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Downlink Cooperative Broadcast Transmission Based on Superposition Coding in a Relaying System for Future Wireless Sensor Networks

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Abstract: This study investigates the superiority of cooperative broadcast transmission over traditional orthogonal schemes when applied in a downlink relaying broadcast channel (RBC). Two proposed cooperative broadcast transmission protocols, one with an amplify-and-forward (AF) relay, and the other with a repetition-based decode-and-forward (DF) relay, are investigated. By utilizing superposition coding (SupC), the source and the relay transmit the private user messages simultaneously instead of sequentially as in traditional orthogonal schemes, which means the channel resources are reused and an increased channel degree of freedom is available to each user, hence the half-duplex penalty of relaying is alleviated. To facilitate a performance evaluation, theoretical outage probability expressions of the two broadcast transmission schemes are developed, based on which, we investigate the minimum total power consumption of each scheme for a given traffic requirement by numerical simulation. The results provide details on the overall system performance and fruitful insights on the essential characteristics of cooperative broadcast transmission in RBCs. It is observed that better overall outage performances and considerable power gains can be obtained by utilizing cooperative broadcast transmissions compared to traditional orthogonal schemes.

Keywords: broadcast channel; relay channel; repetition coding; superposition coding; successive interference cancellation

1. Introduction

In recent years, a wireless sensor network has developed rapidly [1–3] and been widely used in many fields, such as meteorology [4,5]. Relaying has been shown to achieve anti-fading capability in the future wireless sensor network [6]. Initial studies on relaying focus on the single source-destination pair scenario and various cooperative transmission protocols have been proposed [7,8]. The most investigated protocols are the amplify-and-forward (AF) and repetition-coded decode-and-forward (DF) protocols with a half-duplex operation, which fit well into existing systems. Despite the diversity gain provided by these relaying strategies, an extra timeslot for message forwarding is required, which leads to a substantial loss [7].

Regarding this, we consider the broadcast channel (BC) where a source node transmits information to a number of users. In BCs, since the source knows the messages of all users, non-orthogonal schemes that transmit multiple user messages simultaneously may reduce the overall consumed bandwidth and exploit the residual degrees of freedom, then potentially provide better performance. Hence, it is necessary to extend relays to BCs (namely RBC) and investigate how the inherent benefits of BCs can be utilized for efficient relaying.

The investigation on the incorporation of RBCs has attracted some interest recently [9–11]. Various efficient relaying schemes have been proposed for fading RBCs [12–14]. As one of the relaying schemes in the fading RBCs, superposition coding (SupC) can achieve a desirable capacity region by suitable power splitting [15–18]. By utilizing SupC, the source (as well as the relay) transmits the messages from both users' messages simultaneously in a single time slot. Two time slots are needed for each transmission round, thus each user is allowed to occupy the full degrees of freedom of the channel and is assured a diversity gain. In the past few years, the performance of SupC exploited in RBCs has been intensively studied.

More generally, they can be categorized into two distinctive types. The first type considers the case where the relay uses the same power splitting factor (PSF) as the source [19,20]. The optimal power allocations have been proposed and the ergodic capacity of this case has been analyzed. The second type focuses on the case where SupC is utilized only by the source with a partial retransmission at the relay [21,22]. The outage probability of this case has been simulated and the efficiency of the proposed scheme has been confirmed.

Basically, the above schemes are constrained with regard to the PSFs and retransmission. Unconstrained schemes in which the source has possibly different PSFs with the relay and the whole retransmission is utilized at the relay may have better performances. However, the interference at the destination node is higher due to the full dependency of the two diversity signals. It is very challenging to analyze the complex signal-to-noise ratio (SNR) of the closed form expression.

In this study, two cooperative broadcast transmission protocols based on SupC in a downlink RBC is proposed By utilizing random dither at the relay, the two diversity signals at the destination are uncorrelated and the SNR is much easier to analyze. Analytical results on the valid region of the PSF pair are provided. The outage events and theoretical outage probabilities of the AF and DF broadcast schemes are calculated and simulated.

The rest of this paper is organized as follows. Section 2 describes the system model of this study and provides the details of the two proposed cooperative broadcast transmission protocols. The outage events of the two proposed schemes are analyzed in Section 3. Section 4 discusses the power gain and corresponding resource allocation problems. Numerical results are provided in Section 5 to demonstrate the comparable performances of the different protocols. Finally, Section 6 concludes the study.

2. System Model and Proposed Broadcast Transmission Protocols

2.1. System Model

This study investigates the scenario consisting of one source node (N_s), one relay node (N_r), and two users (N_{d_1},N_{d_2}), as shown in Figure 1. Each of the two users receives a different message from the source with the help of the relay. We assume that the maximal ratio combining (MRC) detection is used. As in [7,13], the realistic half-duplex constraint is imposed on the nodes, and a time division multiple access (TDMA) system is assumed. Despite the loss in spectral efficiency due to an extra time slot used for relaying, it will be shown that, in multiuser broadcast (downlink) communication scenarios, this drawback can be mitigated by using nonorthogonal transmission strategies. The transmissions between N_i and N_j, $i \in \{s, r\}$ and $j \in \{r, d_1, d_2\}$, are subject to quasi-static Rayleigh fading and log-distance path loss. We use h_{ij} to denote the complex-valued channel gain of the link between N_i and N_j. The channel gains of the different links are independent. We assume independent and identically distributed (i.i.d), circular symmetric complex-valued additive Gaussian noises at the receiver sides. The details of the AF and DF protocols are given in the following subsections.

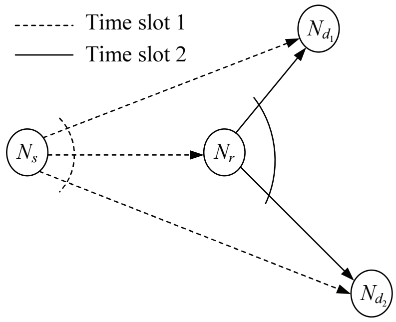


Figure 1. System model of the RBC with two users and one dedicated relay node.

2.2. Description of Proposed DF Broadcast Transmission Protocol

In the DF protocol considered in this study, two time slots are needed to accomplish a one-round communication at the source with a pair of users, in which the signal delivered by the source, as well as the relay, contains the messages of both users by information superposition. The same is true for the AF protocol. The source transmits the messages of N_{d_1} and N_{d_2} simultaneously in the first time slot utilizing two-level SupC [19]. After the relay decodes the signal received from the source, the relay re-encodes the recovered messages using the same codebook used at the source subsequently, the codewords are superimposed and forwarded to the users. The signals received at N_{d_1}, N_{d_2}, and N_r in the first time slot are defined, respectively, as follows:

$$y_{d_1,1} = h_{sd_1}\left(\sqrt{\alpha P_s}x_1 + \sqrt{\bar{\alpha}P_s}x_2\right) + n_{d_1,1}, \tag{1}$$

$$y_{d_2,1} = h_{sd_2}\left(\sqrt{\alpha P_s}x_1 + \sqrt{\bar{\alpha}P_s}x_2\right) + n_{d_2,1}, \tag{2}$$

$$y_{r,1} = h_{sr}\left(\sqrt{\alpha P_s}x_1 + \sqrt{\bar{\alpha}, P_s}x_2\right) + n_{r,1} \tag{3}$$

where x_s corresponds to the symbol transmitted by the source that contains the unit energy codewords x_1 and x_2 (x_1 and x_2 contain the messages to be received by N_{d_1} and N_{d_2}, respectively), P_s is the transmitted power of the source, and $n_{j,1}$ is the additive white Gaussian noises at nodes N_j in the first time slot, where each has variance of $\sigma^2 = \frac{N_0}{2}$ per complex dimension. α is the PSF of the source and indicates the fraction of the power allocated for the transmission of x_1 with the remainder used for x_2; $\bar{\alpha} = 1 - \alpha$. Provided that x_1 and x_2 have been successfully decoded by the relay, the users receive in the second time slot

$$y_{d_1,2} = h_{rd_1}\left(\sqrt{\beta P_r}x_1 + \sqrt{\bar{\beta}P_r}x_2\right) + n_{d_1,2}, \tag{4}$$

$$y_{d_2,2} = h_{rd_2}\left(\sqrt{\beta P_r}x_1 + \sqrt{\bar{\beta}P_r}x_2\right) + n_{d_2,2}, \tag{5}$$

where P_r is the transmitted power of the relay N_r, $n_{j,2}$ is the additive white Gaussian noise (AWGN) at nodes N_j in the second time slot, where each has a variance of $\sigma^2 = \frac{N_0}{2}$ per complex dimension; β is the PSF of the relay; and $\bar{\beta} = 1 - \beta$.

2.3. Description of Proposed AF Broadcast Transmission Protocol

In contrast to the DF protocol, in the AF case, the relay amplifies the received signal (including the noise) by a suitable factor, such that its transmitted power constraint is not affected, and forwards the scaled version to the users in the second time slot. The corresponding received signals by N_{d_1}, N_{d_2}, and N_r in the first time slot are as described in Equations (1)–(3), respectively. The relay amplifying factor is

$$G = \sqrt{\frac{P_r}{P_s|h_{sr}|^2 + N_0}}. \tag{6}$$

N_{d_1} and N_{d_2} receive in the second time slot

$$\begin{aligned} y_{d_1,2} &= h_{rd_1} G y_{r,1} + n_{d_1,2} \\ &= h_{rd_1} G h_{sr}\left(\sqrt{\alpha P_s}x_1 + \sqrt{\bar{\alpha} P_s}x_2\right) + h_{rd_1} G n_{r,1} + n_{d_1,2}, \end{aligned} \tag{7}$$

$$y_{d_2,2} = h_{rd_2} G h_{sr}\left(\sqrt{\alpha P_s}x_1 + \sqrt{\bar{\alpha} P_s}x_2\right) + h_{rd_2} G n_{r,1} + n_{d_2,2}. \tag{8}$$

For both the AF and DF broadcast transmission protocols, the receivers combine the signals received from N_s and N_r using MRC and perform successive interference cancellation (SIC) to recover the messages.

2.4. Notations

For notational convenience, we use

$$\gamma_{ij} = P_i \frac{|h_{ij}|^2}{N_0} \tag{9}$$

to denote the instantaneous SNR of the link between N_i and N_j, $i \in \{s, r\}$, and $j \in \{r, d_1, d_2\}$. In addition, we use

$$\Gamma_{ij} = P_i \cdot PL_{ij} \tag{10}$$

to denote the mean value of γ_{ij}, where PL_{ij} is the path loss of the link from N_i to N_j. It can be easily deduced that the γ_{ij}s are independent exponentially distributed random variables with parameters $1/\Gamma_{ij}$. The coding rates of x_1 and x_2 are denoted by τ_1 and τ_2 throughout the rest of this paper.

3. Outage Analysis of Proposed Cooperative Broadcast Transmission Protocols

3.1. Outage Analysis of Proposed DF Broadcast Transmission Protocol

The main feature of a broadcast transmission with respect to an orthogonal transmission is that the messages aimed at isolated users are superimposed before the transmission, hence we have to detect the user messages from a maximum ratio combination of two independent superimposed signals at the receivers, which is much more complicated to analyze. As was stated in the introduction, we assume a pre-fixed decoding order with SIC at the receivers. Without loss of generality, the message of user 2 (x_2) is decoded first in this study. Although the approach in [23] was proposed and studied in the single-user fading channel scenario, it fits well into multi-user/multi-receiver systems [16,19]. First, as an example, we consider the use of the SIC approach in [23] in the two-user fading BC.

