yx}}(f) \sum_{i=1}^{N} a_i \, \mathrm{e}^{\mathrm{j}2\pi f t_i} \tag{30}$$

with
$S^{\mathrm{T}}_{\mathrm{yx}}(f)$: cross-power-spectral density of echo signal with the excitation signal.

We can write

$$10 \log_{10}\left(\frac{I_{\mathrm{in}}}{I_0}\right) = SL - TL \tag{31}$$

and thus

$$S^{\mathrm{T}}_{\mathrm{xx,in}}(f) = S^{\mathrm{T}}_{\mathrm{xx,TX}}(f) \, \frac{R^2}{1\,\mathrm{m}^2} \quad , \tag{32}$$

$$TS = 10 \log_{10}\left(\frac{\int_{-\infty}^{+\infty} S^{\mathrm{T}}_{\mathrm{xx,TX}}(f) \, \widetilde{TS}_{\mathrm{lin}}(f) \, \mathrm{d}f}{\int_{-\infty}^{+\infty} S^{\mathrm{T}}_{\mathrm{xx,TX}}(f) \, \mathrm{d}f}\right) \tag{33}$$

$$\text{and} \quad \widetilde{TS}(f) = 10 \log_{10}\left(\frac{S^{\mathrm{T}}_{\mathrm{xx,RX,T}}(f)}{S^{\mathrm{T}}_{\mathrm{xx,TX}}(f)}\right) + 2\,TL \quad , \tag{34}$$

(see Appendix 2.3 for detailed calculations)
so as a consequence, we obtain

$$\widetilde{TS}(f) = 20 \log_{10}\left(\frac{S^{\mathrm{T}}_{\mathrm{yx}}(f)}{S^{\mathrm{T}}_{\mathrm{xx,TX}}(f)}\right) + 2\,TL \tag{35}$$

$$\text{and} \quad \widetilde{TS}(f) = 2\left(\Delta\widetilde{TS}(f) + \widetilde{TS}_{\mathrm{const}}(f)\right) \tag{36}$$

with
$\Delta\widetilde{TS}$: relative spectral target strength estimate,
$\widetilde{TS}_{\mathrm{const}}$: constant part of spectral target strength,
which are calculated by

$$\Delta\widetilde{TS}(f) = 10 \log_{10}\left(\frac{S^{\mathrm{T}}_{\mathrm{yx}}(f)}{S_0}\right) + 20 \log_{10}\left(\frac{t_{\mathrm{T}}}{t_0}\right) \tag{37}$$

$$\text{and} \quad \widetilde{TS}_{\mathrm{const}}(f) = 20 \log_{10}\left(\frac{c\, t_0}{2 \cdot 1\,\mathrm{m}}\right) - 10 \log_{10}\left(\frac{S^{\mathrm{T}}_{\mathrm{xx,TX}}(f)}{S_0}\right) \tag{38}$$

with
S_0: power spectral density reference value, may be chosen arbitrarily, in $\mathrm{Pa}^2\,\mathrm{Hz}^{-1}$.

2. Detailed Proof of Relations Between Signal Correlation Functions

2.1. Proof for equation 17

The XCF of p_{RX} and p_{TX} can be calculated by

$$r^{\mathrm{T}}_{\mathrm{yx}}(\tau) = \frac{1}{2\,T_{\mathrm{ref}}} \int_{-\infty}^{+\infty} p_{\mathrm{RX}}(t+\tau)\, p_{\mathrm{TX}}(t)\,\mathrm{d}t \quad , \tag{39}$$

$$= \frac{1}{2\,T_{\mathrm{ref}}} \int_{-\infty}^{+\infty} \left[\sum_{i=1}^{N} a_i\, p_{\mathrm{TX}}(t-t_i+\tau) + p_{\mathrm{uncorr}}(t+\tau) \right] p_{\mathrm{TX}}(t)\,\mathrm{d}t \quad , \tag{40}$$

$$= \frac{1}{2\,T_{\mathrm{ref}}} \sum_{i=1}^{N} a_i \int_{-\infty}^{+\infty} p_{\mathrm{TX}}(t-t_i+\tau)\, p_{\mathrm{TX}}(t)\,\mathrm{d}t \quad , \tag{41}$$

$$= \sum_{i=1}^{N} a_i\, r^{\mathrm{T}}_{\mathrm{xx,TX}}(\tau - t_i) \quad . \tag{42}$$

p_{uncorr} is assumed to be orthogonal to p_{TX} and, therefore, does not have an influence on $r^{\mathrm{T}}_{\mathrm{yx}}(\tau)$.