3.1.1. Selection of the PSF in the Two-User Fading Broadcast Channel

We use the same notations and assumptions as for the dedicated RBC, with the exception that there is no relay node. Two channel thresholds $\|h_1\|$ and $\|h_2\|$ are used to indicate the channel condition required for successful decoding of x_1 and x_2, respectively. Since x_2 (the message of user 2) is decoded first, $\|h_2\|$ denotes the bad channel state [16,23], namely

$$|h_2| \le |h_1|. \tag{11}$$

In addition, we have the following expressions of the message rates τ_1 and τ_2, which are related to the channel thresholds and the PDF α:

$$\tau_2 = \log\left(1 + \frac{\bar{\alpha} P_s |h_2|^2}{1 + \alpha P_s |h_2|^2}\right), \tag{12}$$

$$\tau_1 = \log\left(1 + \alpha P_s |h_1|^2\right). \tag{13}$$

It should be noted that the term $\alpha P_s |h_2|^2$ in the denominator of Equation (12) indicates the noise introduced by x_1 when decoding x_2. The inequality in Equation (11) implies that a receiver can never decode the message of N_{d_1} alone. In other words, the message of N_{d_2} is physically degraded to that of N_{d_1}. After some manipulations, Equations (12) and (13) can be rephrased as

$$P_s |h_2|^2 = \frac{2^{\tau_2} - 1}{1 - \alpha 2^{\tau_2}}, \alpha < \frac{1}{2^{\tau_2}}, \tag{14}$$

$$P_s |h_1|^2 = \frac{2^{\tau_1} - 1}{\alpha}. \tag{15}$$

Combining Equations (11), (14) and (15), a valid range of α is obtained as follows:

$$\alpha \le \frac{2^{\tau_1} - 1}{2^{\tau_1} 2^{\tau_2} - 1} = \alpha^{max}. \tag{16}$$

The notation α^{max} is used instead of $\frac{2^{\tau_1}-1}{2^{\tau_1}2^{\tau_2}-1}$ throughout the rest of this paper.

The above discussion is for the broadcast system without a relay, in which only a single superimposed signal is received at the destinations. The problem is much more complicated in dedicated RBCs because two superimposed signals are received at the destinations and are combined using MRC (as is assumed in this study). Fortunately, a similar conclusion can be drawn for successive decoding over combined superimposed signals as over a single superimposed signal.

3.1.2. Selection of the PSF in the DF Broadcast Transmission

Now, we proceed to consider the DF protocol for the dedicated RBC. As we described in Section 2, the user messages are transmitted over two consecutive time slots. In the first time slot, the source transmits a superposition of x_1 and x_2; the relay and both users listen. The second time slot transmission can be one of three cases depending on whether the relay successfully decodes x_1 and/or x_2.

Case 1: First, suppose a correct recovery of the messages of both users at the relay, which corresponds to the following event:

$$\left[\frac{\bar{\alpha} P_s |h_{sr}|^2}{1 + \alpha P_s |h_{sr}|^2} \ge 2^{\tau_2} - 1\right] \bigcap \left[\alpha P_s |h_{sr}|^2 \ge 2^{\tau_1} - 1\right]. \tag{17}$$

Here, in Equation (17) (and in Equations (18) and (19)), we temporarily use $P_i |h_{ij}|^2$ instead of its abbreviation γ_{ij} for clarity of expression). N_{d_1} will receive a superimposed signal transmitted by the source and the relay as described in Equations (1) and (4). In order to decrease the full dependency of the two diversity signals, random dithering is utilized at the relay. Thus, the codewords x_1 will be replaced by $\tilde{x}_1$. We can write the outage event for decoding of x_2 at N_{d_1} as:

$$\frac{\bar{\alpha} P_s |h_{sd_1}|^2}{1 + \alpha P_s |h_{sd_1}|^2} + \frac{\bar{\beta} P_r |h_{rd_1}|^2}{1 + \beta P_r |h_{rd_1}|^2} < 2^{\tau_2} - 1, \tag{18}$$

and the outage of x_1 provided that x_2 has already been successfully decoded is

$$\alpha P_s |h_{sd_1}|^2 + \beta P_r |h_{rd_1}|^2 < 2^{\tau_1} - 1. \tag{19}$$

In view of the assumption regarding the decoding order, we hope that the decoding of x_2 is physically degraded to that of x_1 in an appropriate sense (The channel realizations that satisfy the successful decoding of x_1 also meet the condition to decode x_2.) as in a conventional broadcast transmission, for which the PSF should be suitably designed.

Theorem 1. *Let*

$$\Phi_1 = \Big\{ (\gamma_1, \gamma_2) : a\gamma_1 + b\gamma_2 < 2^{\tau_1} - 1 \Big\},$$
$$\Phi_2 = \Big\{ (\gamma_1, \gamma_2) : \frac{\bar{a}\gamma_1}{1 + a\gamma_1} + \frac{\bar{b}\gamma_2}{1 + b\gamma_2} < 2^{\tau_2} - 1 \Big\}, \tag{20}$$

where $a, b \in [0, 1]$, $\bar{a} = 1 - a$ and $\bar{b} = 1 - b$, γ_1 and γ_2 are nonnegative random variables. Then, $\Phi_2 \subseteq \Phi_1$ if and only if

$$a \leq \alpha^{max} \text{ and } b \leq \alpha^{max},$$

where α^{max} is as defined in Equation (16).

Proof 1. The proof is shown in Appendix A. □

Theorem 1 indicates that, in Case 1, the decoding of x_2 at each destination is degraded to that of x_1 only when α and β are smaller than α^{max}.

Case 2: Then, consider the case that only x_2 is correctly decoded by the relay. In this case, x_2 is retransmitted by the relay with full power in the second time slot. The outage events corresponding to x_2 and x_1 (conditioned on the successful decoding of x_2) at N_{d_1} are

$$\frac{\bar{\alpha}\gamma_{sd_1}}{1 + \alpha\gamma_{sd_1}} + \gamma_{rd_1} < 2^{\tau_2} - 1 \tag{21}$$

and

$$\alpha\gamma_{sd_1} < 2^{\tau_1} - 1, \tag{22}$$

respectively.

Case 3: When the relay fails to recover the messages of both users, only the signal received from the source can be used to decode x_1 and x_2 at each destination. The outage event for decoding of x_2 at N_{d_1} is

$$\frac{\bar{\alpha}\gamma_{sd_1}}{1 + \alpha\gamma_{sd_1}} < 2^{\tau_2} - 1. \tag{23}$$

The outage event corresponding to x_1 (provided that x_2 was successfully decoded is as described in Equation (22).

Corollary 1. *For the DF broadcast transmission protocol, the decoding of x_2 at each receiver is degraded to that of x_1, if and only if*

$$\alpha \leq \alpha^{max} \text{ and } \beta \leq \alpha^{max}. \tag{24}$$

Proof 2. Theorem 1 serves as the necessity proof. Hence, we only need to provide the sufficiency proof. For the decoding process at the relay as well as at N_{d_1} and N_{d_2} in Case 3, the discussion in Section 3.1.1 has proved the degradedness of x_2 with respect to x_1 when $\alpha < \alpha^{max}$. The proof of Case 1 is well provided by Theorem 1. For Case 2, we only need to prove that for an arbitrary SNR value of the link between the relay and N_{d_1}, the SNR (of the link between the source and N_{d_1}) required for the correct decoding of x_2 is lower than that required for the decoding of x_1 (provided that x_2 has been subtracted from the received signal). The proof is rather straightforward, hence is omitted here for convenience. □

Although the above discussion is focused on the decoding at N_{d_1}, the same result can be obtained for the decoding at N_{d_2}.

3.1.3. Outage Probability of the DF Broadcast Transmission Protocol

Unless elsewhere stated, the PSFs are selected according to Equation (24). Since the three outage cases discussed above are disjoint and γ_{sr}, γ_{sd_1} and γ_{rd_1} are mutually independent, we can write the overall outage probability of N_{d_1} as

$$\begin{aligned}
P_{out,1} &\overset{(a)}{=} \Pr\left\{\mathcal{O}^c_{2,r}\bigcap\mathcal{O}^c_{1|2,r}\right\}\cdot\left(\Pr\left\{\mathcal{O}_{2,d_1;1}\right\}+\Pr\left\{\mathcal{O}^c_{2,d_1;1}\bigcap\mathcal{O}_{1|2,d_1;1}\right\}\right)\\
&+\Pr\left\{\mathcal{O}^c_{2,r}\bigcap\mathcal{O}_{1|2,r}\right\}\cdot\left(\Pr\left\{\mathcal{O}_{2,d_1;2}\right\}+\Pr\left\{\mathcal{O}^c_{2,d_1;2}\bigcap\mathcal{O}_{1|2,d_1;2}\right\}\right)\\
&+\Pr\left\{\mathcal{O}_{2,r}\right\}\cdot\left(\Pr\left\{\mathcal{O}_{2,d_1;3}\right\}+\Pr\left\{\mathcal{O}^c_{2,d_1;3}\bigcap\mathcal{O}_{1|2,d_1;3}\right\}\right)\\
&\overset{(b)}{=} \Pr\left\{\mathcal{O}^c_{1|2,r}\right\}\cdot\Pr\left\{\mathcal{O}_{1|2,d_1;1}\right\}\\
&+\Pr\left\{\mathcal{O}^c_{2,r}\bigcap\mathcal{O}_{1|2,r}\right\}\cdot\Pr\left\{\mathcal{O}_{1|2,d_1;2}\right\}\\
&+\Pr\left\{\mathcal{O}_{2,r}\right\}\cdot\Pr\left\{\mathcal{O}_{1|2,d_1;3}\right\}\\
&\overset{(c)}{=} \Pr\left\{\mathcal{O}^c_{1|2,r}\right\}\cdot\Pr\left\{\mathcal{O}_{1|2,d_1;1}\right\}\\
&+\Pr\left\{\mathcal{O}_{1|2,r}\right\}\cdot\Pr\left\{\mathcal{O}_{1|2,d_1;2}\right\},
\end{aligned}\tag{25}$$

where, to save space, we use $\mathcal{O}_{2,r}$ to denote the outage event of x_2 at N_r, $\mathcal{O}_{1|2,r}$ is the outage event of x_1 at N_r provided that x_2 has already been correctly decoded, $\mathcal{O}_{2,d_1;\theta}$ is the outage event of x_2 at N_{d_1} in Case θ ($\theta = 1, 2, 3$), and $\mathcal{O}_{1|2,d_1;\theta}$ is the outage event of x_1 at N_{d_1} in Case θ provided that x_2 has already been correctly decoded. In addition, $\mathcal{O}^c$ denotes the complementary event. $\mathcal{O}^c_{2,r}$ and $\mathcal{O}^c_{1|2,r}$ correspond to the left and right sides, respectively, of Equation (17); $\mathcal{O}_{2,d_1;\theta}$ with $\theta = 1, 2$, and 3 are as described in Equations (18), (21) and (23), respectively; $\mathcal{O}_{1|2,d_1;1}$ corresponds to Equation (19), $\mathcal{O}_{1|2,d_1;2}$ and $\mathcal{O}_{1|2,d_1;3}$ are the same and correspond to Equation (22). In Equation (25), (a) is the general expression, (b) follows from the degradedness of x_2 with respect to x_1 in the decoding sense, and, in (c), the last two terms of (b) are combined. We further evaluate Equation (25) as in Equation (26) where the calculation of ψ_{MRC} is as stated in Equation (27):

$$\begin{aligned}
P_{out,1} = &\left[1-\exp\left(-\frac{2^{\tau_1}-1}{\alpha\Gamma_{sr}}\right)\right]\cdot\left[1-\exp\left(-\frac{2^{\tau_1}-1}{\alpha\Gamma_{sd_1}}\right)\right]\\
&+\exp\left(-\frac{2^{\tau_1}-1}{\alpha\Gamma_{sr}}\right)\cdot\psi_{MRC}\left(\alpha\gamma_{sd_1},\beta\gamma_{rd_1},\tau_1\right),
\end{aligned}\tag{26}$$

where the calculation of ψ_{MRC} is [7,8]

$$\Psi_{MRC}(\gamma_{sd},\gamma_{rd},\tau) = \begin{cases} 1-\exp\left(\frac{1-2^\tau}{\Gamma_{rd}}\right)-\frac{\Gamma_{sd}}{\Gamma_{sd}-\Gamma_{rd}}\exp\left(\frac{1-2^\tau}{\Gamma_{sd}}\right)\cdot\left[1-\exp\left(\frac{\Gamma_{sd}-\Gamma_{rd}}{\Gamma_{sd}}\frac{1-2^\tau}{\Gamma_{rd}}\right)\right], & \Gamma_{sd}\neq\Gamma_{rd},\\ 1-\exp\left(\frac{1-2^\tau}{\Gamma_{rd}}\right)+\frac{1-2^\tau}{\Gamma_{rd}}\exp\left(\frac{1-2^\tau}{\Gamma_{sd}}\right), & \Gamma_{sd}=\Gamma_{rd}.\end{cases}\tag{27}$$

To obtain the outage probability of N_{d_2}, we consider the outage events first. The outage event of x_2 at N_{d_2} in each of the above-mentioned three cases can be obtained by substituting γ_{sd_2} and γ_{rd_1} for γ_{sd_1} and γ_{rd_1}, respectively, in the corresponding outage events of x_2 at N_{d_1}. We do not repeat the similar process as for calculating the outage probability of N_{d_1} and directly give the outage probability of N_{d_2} as

$$
\begin{aligned}
P_{out,2} &= \Pr\left\{\alpha\gamma_{sr} \geq 2^{\tau_1}-1\right\} \cdot \Pr\left\{\frac{\bar{\alpha}\gamma_{sd_2}}{1+\alpha\gamma_{sd_2}} + \frac{\bar{\beta}\gamma_{rd_2}}{1+\beta\gamma_{rd_2}} < 2^{\tau_2}-1\right\} \\
&+ \Pr\left\{\alpha\gamma_{sr} < 2^{\tau_1}-1\right\} \cdot \Pr\left\{\frac{\bar{\alpha}\gamma_{sr}}{1+\alpha\gamma_{sr}} \geq 2^{\tau_2}-1\right\} \cdot \Pr\left\{\frac{\bar{\alpha}\gamma_{sd_2}}{1+\alpha\gamma_{sd_2}} + \gamma_{rd_2} < 2^{\tau_2}-1\right\} \\
&+ \Pr\left\{\frac{\bar{\alpha}\gamma_{sr}}{1+\alpha\gamma_{sr}} < 2^{\tau_2}-1\right\} \cdot \Pr\left\{\frac{\bar{\alpha}\gamma_{sd_2}}{1+\alpha\gamma_{sd_2}} < 2^{\tau_2}-1\right\} \\
&= \exp\left(-\frac{2^{\tau_1}-1}{\alpha\Gamma_{sr}}\right) \cdot \underbrace{\int\int_{B_1} f_{sd_2}(\gamma_{sd_2}) f_{rd_2}(\gamma_{rd_2}) d\gamma_{sd_2} d\gamma_{rd_2}}_{\psi_1(\gamma_{sd_2},\gamma_{rd_2},\alpha,\beta,\tau_2)} \\
&+ \left[\exp\left(-\frac{2^{\tau_2}-1}{(1-\alpha 2^{\tau_2})\Gamma_{sr}}\right) - \exp\left(-\frac{2^{\tau_1}-1}{\alpha\Gamma_{sr}}\right)\right] \cdot \underbrace{\int\int_{B_2} f_{sd_2}(\gamma_{sd_2}) f_{rd_2}(\gamma_{rd_2}) d\gamma_{sd_2} d\gamma_{rd_2}}_{\psi_2(\gamma_{sd_2},\gamma_{rd_2},\alpha,\tau_2)} \\
&+ \left[1-\exp\left(-\frac{2^{\tau_2}-1}{(1-\alpha 2^{\tau_2})\Gamma_{sr}}\right)\right] \cdot \left[1-\exp\left(-\frac{2^{\tau_2}-1}{(1-\alpha 2^{\tau_2})\Gamma_{sd_2}}\right)\right],
\end{aligned}
\tag{28}
$$

where

$$
\begin{aligned}
B_1 &\equiv \left\{(\gamma_{sd_2},\gamma_{rd_2}) : \frac{\bar{\alpha}\gamma_{sd_2}}{1+\alpha\gamma_{sd_2}} + \frac{\bar{\beta}\gamma_{rd_2}}{1+\beta\gamma_{rd_2}} < 2^{\tau_2}-1\right\}, \\
B_2 &\equiv \left\{(\gamma_{sd_2},\gamma_{rd_2}) : \frac{\bar{\alpha}\gamma_{sd_2}}{1+\alpha\gamma_{sd_2}} + \gamma_{rd_2} < 2^{\tau_2}-1\right\}.
\end{aligned}
\tag{29}
$$

Using the results in Appendix B, we can expand ψ_1 and ψ_2 to obtain

$$
\begin{aligned}
\psi_1\left(\gamma_{sd_2},\gamma_{rd_2},\alpha,\beta,\tau_2\right) &= \Pr\left\{(\gamma_{sd_2},\gamma_{rd_2}) \in B_1\right\} \\
&= \int_0^l f_{rd_2}(\gamma_{rd_2}) \int_0^{\hat{p}(\gamma_{rd_2})} f_{sd_2}(\gamma_{sd_2}) d\gamma_{sd_2} d\gamma_{rd_2} \\
&= \int_0^l \frac{1}{\Gamma_{rd_2}} \exp\left(-\frac{\gamma_{rd_2}}{\Gamma_{rd_2}}\right)\left[1-\exp\left(-\frac{\hat{p}(\gamma_{rd_2})}{\Gamma_{sd_2}}\right)\right] d\gamma_{rd_2},
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
\psi_2\left(\gamma_{sd_2},\gamma_{rd_2},\alpha,\beta,\tau_2\right) &= \Pr\left\{(\gamma_{sd_2},\gamma_{rd_2}) \in B_2\right\} \\
&= \int_0^{2^{\tau_2}-1} f_{rd_2}(\gamma_{rd_2}) \int_0^{\tilde{p}(\gamma_{rd_2})} f_{sd_2}(\gamma_{sd_2}) d\gamma_{sd_2} d\gamma_{rd_2} \\
&= \int_0^{\tilde{l}} \frac{1}{\Gamma_{rd_2}} \exp\left(-\frac{\gamma_{rd_2}}{\Gamma_{rd_2}}\right)\left[1-\exp\left(-\frac{\tilde{p}(\gamma_{rd_2})}{\Gamma_{sd_2}}\right)\right] d\gamma_{rd_2},
\end{aligned}
\tag{31}
$$

where in Equations (30) and (31), $\hat{p}(\gamma_{rd_2})$ and $\tilde{p}(\gamma_{rd_2})$ are notations as defined in Appendix B and $\tilde{l}$ equals to $2^{\tau_2}-1$.

3.2. Outage Analysis of Proposed AF Broadcast Transmission Protocol

Since the AF relay always retransmits an amplified version of its observations, no classified discussion is needed for the second time slot transmission. According to Equations (1) and (7) and the amplification gain G in Equation (6), the outage event corresponding to the decoding of x_2 at N_{d_1} is

$$
\frac{\bar{\alpha}\gamma_{sd_1}}{1+\alpha\gamma_{sd_1}} + \frac{\bar{\alpha}\gamma_{sr}\gamma_{rd_1}}{\alpha\gamma_{sr}\gamma_{rd_1}+\gamma_{sr}+\gamma_{rd_1}+1} < 2^{\tau_2}-1
\tag{32}
$$

and that of x_1 conditioned on the successful decoding of x_2 is

$$\alpha\gamma_{sd_1} + \frac{\alpha\gamma_{sr}\gamma_{rd_1}}{\gamma_{sr} + \gamma_{rd_1} + 1} < 2^{\tau_1} - 1. \tag{33}$$

For clarity of expression, we introduce the notation

$$\gamma_{AF} = \frac{\gamma_{sr}\gamma_{rd_1}}{\gamma_{sr} + \gamma_{rd_1} + 1}$$

and the outage events in Equations (32) and (33) can be rephrased as

$$\frac{\bar{\alpha}\gamma_{sd_1}}{1 + \alpha\gamma_{sd_1}} + \frac{\bar{\alpha}\gamma_{AF}}{\alpha\gamma_{AF} + 1} < 2^{\tau_2} - 1 \tag{34}$$

and

$$\alpha\gamma_{sd_1} + \alpha\gamma_{AF} < 2^{\tau_1} - 1, \tag{35}$$

respectively.

Note that γ_{AF} is the harmonic mean of the two exponential random variables γ_{sr} and γ_{rd_2}. It is well recognized in the literature [24,25] that, at high values of Γ_{sr} and Γ_{rd_2}, γ_{AF} can be approximated by an exponential random variable with the parameter $\frac{1}{\Gamma_{sr}} + \frac{1}{\Gamma_{rd_2}}$. Hence, if we are considering the high SNR approximation of the outage behavior, according to Theorem 1 and the outage events defined in Equations (34) and (35), x_2 is degraded to x_1 (from the decoding sense) when $\alpha \leq \alpha^{max}$. In fact, this conclusion applies to the whole SNR region because, for all possible values of γ_{sr} and γ_{rd_2}, γ_{AF} as a whole can be treated as a nonnegative random variable, which coincides with the assumption of Theorem 1. Therefore, with the AF protocol, the decoding of x_2 is degraded to that of x_1 if and only if $\alpha \leq \alpha^{max}$. A more systematic proof of this is provided in Appendix C.

In the following, we focus on the case of $0 < \alpha \leq \alpha^{max}$. (When α equals to zero, the problem degrades to a communication of the source with N_{d_2} only, which is unexpected.) Then, the outage event of N_{d_1} is simply the one in Equation (33) and the outage event of N_{d_2} is directly obtained as

$$\frac{\bar{\alpha}\gamma_{sd_2}}{1 + \alpha\gamma_{sd_2}} + \frac{\bar{\alpha}\gamma_{sr}\gamma_{rd_2}}{\alpha\gamma_{sr}\gamma_{rd_2} + \gamma_{sr} + \gamma_{rd_2} + 1} < 2^{\tau_2} - 1. \tag{36}$$

Following the same line of discussion as in Appendix D and in Section 5.1.2, the outage probabilities of N_{d_1} and N_{d_2} have the same formulation as that of conventional AF relaying as

$$P_{out,u} = \int_0^{l_u} \frac{1}{\Gamma_{sd_1}} \exp\left(-\frac{\gamma_{sd_1}}{\Gamma_{sd_1}}\right) \cdot (I_1 + I_2 + I_3) d\gamma_{sd_1} \tag{37}$$

with I_1, I_2, and I_3 defined in Equation (50)–(52) and $u = 1, 2$. For the outage expression of N_{d_1}, the notations in Equations (37) and (50)–(52) are specified as follows:

$$\begin{aligned} u &= 1; \\ l_1 &= \frac{2^{\tau_1} - 1}{\alpha}; \\ m(\gamma_{sd_1}) &= \frac{2^{\tau_1} - 1}{\alpha} - \gamma_{sd_1}; \\ n(\gamma) &= \frac{(\gamma + 1)m(\gamma_{sd_1})}{\gamma - m(\gamma_{sd_1})}, \gamma = \gamma_{sr}, \gamma_{rd_1}; \\ \gamma^* &= m(\gamma_{sd_1}) + \sqrt{[m(\gamma_{sd_1})]^2 + m(\gamma_{sd_1})}, \end{aligned} \tag{38}$$

and for N_{d_2}

$$
\begin{aligned}
u &= 2; \\
l_2 &= \frac{2^{\tau_2}-1}{1-\alpha 2^{\tau_2}}; \\
m(\gamma_{sd_2}) &= \frac{2^{\tau_2}-1-\frac{\bar{\alpha}\gamma_{sd_2}}{1+\alpha\gamma_{sd_2}}}{\bar{\alpha}-\alpha\left(2^{\tau_2}-1-\frac{\bar{\alpha}\gamma_{sd_2}}{1+\alpha\gamma_{sd_2}}\right)}; \\
n(\gamma) &= \frac{(\gamma+1)m(\gamma_{sd_2})}{\gamma-m(\gamma_{sd_2})}, \gamma=\gamma_{sr}, \gamma_{rd_1}; \\
\gamma^* &= m(\gamma_{sd_2})+\sqrt{[m(\gamma_{sd_2})]^2+m(\gamma_{sd_2})}.
\end{aligned} \tag{39}
$$

4. Power Gain and Resource Allocation

In order to evaluate the overall system performance of different schemes, we consider the power gain of the broadcast protocols over the orthogonal schemes. To calculate the power gain, we first need to obtain the minimum overall transmit power required by each scheme such that a given transmission rate pair can be achieved subject to the outage probabilities $P^{th}_{out,1}$ and $P^{th}_{out,2}$ for N_{d_1} and N_{d_2}, respectively. We use R_1 and R_2 to denote the effective information rates of N_{d_1} and N_{d_2}, respectively, and P_t for the overall transmit power.

Note that, for the orthogonal schemes, the channel resources assigned to the transmission of different users' messages may be different, as illustrated in Figure 2. Only the AF protocols are included for brevity, and the DF counterparts have similar transmission structures. Without loss of generality, the whole channel block per round of transmission is assumed to be 1; δ_{dt} and δ_{af} are used to denote the portion of the channel block allocated for the transmission of x_1 with the remainder for x_2. In addition, a power allocation between the two users is allowed. For the cooperative broadcast transmission protocols, the optimal power allocation of the power among different users is achieved by simply optimizing the PSFs. We use ζ to denote the ratio of the total power assigned for source transmitting and $\bar{\zeta} = 1-\zeta$ for relay forwarding. For the baseline schemes, we reuse the notations α and $\bar{\alpha}$ (β and $\bar{\beta}$) to indicate the percentage of the total source (relay) power used to send x_1 and x_2, respectively. The simplified expressions of AF and DF followed by 'Orthogonal' or 'Broadcast' (see Figure 2) will be used instead of their lengthy versions.