2.2. Proof for equation 18

The relation between the target echo ACF $r^{\mathrm{T}}_{\mathrm{xx,RX,T}}(\tau)$ and $r^{\mathrm{T}}_{\mathrm{yx}}(\tau+t_i)$ is derived by

$$r^{\mathrm{T}}_{\mathrm{xx,RX,T}}(\tau) = \frac{1}{2\,T_{\mathrm{ref}}} \int_{-\infty}^{+\infty} p_{\mathrm{RX,T}}(t+\tau)\, p_{\mathrm{RX,T}}\,\mathrm{d}t \quad , \tag{43}$$

$$= \frac{1}{2\,T_{\mathrm{ref}}} \int_{-\infty}^{+\infty} \sum_{i=1}^{N} a_i\, p_{\mathrm{TX}}(t-t_i+\tau) \sum_{k=1}^{N} a_k\, p_{\mathrm{TX}}(t-t_k)\,\mathrm{d}t \quad , \tag{44}$$

$$= \sum_{i=1}^{N} a_i \sum_{k=1}^{N} a_k \frac{1}{2T_{\mathrm{ref}}} \int_{-\infty}^{+\infty} p_{\mathrm{TX}}(t-t_i+\tau)\, p_{\mathrm{TX}}(t-t_k)\,\mathrm{d}t \quad , \tag{45}$$

$$= \sum_{i=1}^{N} a_i \sum_{k=1}^{N} a_k \frac{1}{2T_{\mathrm{ref}}} \int_{-\infty}^{+\infty} p_{\mathrm{TX}}(t-t_i+t_k+\tau)\, p_{\mathrm{TX}}(t)\,\mathrm{d}t \quad , \tag{46}$$

$$= \sum_{i=1}^{N} a_i \sum_{k=1}^{N} a_k\, r^{\mathrm{T}}_{\mathrm{xx,TX}}(\tau - t_i + t_k) \quad , \tag{47}$$

$$= \sum_{i=1}^{N} a_i\, r^{\mathrm{T}}_{\mathrm{yx}}(\tau + t_i) \quad . \tag{48}$$

Equation 17 is used in the last step.

2.3. Proof for equation 32

From equation 31 follows

$$10\,\log_{10}\left(\frac{I_{\mathrm{in}}}{I_{\mathrm{TX}}}\right) = TL \tag{49}$$

$$\text{and}\quad 10\,\log_{10}\left(\frac{\int_{-\infty}^{+\infty} S^{\mathrm{T}}_{\mathrm{xx,in}}(f)\,\mathrm{d}t}{\int_{-\infty}^{+\infty} S^{\mathrm{T}}_{\mathrm{xx,TX}}(f)\,\mathrm{d}t}\right) = 10\,\log_{10}\left(\frac{R^2}{1\,\mathrm{m}^2}\right), \tag{50}$$

where TL does not depend on frequency and the relation must be true for any arbitrary $S^{\mathrm{T}}_{\mathrm{xx,in}}(f)$. Hence, the relation

$$10\ \log_{10}\left(\frac{S^{\mathrm{T}}_{\mathrm{xx,in}}(f)}{S^{\mathrm{T}}_{\mathrm{xx,TX}}(f)}\right) = 10\ \log_{10}\left(\frac{R^2}{1\,\mathrm{m}^2}\right) \tag{51}$$

must be true.

Abbreviations

The following abbreviations are used in this manuscript:

TS	Target Strength
RMS	Root Mean Square
ACF	Auto-Correlation Function
XCF	Cross-Correlation Function