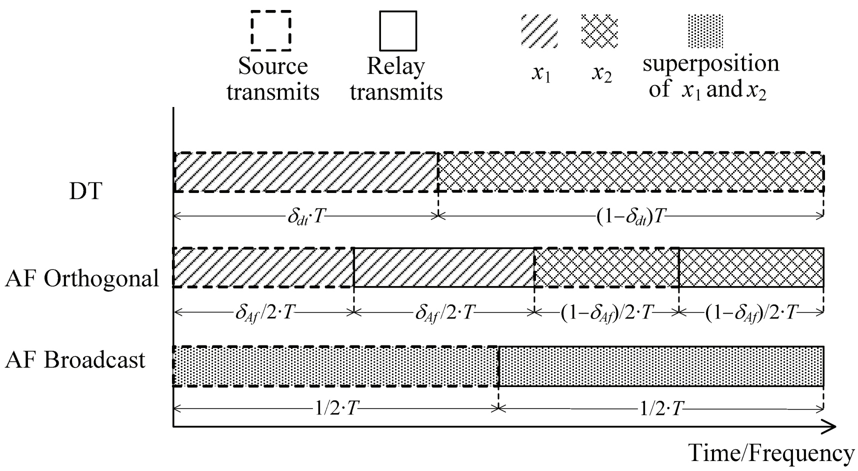


Figure 2. Comparison between the cooperative broadcast transmission and conventional transmission schemes when assigning orthogonal channels to different users.

Figure 2 shows that the conversion between the effective information rates and the code rates in the DT protocol are

$$\tau_1 = \frac{R_1}{\delta_{dt}}, \tau_2 = \frac{R_2}{1-\delta_{dt}}; \tag{40}$$

and the practical transmit powers of the source corresponding to x_1 and x_2 are

$$P_s = \frac{\alpha P_t}{\delta_{dt}} \text{ and } P_s = \frac{\bar{\alpha} P_t}{1-\delta_{dt}}, \tag{41}$$

respectively. Similarly, we have for the AF Orthogonal

$$\tau_1 = \frac{2R_1}{\delta_{af}}, P_s = \frac{2\alpha\zeta P_t}{\delta_{af}}, P_r = \frac{2\beta\bar{\zeta} P_t}{\delta_{af}} \tag{42}$$

for transmission of x_1 and

$$\tau_2 = \frac{2R_2}{1-\delta_{af}}, P_s = \frac{2\bar{\alpha}\zeta P_t}{1-\delta_{af}}, P_r = \frac{2\bar{\beta}\bar{\zeta} P_t}{1-\delta_{af}} \tag{43}$$

for x_2. For the AF Broadcast, we have

$$\tau_1 = 2R_1, \tau_2 = 2R_2, P_s = 2\zeta P_t, P_r = 2\bar{\zeta} P_t. \tag{44}$$

Finally, the cases of the DF Orthogonal and DF Broadcast are akin to their AF counterparts and are omitted here for the sake of brevity.

With the above descriptions, the calculation of the minimum overall transmit power is to find the optimal values of α, β, ζ, and δ_{dt} (or δ_{af}) that minimize P_t for given values of R_1 and R_2 (or τ_1 and τ_2). To solve the resource allocation problem analytically is quite difficult due to the complex outage expressions. The situation may be alleviated by using the high SNR approximation of outage behavior. However, the main purpose of this study is to examine the potential advantages of cooperative broadcast transmissions over conventional orthogonal schemes and a specialized investigation in the high SNR regime is beyond the scope of this study. Fortunately, all the variables to be optimized have a valid range of $[0, 1]$, which makes it practically feasible to solve the optimization problem by using a numerical search. By using the minimum P_t for certain transmission rate pairs of interest (R_1, R_2) as well as the target outage probabilities $P_{out,1}^{th}$ and $P_{out,2}^{th}$, the power gain of the broadcast protocols over the baseline schemes can be obtained.

5. Numerical Results

In this section, we present some numerical results to compare the outage performances and power consumptions of the cooperative broadcast transmission protocols with those of the other schemes. We consider a two-dimensional model as in Figure 3, where θ_1 is the angle of the line $N_{d_1} - N_s - N_r$, θ_2 is the angle of the line $N_{d_2} - N_s - N_r$, and d_{ij} denotes the Euclidean distance between N_i and N_j. Without loss of generality, we use d_{sd_2} as a reference distance and consider a number of scenarios with different values of d_{sd_1}, d_{sr}, and θ_1, θ_2, d_{rd_1} and d_{rd_2} can be determined by the triangle equalities

$$d_{rd_1}{}^2 = d_{sr}{}^2 + d_{sd_1}{}^2 - 2d_{sr}d_{sd_1}\cos(\theta_1), \tag{45}$$

$$d_{rd_2}{}^2 = d_{sr}{}^2 + d_{sd_2}{}^2 - 2d_{sr}d_{sd_2}\cos(\theta_2). \tag{46}$$

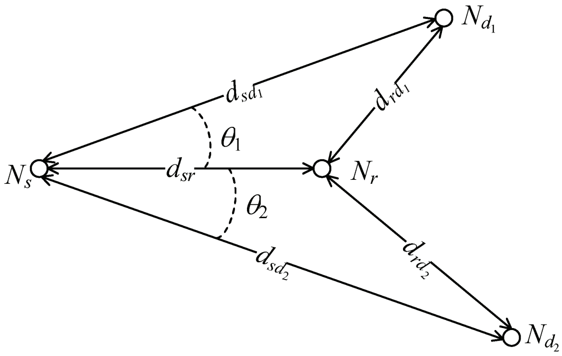


Figure 3. Geometry model of a relay broadcast channel (RBC).

Since a log-distance path loss model is assumed, we have $PL_{ij} = PL_{sd_2} \cdot \left(\frac{d_{ij}}{d_{sd_2}}\right)^{\eta}$, where η is the path loss exponent. Throughout this, we use $\eta = 3.75$ unless stated otherwise.

5.1. Outage Probability of Conventional Relaying with Orthogonal Multiplexing

First, we give the outage probability of conventional relaying with orthogonal multiplexing. The orthogonal AF and DF schemes have the same assumptions as the broadcast transmission protocols in this study, namely half-duplex operation and MRC detection.

5.1.1. Conventional DF Relaying with Orthogonal Multiplexing

The outage probabilities of conventional DF relaying with orthogonal multiplexing at N_{d_1} and N_{d_2} can be shown as [7,8]

$$\begin{aligned} P_{out,1} = & \left[1 - \exp\left(-\frac{2^{\tau_1} - 1}{\Gamma_{sd_1}}\right)\right] \cdot \left[1 - \exp\left(-\frac{2^{\tau_1} - 1}{\Gamma_{sr}}\right)\right] \\ & + \exp\left(-\frac{2^{\tau_1} - 1}{\Gamma_{sr}}\right) \cdot \psi_{MRC}\left(\gamma_{sd_1}, \gamma_{rd_1}, \tau_1\right), \end{aligned} \tag{47}$$

$$\begin{aligned} P_{out,2} = & \left[1 - \exp\left(-\frac{2^{\tau_2} - 1}{\Gamma_{sd_2}}\right)\right] \cdot \left[1 - \exp\left(-\frac{2^{\tau_2} - 1}{\Gamma_{sr}}\right)\right] \\ & + \exp\left(-\frac{2^{\tau_2} - 1}{\Gamma_{sr}}\right) \cdot \psi_{MRC}\left(\gamma_{sd_2}, \gamma_{rd_2}, \tau_2\right). \end{aligned} \tag{48}$$

5.1.2. Conventional AF Relaying with Orthogonal Multiplexing

The outage probabilities of conventional DF relaying with orthogonal multiplexing at N_{d_1} is [7]. (The analytical outage expression is derived in Appendix D)

$$P_{out,1} = \int_0^{2^{\tau_1}-1} \frac{1}{\Gamma_{sd_1}} \exp\left(-\frac{\gamma_{sd_1}}{\Gamma_{sd_1}}\right) \cdot (I_1 + I_2 + I_3) d\gamma_{sd_1}, \tag{49}$$

where I_1, I_2, and I_3 correspond to the probabilities of the events A_1, A_2, and A_3, respectively, and can be obtained as in (50)–(52):

$$\begin{aligned} I_1 &= 1 - \int_{m(\gamma_{sd_1})}^{\infty} f_{sr}(\gamma_{sr}) d\gamma_{sr} \cdot \int_{m(\gamma_{sd_1})}^{\infty} f_{rd_1}(\gamma_{rd_1}) d\gamma_{rd_1} \\ &= 1 - \exp\left(-\frac{m(\gamma_{sd_1})}{\Gamma_{sr}}\right) \cdot \exp\left(-\frac{m(\gamma_{sd_1})}{\Gamma_{rd_1}}\right), \end{aligned} \tag{50}$$

$$
\begin{aligned}
I_2 &= \int_{m(\gamma_{sd_1})}^{\gamma^*(\gamma_{sd_1})} f_{sr}(\gamma_{sr}) \cdot \int_{\gamma_{sr}}^{n(\gamma_{sr})} f_{rd_1}(\gamma_{rd_1}) d\gamma_{rd_1} d\gamma_{sr} \\
&= \int_{m(\gamma_{sd_1})}^{\gamma^*(\gamma_{sd_1})} \frac{1}{\Gamma_{sr}} \exp\left(-\frac{\gamma_{sr}}{\Gamma_{sr}}\right) \cdot \left[\exp\left(-\frac{\gamma_{sr}}{\Gamma_{rd_1}}\right) - \exp\left(-\frac{n(\gamma_{sr})}{\Gamma_{rd_1}}\right)\right] d\gamma_{sr},
\end{aligned} \tag{51}
$$

$$
\begin{aligned}
I_3 &= \int_{m(\gamma_{sd_1})}^{\gamma^*(\gamma_{sd_1})} f_{rd_1}(\gamma_{rd_1}) \int_{\gamma_{rd_1}}^{n(\gamma_{rd_1})} f_{sr}(\gamma_{sr}) d\gamma_{sr} d\gamma_{rd_1} \\
&= \int_{m(\gamma_{sd_1})}^{\gamma^*(\gamma_{sd_1})} \frac{1}{\Gamma_{rd_1}} \exp\left(-\frac{\gamma_{rd_1}}{\Gamma_{rd_1}}\right) \cdot \left[\exp\left(-\frac{\gamma_{rd_1}}{\Gamma_{sr}}\right) - \exp\left(-\frac{n(\gamma_{rd_1})}{\Gamma_{sr}}\right)\right] d\gamma_{rd_1}.
\end{aligned} \tag{52}
$$

The outage probability of N_{d_2} can be obtained in a straightforward manner by substituting τ_2, Γ_{sd_2}, Γ_{rd_2}, γ_{sd_2}, and γ_{rd_2} for τ_1, Γ_{sd_1}, Γ_{rd_1}, γ_{sd_1}, and γ_{rd_1} in (49)–(52), respectively; the details are omitted here due to space limitations.

5.2. Comparison with Other Schemes

Figures 4 and 5 shows the outage probabilities of the different DF protocols at N_{d_1} and N_{d_2}. These DF protocols include the PSF-unconstrained broadcast, PSF-constrained broadcast (Here, PSF-constrained protocol means that the relay uses the same PSF with the source. However, in the PSF-unconstrained protocol, the PSFs of the source and relay may be different) and orthogonal DF protocols. The source and the relay have the same transmit power, namely $P_s = P_r$. Since repetition coded relay is assumed for all the schemes, the transmission durations of the source and the relay are equal to each other, hence $P_t = \frac{P_s+P_r}{2}$. It is also assumed that the channel resources (time/frequency) are equally occupied by the users. To have a fair comparison among different protocols, the code rates of the orthogonal cooperative schemes are double those of the broadcast schemes. The analytical results are obtained using the outage expressions in Equations (26), (28), (47) and (48). In addition, Monte Carlo simulations have been provided for these protocols to validate the analytical results. Obviously, the analytical and simulation results match very well.

Figures 4 and 5 clearly indicate that, for the weaker user (N_{d_2}), a gain of about 3 dB is provided by the PSF-constrained broadcast transmission over the orthogonal scheme, whereas the stronger user (N_{d_1}) suffers a 3 dB degraded performance. It can be seen that, by appropriately adjusting the PSFs in the PSF-unconstrained broadcast schemes, the outage behavior of N_{d_1} can be largely improved. Figures 4 and 5 show that there is only a slight loss in the outage performance of N_{d_1} and still a gain of about 1.5 dB is achieved by N_{d_2} in the PSF-unconstrained broadcast transmission.

Similar results can be obtained in the comparisons between the AF broadcast and orthogonal protocols, as shown in Figures 6 and 7. In the AF broadcast scheme with $\alpha = 0.5$, N_{d_2} achieves a gain of about 3 dB and N_{d_1} suffers a 3dB degraded performance over the orthogonal scheme. However, in the AF broadcast scheme with $\alpha = 0.9$, both (N_{d_1}) and (N_{d_2}) obtain enhanced performances compared with the orthogonal scheme. About 2 dB and 1 dB gains are provided by the AF broadcast scheme with $\alpha = 0.9$ for (N_{d_1}) and (N_{d_2}), respectively.

Generally, provided that the stronger user's outage performance satisfies its target error probability, suitable α and β may be selected such that a higher gain can be achieved by the weaker user, which constitutes a better overall system performance.

Figure 8 shows the case when both users have the same transmission rate and outage constraint with the power and channel resources being optimally allocated such that the overall transmit power P_t required to satisfy the target outage probability for each transmission scheme is minimized. The transmission rates R_1 and R_2 are as defined in Section 4. The cooperative broadcast schemes generally maintain an advantage over the other collaborative strategies and the power gains become more evident in the higher rate region. As the rate increases, the noncooperative transmission gradually begins to dominates the cooperative methods. This is due to the low spectral efficiency of the repetition coded relay. Although the broadcast schemes are superior from the perspective that the transmission of each user's message employs the whole time slot, for which x_1 and x_2 can have lower code rates and the half-duplex penalty of conventional relaying is alleviated, when we treat x_1 and x_2 as a whole, they also suffer from the drawback of the repetition-based relay. Hence, the broadcast transmission schemes are expected to provide the most benefits in relatively (but not extremely) high rate regions.

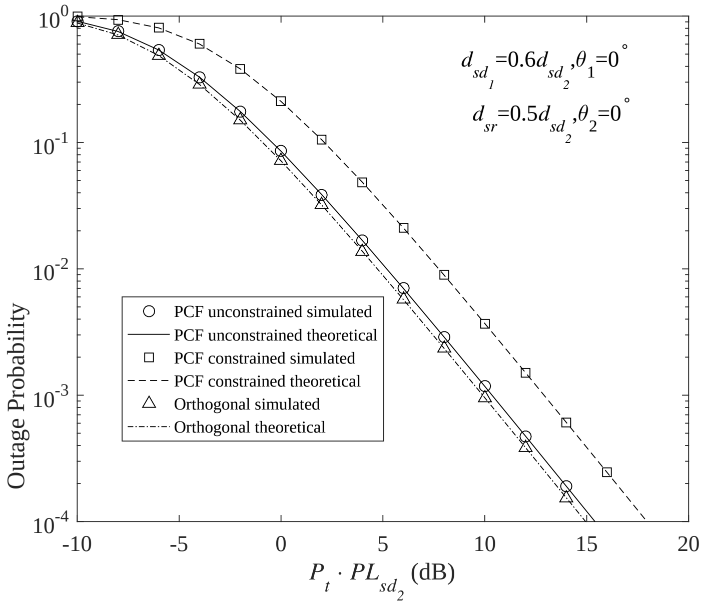


Figure 4. Outage probability of different DF protocols at N_{d_1} for code rates $\tau_1 = \tau_2 = 1$ b/s/Hz ($\tau_1 = \tau_2 = 2$ b/s/Hz for the orthogonal multiplexing), PCFs of the PCF-constrained protocol $\alpha = 0.5\alpha^{max}$ and $\beta = 0.5\alpha^{max}$, PCFs of the PCF-unconstrained protocol $\alpha = 0.9\alpha^{max}$ and $\beta = 0.5\alpha^{max}$, and transmit powers $P_r = P_s$.

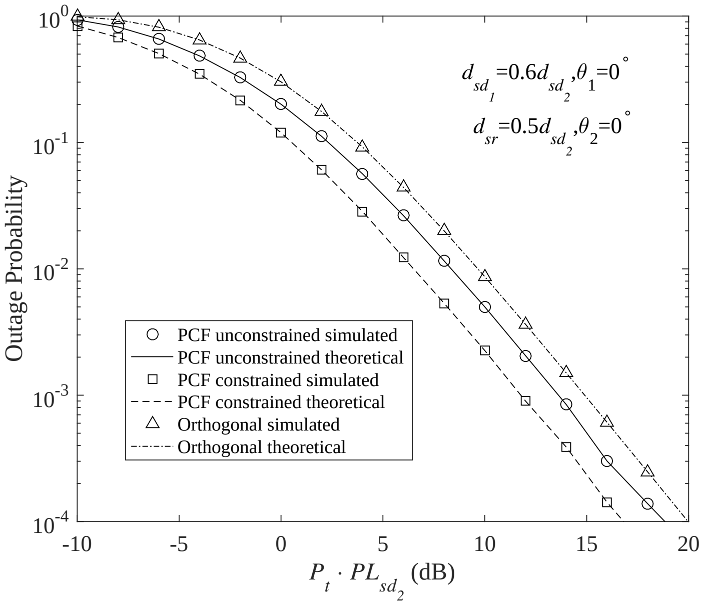


Figure 5. Outage probability of different DF protocols at N_{d_2} for code rates $\tau_1 = \tau_2 = 1$ b/s/Hz ($\tau_1 = \tau_2 = 2$ b/s/Hz for the orthogonal multiplexing), PCFs of the PCF-constrained protocol $\alpha = 0.5\alpha^{max}$ and $\beta = 0.5\alpha^{max}$, PCFs of the PCF-unconstrained protocol $\alpha = 0.9\alpha^{max}$ and $\beta = 0.5\alpha^{max}$, and transmit powers $P_r = P_s$.

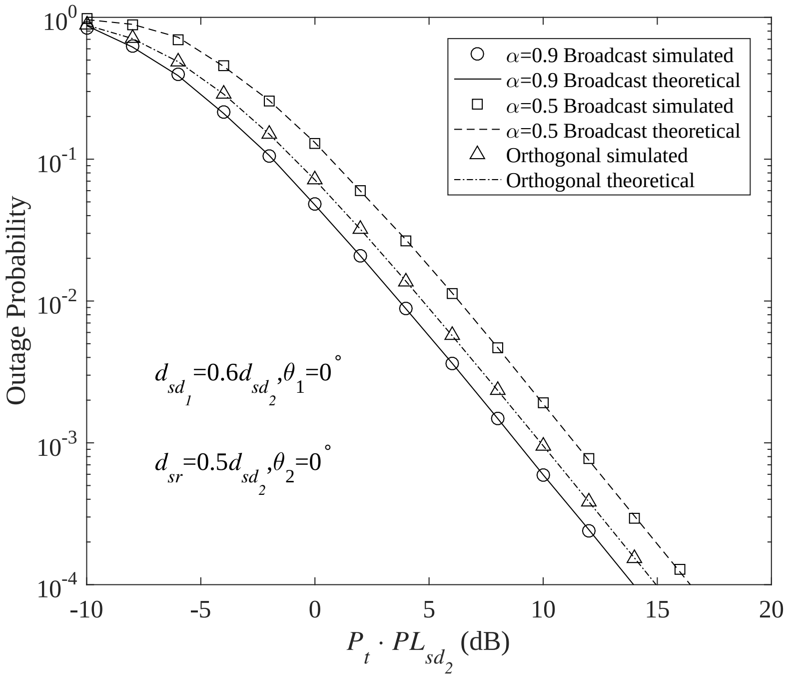


Figure 6. Outage probability of different AF protocols at N_{d_1} for code rates $\tau_1 = \tau_2 = 1$ b/s/Hz ($\tau_1 = \tau_2 = 2$ b/s/Hz for the orthogonal multiplexing), and transmit powers $P_r = P_s$.

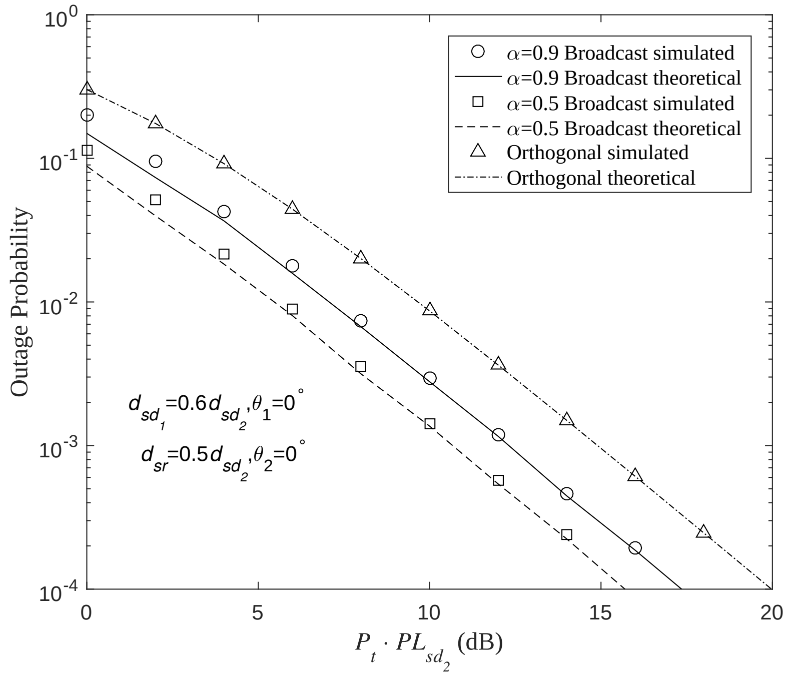


Figure 7. Outage probability of different AF protocols at N_{d_2} for code rates $\tau_1 = \tau_2 = 1$ b/s/Hz ($\tau_1 = \tau_2 = 2$ b/s/Hz for the orthogonal multiplexing), and transmit powers $P_r = P_s$.

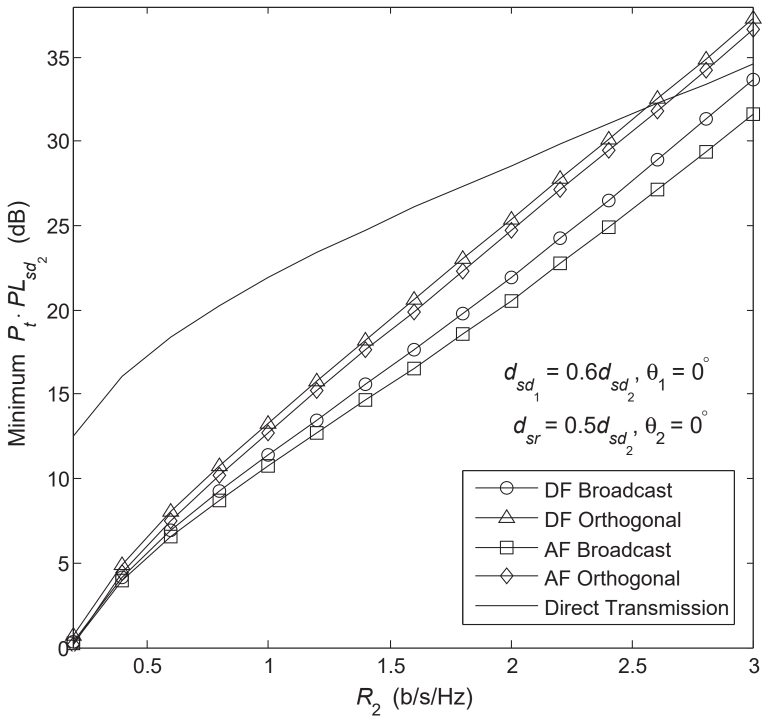


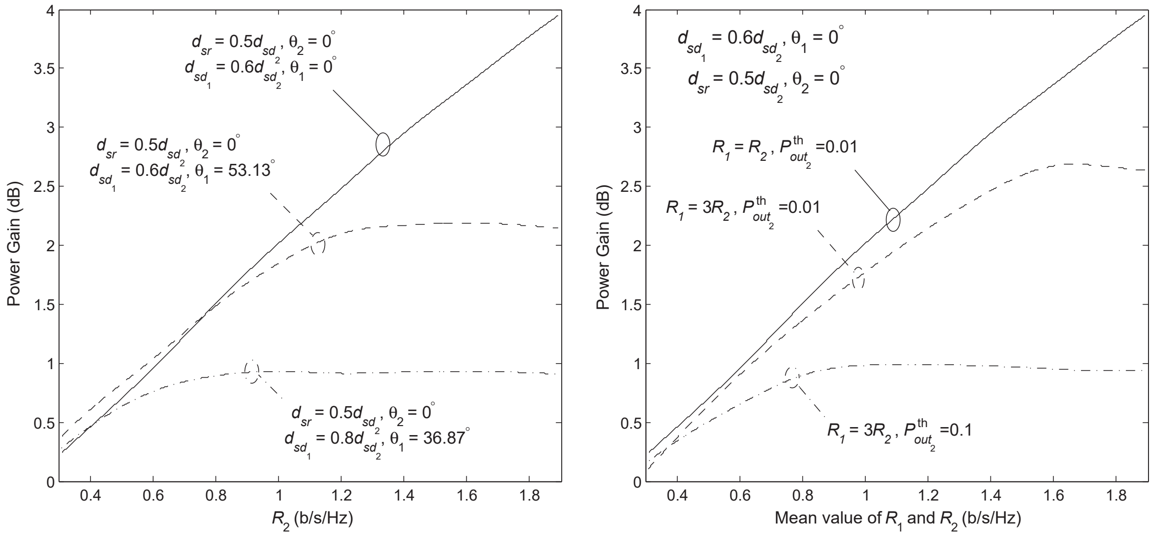
Figure 8. Minimum overall transmit power versus transmission rate with $R_1 = R_2$ and $P_{out}^{th_1} = P_{out}^{th_2} = 0.01$.

5.3. Effect of Disparity in Channel Qualities and Desired Performances of Users

As it is well known [15], the superiority of broadcast transmissions over those with orthogonal multiplexing is due to the disparity in the user channel qualities. Generally, the degree of disparity is affected by two aspects; one is the distinct channel attenuations suffered by different users' messages and the other is the disparity in the desired transmission rates and the outage probabilities, which determines the channel quality required by each user to satisfy its target performance.

Figure 9a shows the power gain versus the rate for various network geometries. Only the power gain of the AF Broadcast compared with the AF Orthogonal is shown for simplicity. The case of the DF Broadcast is similar. As it can be expected, as the disparity in the user's channel qualities decreases, the power gain drops in most of the rate region. The exception occurring in the small rate region can be explained by the fact that the superiority of the AF Broadcast over the AF Orthogonal with regard to spectral efficiency becomes less evident in the lower rate region, whereas a reduction in channel disparity mitigates the disadvantage of the AF Broadcast relative to the AF Orthogonal due to its inability regarding time allocation.

Figure 9b shows the power gain versus the mean transmission rate for various requirements of rate and outage by both users. Requiring a worse quality of service for N_{d_2} is equivalent to improving its channel quality relative to N_{d_1}. Similar trends of the power gain are observed in Figure 9a,b.



(**a**) (**b**)

Figure 9. Power gain of AF Broadcast over AF Orthogonal versus rate. In (**a**) $R_1 = R_2$ and $P_{out,1}^{th} = P_{out,2}^{th} = 0.01$, the three curves from top to bottom correspond to the disparity in channel qualities of users from large to small, while (**b**) shows the three cases when N_{d_2} has the same transmission rate and outage as N_{d_1}, larger outage probability than N_{d_1}, and lower transmission rate and larger outage probability than N_{d_1}.

In all of the above-mentioned results, we only include the scenario in which both users' source-to -destination links are statistically worse than the source-relay link, namely $d_{sr} < d_{sd_1}$ and $d_{sr} < d_{sd_2}$ (Case 1). A different case occurs when $d_{sr} \geq d_{sd_1}$ and $d_{sr} < d_{sd_2}$ (Case 2). In Case 2, rather than using AF Orthogonal (DF Orthogonal), a mixed strategy with N_{d_1} using a direct transmission and N_{d_2} using AF relaying (DF relaying) is preferred in moderately higher rate regions (It should be noted that the mixed schemes are also possible to provide improvement in Case 1, while our purpose is to illustrate the main observations through the selected scenarios, not to cover all situations). Figure 10 shows the power gain of the AF broadcast in comparison with the AF Orthogonal and the mixed AF scheme. It can be seen that the power gain is affected by the transmission rate and the disparity in channel qualities in a similar manner as in Case 1; the only difference is that the power gain (when compared with the mixed scheme) first reaches a peak value as R_2 increases and then drops until the AF Broadcast is inadequate. The coordinate of the point at which the mixed scheme surpasses the AF Orthogonal has been marked for each channel condition in Figure 10. The conversion points move from right to left as the channel disparity increases, which indicates that, in Case 2 with a large disparity in the channel qualities of users, it is preferred to make the relay serve the weak user only (When the relay forwards the weak user's message only, the AF (as well as DF) Orthogonal scheme degrades to a mixed scheme.) is better than to use more power to transmit the weak user's message. The amplified signal received from the relay has a limited contribution to the decoding of x_1 at N_{d_1} considering that the direct link of N_{d_1} is better than the source-to-relay link. Moreover, having the relay retransmit x_2 only largely enhances the successful detection of x_2 at N_{d_2} (as well as at N_{d_1}); otherwise, much more power has to be used to ensure that x_2 is received. Again, by comparing Figure 10a,b, we see that the largest power gain is obtained at moderate rate values.

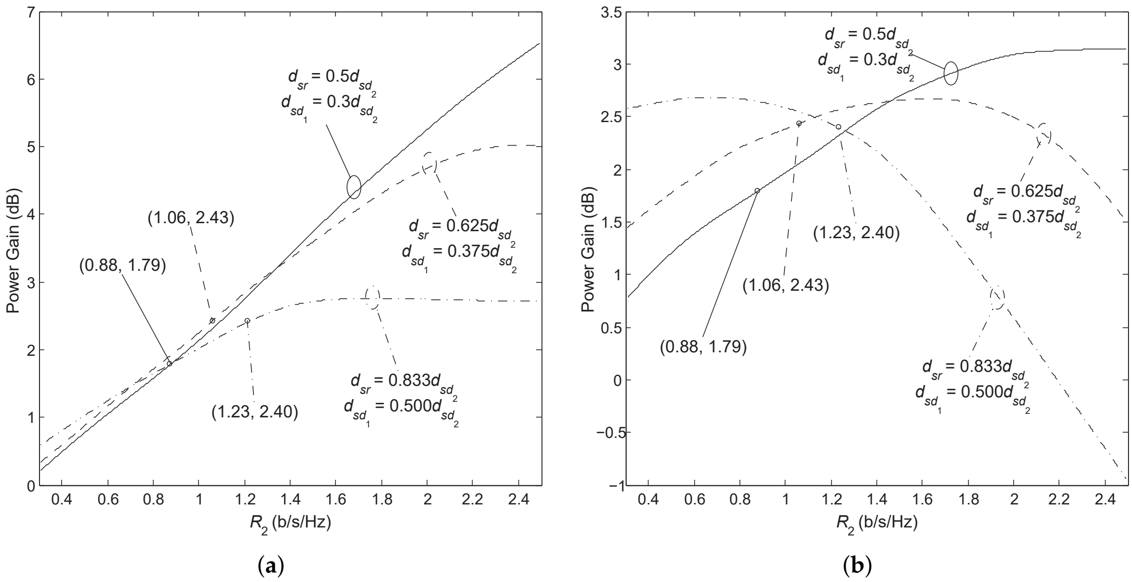


Figure 10. Power gain of AF Broadcast versus the R_2 with $R_1 = R_2$ and $P^{th}_{out,1} = P^{th}_{out,2} = 0.01$. The three curves from top to bottom in (**a**) correspond to the disparity in the channel qualities of users from high to low, all three cases have $\theta_1 = \theta_2 = 0°$. In (**a**), the AF Broadcast is compared with the AF Orthogonal, whereas in (**b**) compared with a mixed AF scheme.

5.4. Comparison between AF Broadcast and DF Broadcast

In Figures 4–8, we notice that the AF Broadcast provides better system performance than the DF broadcast. Now, we compare the outage event of the AF Broadcast described in Section 3.2 with that of the DF Broadcast described in Section 3.1; three cases can occur. When both users' messages are fully recovered by the relay, the outage event of the DF Broadcast is a strict subset of the outage event of the AF Broadcast. If the relay fails to decode both x_1 and x_2, the outage event of the DF Broadcast covers that of the AF Broadcast. Moreover, when the relay decodes x_2 only, neither of the outage events of the AF Broadcast and DF Broadcast included in the other. An inherent property of a dedicated-RBC is the high loaded source-to-relay link. Specifically, in this study, the messages of the two users are delivered by the source to the relay through a single source-to-relay link, which increases the chances that the relay fails to recover the user messages. Hence, a no-worse performance is expected from the AF Broadcast than the DF Broadcast, especially when the relay is in close proximity to the destination nodes. However, with a better source-to-relay link, the probability of the successful decoding of x_1 and x_2 by the relay is increased and, at the same time, the negative impact of the noise amplification on the performance of the AF Broadcast is reduced. Finally, by averaging over all channel realizations, the possibility of the AF Broadcast to perform worse than the DF Broadcast is quite low.

Figure 11 shows the power gain of the AF Broadcast over the DF Broadcast. For comparison purposes, we choose θ_1 and θ_2 such that the three cases have the same values of d_{sd_1}, d_{sd_2}, d_{rd_1}, and d_{rd_2}. It can be seen that the AF Broadcast has a better performance for most of the cases and performs slightly worse than the DF Broadcast only in the low-rate regime. The power gain decreases with increasing proximity of the relay to the source.

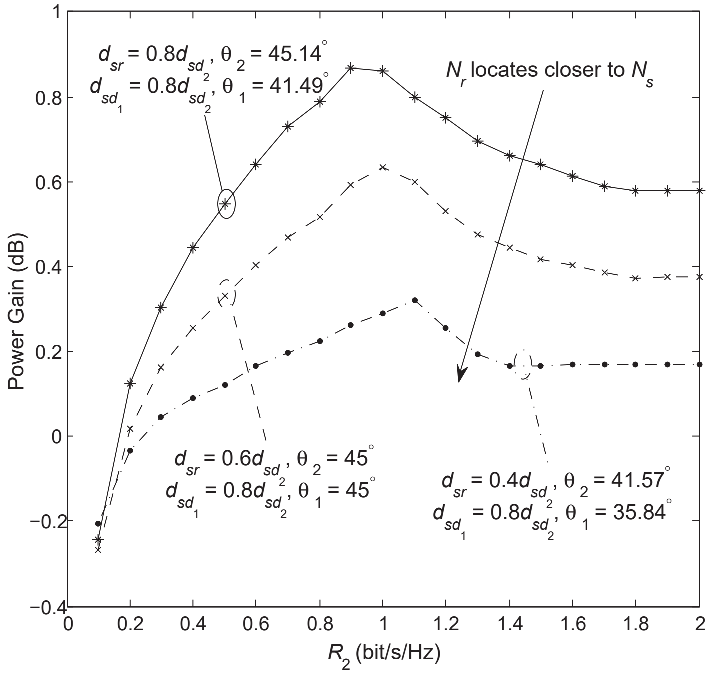


Figure 11. Power gain of AF Broadcast over DF Broadcast versus rate with $R_1 = R_2$ and $P_{out}^{th_1} = P_{out}^{th_2} = 0.01$.

6. Conclusions

In this study, two cooperative broadcast transmission protocols have been considered for the two-user dedicated RBC. By utilizing SupC, the messages of multiple users can be conveyed simultaneously over the same channel block, and by using a portion of the power used for the transmission of each user's message, a trade-off between the users' performances is achieved. We have shown that the bad user's outage behavior can be considerably improved with by a slight increase in the outage probability of the good user, which constitutes a better overall system performance.

The numerical results of a number of scenarios of interest demonstrated that the investigated broadcast transmission strategies generally provide better performances. In addition, the power gain achieved by the cooperative broadcast transmission is largely affected by the level of disparity in the channel qualities and in the quality-of-service requirements of the users, which implies that it is nontrivial to determine the target of cooperation in practical applications; this is an interesting subject for future research. Moreover, it was observed that the broadcast schemes are advantageous in the low to moderate rate regions and provide the furthest gain at certain moderate rates. The comparison between the AF and DF broadcast transmission protocols indicate that a good source-to-relay link is more crucial for the dedicated-RBC than for the conventional relay systems when a regenerative relay is used.

There is additional complexity associated with our cooperative broadcast schemes. First, the utilization of SupC makes the decoding at the relay and at the good user slightly more complex, compared to applications without SupC. In addition, a user pairing procedure is needed prior to the initiation of the communication when applied in a system with more users. Despite the significant improvement provided by our schemes, more efficient protocols should be explored in the future. Furthermore, there are many channel circumstances in addition to those considered in this study that warrant further investigation. Inspired by the results described in Sections 5.3 and 5.4, more comprehensive studies on the impact of the geometry and quality-of-service requirements on the comparable performances of different protocols are needed to provide guidance for practical applications.

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Appendix A. Proof of Theorem 1

To proceed, we introduce two auxiliary sets Θ_1 and Θ_1, which are functions of ω and are defined as follows:

$$\Theta_1(\omega) = \{\gamma : \omega\gamma < t_1\}, \Theta_2(\omega) = \left\{\gamma : \frac{\bar{\omega}\gamma}{1+\omega\gamma} < t_2\right\},$$

where $\omega \in [0,1]$, γ indicates any nonnegative random variables, and $t_1, t_2 \geq 0$. It can be simply verified that $\Theta_2 \subseteq \Theta_1$ if and only if $w \leq \frac{t_1}{(t_1+1)(t_2+1)-1}$. In the following, we use $\hat{t}$ to denote $\frac{t_1}{(t_1+1)(t_2+1)-1}$. Then, we consider the following two sets:

$$\begin{aligned} \Phi_1 &= \left\{ (\gamma_1, \gamma_2) : a\gamma_1 + b\gamma_2 < t_1 \right\}, \\ \Phi_2 &= \left\{ (\gamma_1, \gamma_2) : \frac{\bar{a}\gamma_1}{1+a\gamma_1} + \frac{\bar{b}\gamma_2}{1+b\gamma_2} < t_2 \right\}. \end{aligned} \tag{A1}$$

When $a > \hat{t}$, it is obvious that $\Theta_2(a) \nsubseteq \Theta_1(a)$. Then, by taking $\gamma_2 = 0$ for Φ_1 and Φ_2, we obtain $\Phi_2 \nsubseteq \Phi_1$. The same is true when $b > \hat{t}$. Hence, it is necessary for a and b to be no larger than $\hat{t}$ such that $\Phi_2 \subseteq \Phi_1$.

The regions of (γ_1, γ_2) defined by Φ_1 and Φ_2 can be equivalently expressed as in Equations (A3) and (A4); on top of the next page, where μ and ν are auxiliary variables. Suppose $0 < \alpha \leq \hat{t}$. Then, for arbitrarily fixed μ, $\mu \in [0,1]$, Φ_1 in Equation (A3) has the following equivalent expression:

$$\Phi_1 = \underbrace{\left\{ (\gamma_1, \gamma_2) : a\gamma_1 < \mu t_1 \right\}}_{\Phi_{1a}} \bigcap \underbrace{\left\{ (\gamma_1, \gamma_2) : b\gamma_2 < \nu t_1 \right\}}_{\Phi_{1b}},$$

where $\nu = 1 - \mu$. For the same μ and ν, Φ_2 in Equation (A4) has the following equivalent expression:

$$\Phi_2 = \underbrace{\left\{ (\gamma_1, \gamma_2) : \frac{\bar{a}\gamma_1}{1+a\gamma_1} < \mu t_2 \right\}}_{\Phi_{2a}} \bigcap \underbrace{\left\{ (\gamma_1, \gamma_2) : \frac{\bar{b}\gamma_2}{1+b\gamma_2} < \nu t_2 \right\}}_{\Phi_{2b}}. \tag{A2}$$

It can be simply verified that when $a, b \leq \hat{t}$, $\Phi_{2a} \subseteq \Phi_{1a}$ and $\Phi_{2b} \subseteq \Phi_{1b}$, based on which we definitely have $\{\Phi_{2a} \bigcap \Phi_{2b}\} \subseteq \{\Phi_{1a} \bigcap \Phi_{2b}\}$. Thus, $a, b \leq \hat{t}$ is a sufficient condition for Φ_2 to be a subset of Φ_1.

Recall that $2^{\tau_1} - 1 \geq 0$ and $2^{\tau_2} - 1 \geq 0$, Theorem 1 is proved:

$$\Phi_1 = \left\{ (\gamma_1, \gamma_2) : a\gamma_1 < \mu t_1, b\gamma_2 < \nu t_1, 0 \leq \mu \leq 1, \nu = 1 - \mu \right\}, \tag{A3}$$

$$\Phi_2 = \left\{ (\gamma_1, \gamma_2) : \frac{\bar{a}\gamma_1}{1+a\gamma_1} < \mu t_2, \frac{\bar{b}\gamma_2}{1+b\gamma_2} < \nu t_2, 0 \leq \mu \leq 1, \nu = 1 - \mu \right\}. \tag{A4}$$

Appendix B. Characterization of B_1 and B_2

We characterize the sets B_1 and B_2, such that the integrals ψ_1 and ψ_2 can be directly calculated. First, we consider B_1 as defined in Equation (29), which can be equivalently expressed as in Equation (A5):

$$B_1 \equiv \begin{cases} \left\{(\gamma_{sd_2}, \gamma_{rd_2}) : \frac{\bar{\alpha}\gamma_{sd_2}}{1+\alpha\gamma_{sd_2}} < 2^{\tau_2} - 1 - \frac{\bar{\beta}\gamma_{rd_2}}{1+\beta\gamma_{rd_2}}, \gamma_{rd_2} < \frac{2^{\tau_2}-1}{1-\beta 2^{\tau_2}}\right\}, \beta < \frac{1}{2^{\tau_2}}, \\ \left\{(\gamma_{sd_2}, \gamma_{rd_2}) : \frac{\bar{\alpha}\gamma_{sd_2}}{1+\alpha\gamma_{sd_2}} < 2^{\tau_2} - 1 - \frac{\bar{\beta}\gamma_{rd_2}}{1+\beta\gamma_{rd_2}}\right\}, \ \beta \geq \frac{1}{2^{\tau_2}}. \end{cases} \tag{A5}$$

For brevity, we introduce the following notation:

$$p(\gamma_{rd_2}) = 2^{\tau_2} - 1 - \frac{\bar{\beta}\gamma_{rd_2}}{1+\beta\gamma_{rd_2}}. \tag{A6}$$

It can be simply verified that $p(\gamma_{rd_2}) > 0$ with the constraint on γ_{rd_2} as stated in Equation (A5). The first constraint in Equation (A5) can be equivalently expressed as

$$\left[\bar{\alpha} - \alpha p(\gamma_{rd_2})\right] \gamma_{sd_2} < p(\gamma_{rd_2}), \tag{A7}$$

which falls into two cases based on whether $\bar{\alpha}$ is larger than $\alpha p(\gamma_{rd_2})$. When $\bar{\alpha} \leq \alpha p(\gamma_{rd_2})$, Equation (A7) establishes for all valid values of γ_{sd_2}.

When $\bar{\alpha} > \alpha p(\gamma_{rd_2})$, Equation (A7) degrades to

$$\gamma_{sd_2} < \frac{p(\gamma_{rd_2})}{\bar{\alpha} - \alpha p(\gamma_{rd_2})}. \tag{A8}$$

Now, we consider the condition $\bar{\alpha} \leq \alpha p(\gamma_{rd_2})$, which can be rephrased as

$$\left[\bar{\beta} - \beta\left(2^{\tau_2} - \frac{1}{\alpha}\right)\right] \gamma_{rd_2} \leq 2^{\tau_2} - \frac{1}{\alpha}. \tag{A9}$$

It is obvious from Equation (A9) when $\alpha < \frac{1}{2^{\tau_2}}$, $\bar{\alpha} \leq \alpha p(\gamma_{rd_2})$ is always false; when $\alpha \geq \frac{1}{2^{\tau_2}}$, if $\bar{\beta} - \beta(2^{\tau_2} - \frac{1}{\alpha}) \leq 0$ (i.e. $\beta \geq \frac{1}{2^{\tau_2}+1-1/\alpha}$), $\bar{\alpha} \leq \alpha p(\gamma_{rd_2})$ establishes for all valid γ_{rd_2}, else $\bar{\alpha} \leq \alpha p(\gamma_{rd_2})$ only when

$$\gamma_{rd_2} \leq \frac{2^{\tau_2} - \frac{1}{\alpha}}{\bar{\beta} - \beta\left(2^{\tau_2} - \frac{1}{\alpha}\right)}. \tag{A10}$$

Based on all above discussions and the fact that $\alpha \leq \alpha^{max}$, $\beta \leq \alpha^{max}$, and $\alpha^{max} \leq \frac{1}{2^{\tau_2}}$ (the equality is valid only when τ_2 equals to zero), we have, for the case of interest of $\tau_2 > 0$ (When $\tau_2 = 0$, there is no message to be transmitted to N_{d_2}, and the PSFs should be assigned such that $\alpha = 1$ and $\beta = 1$, namely all the power are allocated for transmission of N_{d_1}'s message; as a result, the BRC degrades to a conventional relay channel. Similarly, in Appendix C, we have the assumptions that $\alpha > 0$, $\tau_2 > 0$, and $\tau_1 > 0$.), that

$$B_1 \equiv \left\{(\gamma_{sd_2}, \gamma_{rd_2}) : \gamma_{sd_2} < \hat{p}(\gamma_{rd_2}), \gamma_{rd_2} < l\right\}, \tag{A11}$$

where, for convenience, we have used the following notations:

$$\hat{p}(\gamma_{rd_2}) = \frac{p(\gamma_{rd_2})}{\bar{\alpha} - \alpha p(\gamma_{rd_2})}, \ l = \frac{2^{\tau_2} - 1}{1 - \beta 2^{\tau_2}}. \tag{A12}$$

Then, we consider B_2 as defined in Equation (29). In fact, B_2 is a special case of B_1 with $\beta = 0$, hence B_2 can be equivalently expressed as

$$B_2 \equiv \left\{ (\gamma_{sd_2}, \gamma_{rd_2}) : \gamma_{sd_2} < \tilde{p}(\gamma_{rd_2}), \gamma_{rd_2} < 2^{\tau_2} - 1 \right\}, \tag{A13}$$

where

$$\tilde{p}(\gamma_{rd_2}) = \frac{2^{\tau_2} - 1 - \gamma_{rd_2}}{\bar{\alpha} - \alpha(2^{\tau_2} - 1 - \gamma_{rd_2})}. \tag{A14}$$

Appendix C. Degradeness Condition in the AF Protocol

We consider the outage events in Equations (32) and (33), and denote them using the notations $\mathcal{O}_{2,d_1}$ and $\mathcal{O}_{1|2,d_1}$, respectively. We focus on the scenario when $\alpha > 0$, $\tau_2 > 0$, and $\tau_1 > 0$, and the problem falls into two cases based on whether α is smaller than $\frac{1}{2^{\tau_2}}$. First, we consider the case of $\alpha \geq \frac{1}{2^{\tau_2}}$. It can be simply verified that

$$\frac{\bar{\alpha}\gamma_{sd_1}}{1 + \alpha\gamma_{sd_1}} < 2^{\tau_2} - 1$$

is a certain event by considering its equivalent event

$$(1 - \alpha 2^{\tau_2})\gamma_{sd_1} < 2^{\tau_2} - 1.$$

Thus, the outage event $\mathcal{O}_{2,d_1}$ can be equivalently expressed as

$$\left[\text{all valid } \gamma_{sd_1} \right] \bigcap \left[\frac{\bar{\alpha}\gamma_{sr}\gamma_{rd_1}}{\alpha\gamma_{sr}\gamma_{rd_1} + \gamma_{sr} + \gamma_{rd_1} + 1} < 2^{\tau_2} - 1 - \frac{\bar{\alpha}\gamma_{sd_1}}{1 + \alpha\gamma_{sd_1}} \right]. \tag{A15}$$

In addition, the outage event $\mathcal{O}_{1|2,d_1}$ has the following equivalent expression

$$\left[\gamma_{sd_1} < \frac{2^{\tau_1} - 1}{\alpha} \right] \bigcap \left[\frac{\gamma_{sr}\gamma_{rd_1}}{\gamma_{sr} + \gamma_{rd_1} + 1} < \frac{2^{\tau_1} - 1}{\alpha} - \gamma_{sd_1} \right]. \tag{A16}$$

Definitely, from Equations (A15) and (A16), $\mathcal{O}_{2,d_1}$ is not a subevent of $\mathcal{O}_{1|2,d_1}$. Hence, x_2 is not degraded to x_1 when $\alpha \geq \frac{1}{2^{\tau_2}}$. Now, we consider the case of $0 < \alpha < \frac{1}{2^{\tau_2}}$. Following the same line of discussion as in Appendix D and Section 5.1.2, the outage events $\mathcal{O}_{1|2,d_1}$ can be decomposed and rephrased as

$$\begin{aligned} &\left\{ \gamma_{sd_1} < l_1 \right\} \bigcap \left\{ \left[\gamma_{sr} \leq m(\gamma_{sd_1}) \bigcup \gamma_{rd_1} \leq m(\gamma_{sd_1}) \right] \right. \\ &\bigcup \left[\left(m(\gamma_{sd_1}) < \gamma_{sr} < \gamma^* \right) \bigcap \left(\gamma_{sr} < \gamma_{rd_1} < n(\gamma_{sr}) \right) \right] \\ &\left. \bigcup \left[\left(m(\gamma_{sd_1}) < \gamma_{rd_1} < \gamma^* \right) \bigcap \left(\gamma_{rd_1} \leq \gamma_{sr} < n(\gamma_{rd_1}) \right) \right] \right\}, \end{aligned} \tag{A17}$$

where l_1, $m(\gamma_{sd_1})$, n, and γ^* are as defined in Equation (38). Here, we restate them for the coherence of the discussion:

$$l_1 = \frac{2^{\tau_1} - 1}{\alpha}, \tag{A18}$$

$$m(\gamma_{sd_1}) = \frac{2^{\tau_1} - 1}{\alpha} - \gamma_{sd_1}, \tag{A19}$$

$$n(\gamma) = \frac{(\gamma + 1)m(\gamma_{sd_1})}{\gamma - m(\gamma_{sd_1})}, \gamma = \gamma_{sr}, \gamma_{rd_1}, \tag{A20}$$

$$\gamma^* = m(\gamma_{sd_1}) + \sqrt{[m(\gamma_{sd_1})]^2 + m(\gamma_{sd_1})}. \tag{A21}$$

In addition, $\mathcal{O}_{2,d_1}$ can be expressed in the same form as in Equation (A17), with the difference that

$$l_1 = \frac{2^{\tau_2} - 1}{1 - \alpha 2^{\tau_2}}, \tag{A22}$$

$$m(\gamma_{sd_1}) = \frac{2^{\tau_2} - 1 - \frac{\bar{\alpha}\gamma_{sd_1}}{1+\alpha\gamma_{sd_1}}}{\bar{\alpha} - \alpha\left(2^{\tau_2} - 1 - \frac{\bar{\alpha}\gamma_{sd_1}}{1+\alpha\gamma_{sd_1}}\right)}. \tag{A23}$$

For $\mathcal{O}_{2,d_1}$ to be a subevent of $\mathcal{O}_{1|2,d_1}$, an obvious condition that needs to be satisfied is that

$$\frac{2^{\tau_2} - 1}{1 - \alpha 2^{\tau_2}} \leq \frac{2^{\tau_1} - 1}{\alpha}.$$

With some algebraic manipulations, it can be proved that the above condition is true only when $\alpha \leq \alpha^{max}$ under the assumption that $0 < \alpha < \frac{1}{2^{\tau_2}}$. To proceed, we assumed that $\alpha \leq \alpha^{max}$. An interesting observation is that both $n(\gamma)$ and γ^* are monotonously increasing function of $m(\gamma_{sd_1})$.

Define

$$y(\gamma_{sd_1}) = \frac{2^{\tau_1} - 1}{\alpha} - \gamma_{sd_1} - \frac{2^{\tau_2} - 1 - \frac{\bar{\alpha}\gamma_{sd_1}}{1+\alpha\gamma_{sd_1}}}{\bar{\alpha} - \alpha\left(2^{\tau_2} - 1 - \frac{\bar{\alpha}\gamma_{sd_1}}{1+\alpha\gamma_{sd_1}}\right)}. \tag{A24}$$

It is intuitive to see that $\mathcal{O}_{2,d_1}$ will be a subevent of $\mathcal{O}_{1|2,d_1}$ if $y(\gamma_{sd_1}) \geq 0$ is true for all $\gamma_{sd_1} < l_1$. In order to find the minimal $y(\gamma_{sd_1})$, expression Equation (A24) is differentiated with respect to γ_{sd_1}. The γ_{sd_1} value that minimizes $y(\gamma_{sd_1})$ is

$$\gamma_{sd_1}^* = \frac{2^{\tau_2} - 1}{\bar{\alpha} + 1 - \alpha 2^{\tau_2}}.$$

Obviously, $\gamma_{sd_1}^* < l_1$. Correspondingly,

$$y(\gamma_{sd_1}^*) = \frac{(2^{\tau_1} + 1)[2 - \alpha(2^{\tau_2} + 1)]}{\alpha(\bar{\alpha} + 1 - \alpha 2^{\tau_2})}.$$

It can be validated by some manipulations that $y(\gamma_{sd_1}^*)$ is nonnegative under the assumption that $\alpha \leq \alpha^{max}$.

In conclusion, with the AF broadcast transmission protocol, message x_2 is degraded to x_1 (from the decoding sense) if and only if the condition $\alpha \leq \alpha^{max}$ is satisfied.

Appendix D. Outage Event of Conventional AF Relaying

Here, we characterize the outage event of a conventional AF protocol in a more intuitive format, such that the analytic outage expression can be easily written. We use N_{d_1} as an example and the case of N_{d_2} can be dealt with similarly. As in [7], the outage event of N_{d_1} is given as

$$\gamma_{sd_1} + \frac{\gamma_{sr}\gamma_{rd_1}}{\gamma_{sr} + \gamma_{rd_1} + 1} < 2^{\tau_1} - 1. \tag{A25}$$

We define

$$t = m(\gamma_{sd_1}) = 2^{\tau_1} - 1 - \gamma_{sd_1}, \tag{A26}$$

$$\gamma^*(\gamma_{sd_1}) = m(\gamma_{sd_1}) + \sqrt{m(\gamma_{sd_1})^2 + m(\gamma_{sd_1})}.$$

Then, we consider the event defined in Equation (A25), which can be equivalently expressed as

$$\left[\gamma_{sd_1} < 2^{\tau_1} - 1\right] \bigcap \left[\frac{\gamma_{sr}\gamma_{rd_1}}{\gamma_{sr} + \gamma_{rd_1} + 1} < 2^{\tau_1} - 1 - \gamma_{sd_1}\right]. \tag{A27}$$

The inequality

$$\frac{\gamma_{sr}\gamma_{rd_1}}{\gamma_{sr} + \gamma_{rd_1} + 1} < t, \tag{A28}$$

$\forall t > 0$, has the following two equivalent expressions:

$$\gamma_{rd_1}(\gamma_{sr} - t) < (\gamma_{sr} + 1)t, \tag{A29}$$

$$\gamma_{sr}(\gamma_{rd_1} - t) < (\gamma_{rd_1} + 1)t. \tag{A30}$$

It is obvious from Equations (A29) and (A30) that the inequality in Equation (A28) occurs when either of γ_{sr} and γ_{rd_1} is smaller than t, then we have A_1 in Equation (A31) (provided that $2^{\tau_1} - 1 - \gamma_{sd_1} > 0$):

$$A_1 \equiv \left[\gamma_{sr} \le m(\gamma_{sd_1})\right], \bigcup \left[\gamma_{rd_1} \le m(\gamma_{sd_1})\right], \tag{A31}$$

Furthermore, we consider the case $\gamma_{sr} > t$ and $\gamma_{rd_1} > t$. In this case, the inequalities in Equations (A29) and (A30) convert to the following two events

$$\left[\gamma_{sr} > t \bigcap \gamma_{sr} < \gamma_{rd_1} < \frac{(\gamma_{sr}+1)t}{\gamma_{sr} - t}\right] \bigcup \left[t < \gamma_{rd_1} < \min\left(\gamma_{sr}, \frac{(\gamma_{sr}+1)t}{\gamma_{sr} - t}\right)\right] \tag{A32}$$

$$\left[\gamma_{rd_1} > t \bigcap \gamma_{rd_1} < \gamma_{sr} < \frac{(\gamma_{rd_1}+1)t}{\gamma_{rd_1} - t}\right] \bigcup \left[t < \gamma_{sr} < \min\left(\gamma_{rd_1}, \frac{(\gamma_{rd_1}+1)t}{\gamma_{rd_1} - t}\right)\right] \tag{A33}$$

respectively, the *intersection* of which forms the event defined by Equation (A28), which can be further expressed as an union of two disjoint events based on the relationship between γ_{sr} and γ_{rd_1} as is shown in Equations (A32) and (A33). First, consider if $\gamma_{sr} < \gamma_{rd_1}$, the intersection of Equations (A32) and (A33) results in the following event

$$\left[\gamma_{sr} < \gamma_{rd_1} < \frac{(\gamma_{sr}+1)t}{\gamma_{sr} - t}\right] \bigcap \left[t < \gamma_{sr} < \min\left(\gamma_{rd_1}, \frac{(\gamma_{rd_1}+1)t}{\gamma_{rd_1} - t}\right)\right]. \tag{A34}$$

Since $\gamma_{sr} < \gamma_{rd_1}$, it can be simply verified that $\frac{(\gamma_{sr}+1)t}{\gamma_{sr}-t} < \frac{(\gamma_{rd_1}+1)t}{\gamma_{rd_1}-t}$, $\forall t > 0$ through the general discussion on monotonic functions. Then, we know that γ_{rd_1} is smaller than $\frac{(\gamma_{rd_1}+1)t}{\gamma_{rd_1}-t}$. Thus, we obtain the valid range of γ_{sr} as $t < \gamma_{sr} < \frac{(\gamma_{sr}+1)t}{\gamma_{sr}-t}$. With the inequality $\gamma_{sr} < \frac{(\gamma_{sr}+1)t}{\gamma_{sr}-t}$ and the fact that $\gamma_{sr} \geq 0$, we have $\gamma_{sr} < t + \sqrt{t^2+t}$. From all above discussions, Equation (A34) converts to

$$\left[t < \gamma_{sr} < t + \sqrt{t^2+t}\right] \bigcap \left[\gamma_{sr} < \gamma_{rd_1} < \frac{(\gamma_{sr}+1)t}{\gamma_{sr}-t}\right]. \tag{A35}$$

By using $t = 2^{\tau_1} - 1 - \gamma_{sd_1}$, Equation (A36) is obtained:

$$\begin{aligned} A_2 \equiv & \left[m(\gamma_{sd_1}) < \gamma_{sr} < \gamma^*(\gamma_{sd_1})\right] \\ & \bigcap \left[\gamma_{sr} < \gamma_{rd_1} < \frac{(\gamma_{sr}+1)m(\gamma_{sd_1})}{\gamma_{sr}-m(\gamma_{sd_1})}\right]. \end{aligned} \tag{A36}$$

A similar discussion can be conducted for the case of $\gamma_{sr} \geq \gamma_{rd_1}$ and A_3 in Equation (A37) can be obtained. Details on this are omitted here for the sake of brevity:

$$\begin{aligned} A_3 \equiv & \left[m(\gamma_{sd_1}) < \gamma_{rd_1} < \gamma^*(\gamma_{sd_1})\right] \\ & \bigcap \left[\gamma_{rd_1} < \gamma_{sr} < \frac{(\gamma_{rd_1}+1)m(\gamma_{sd_1})}{\gamma_{rd_1}-m(\gamma_{sd_1})}\right]. \end{aligned} \tag{A37}$$

In conclusion, the outage event of Nd_1 can be expressed as

$$\left[\gamma_{sd_1} < 2^{\tau_1} - 1\right] \bigcap \left[\bigcup_{i=1,2,3} A_i\right], \tag{A38}$$

with A_1,A_2,A_3 are defined in Equations (A31), (A36) and (A37), respectively.

Thus, the outage probabilities of conventional DF relaying with orthogonal multiplexing at N_{d_1} can be calculated as Equations (49)–(52).

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