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Entropy Characteristic on Harmonious Unifying Hybrid Preferential Networks

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Abstract: Based on the harmonious unifying hybrid preferential model (*HUHPM*) network proposed by our group, the entropy characteristic of an un-weighted *HUHPM-BA* network and a weighted *HUHPM-BBV* network are investigated as the total hybrid ratio *d/r* is changed. We derive and compute the general relation of the power exponent of the degree distribution with the entropy by using the Boltzmann-Gibbs entropy (*BGS*) and the Tsallis non-extensive entropy (S_q). It is found that the *BGS* decreases as *d/r* increases and the current of the *BGS* along with hybrid ratio *d/r* or exponent γ of power-law is in agreement between numerical simulation and theoretical analysis. The relationship between the S_q and characteristic parameter q under different *d/r* is also given. And the S_q approaches to the *BGS* when $q \to 1$. These results can provide a better understanding for evolution characteristic in growing complex networks and further applications in network engineering are of prospective potential.

Keywords: Entropy, harmonious unifying hybrid preferential model, un-weighted network, weighted network

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1. Introduction

The discovery of the ubiquity of small world (*SW*) [1] and scale-free (*SF*) networks [2, 3] has led to many exciting insights into fundamental underlying principles that govern complex systems. It has been realized that, despite functional diversity, most real world networks like WWW and Internet systems share important structural features, e.g: small average path length, high clustering, and scale-free degree distribution. A number of network models have been proposed to embody the fundamental characteristics [4-15]. Among them our group led by Fang has proposed the harmonious unifying hybrid preferential model (*HUHPM*) to a certain class of complex networks, which is controlled by a total hybrid ratio *d/r* of deterministic preferential attachment (*DPA*) to random preferential attachment (*RPA*) [4, 10-12]. The unified hybrid ratio can be defined as

$$d/r = \frac{\text{Time intervals } d \text{ of } DPA}{\text{Time intervals } r \text{ of } RPA}$$

where d is the number of time intervals (steps) for *DPA*, and r is the number of *RPA*. Within this model we study both numerically and theoretically. It was found that the *HUHPM* has a key universal topological property and dynamical synchronizability, and one of most important features is that the exponents of power γ (node degree, node strength, and edged weight) are sensitive to the *d/r* or depends on the *d/r* strongly [11,12].

As is well known, statistical mechanics have made a great contribution towards the main advances of topology and dynamics in complex networks, as emphasized by Albert and Barabási [3]. People raise a question here: Entropy as another very important characteristic quantity for statistical mechanics, what role does the entropy play in complex networks? Entropy emerges as a classical thermodynamical concept in the 19th century with Clausius but it is due to the work of Boltzmann and Gibbs that the idea of entropy becomes a cornerstone of statistical mechanics. As a result we have that the entropy of a complex system is given by the so-called Boltzmann-Gibbs entropy (*BGS*)

$$BGS = -k\sum_{i=1}^{\infty} P(i)\ln P(i) \tag{1}$$

with the normalization condition

$$\sum_{i=1}^{\infty} P(i) = 1,$$

where $P(i)$ is the probability for the system to be in the i-th microstate, and k is a Boltzmann constant. Without loss of generality one can also support normalization $k = 1$. In 1988 Constantino Tsallis introduced the non-extensive entropy S_q [16, 17]

$$S_q = -\frac{1}{1-q}\{1 - \sum_{i=1}^{\infty}[P(i)]^q\} \tag{2}$$

as a generalization of the *BGS*, where q is a characteristic parameter. It can be regarded as a minimal extension of Shannon entropy, to which it reduces when $q \rightarrow 1$. Parameter q describes therefore summarily all effects [16, 17].

In this paper we use the *BGS* to measure the relationship of entropy with *d/r*, as well as power exponent γ. Through the numerical simulation and theoretical analysis we validate the correctness of our results in reference [4]. It is found that as *d/r* increases the *BGS* and S_q decrease, which implies that the self-organization of *HUHPM* is enhanced as hybrid ratio *d/r* increases. The relationship between the S_q under different *d/r* is also derived. And the S_q approaches to the *BGS* when $q \to 1$. These results can provide a better understanding for evolution characteristic in growing complex networks.

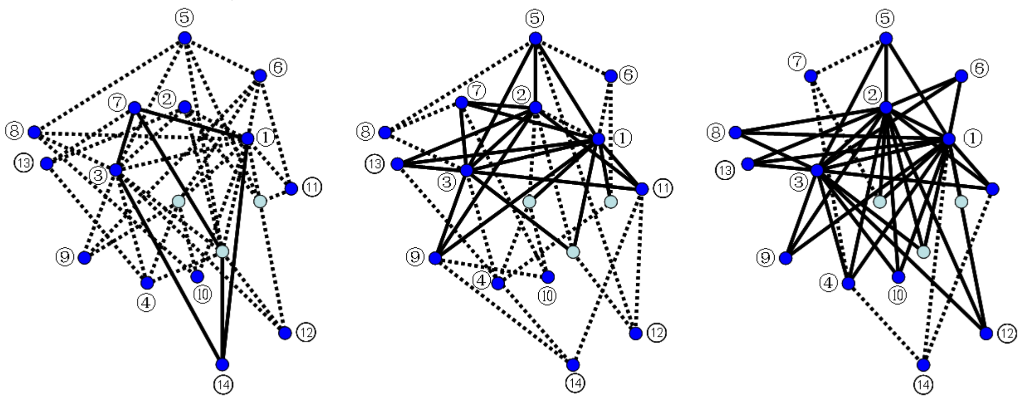
The whole paper is organized as follows. In the Section 2, some hybrid preferential models are firstly described. Then in the Section 3 and Section 4, we respectively investigate two kinds of entropy in different *HUHPM* networks. Theoretical results are verified by several simulation results. We conclude the paper and remark in Section 5.

2. Basic network models: the *HUHPM*

Based on the Barabási and Albert (*BA*) scale-free (*SF*) network [2, 3], Fang and Liang [4] proposed a hybrid preferential model (*HPM*). Furthermore, our group develops the idea to *HUHPM* model. The basic concept and method for the *HUHPM* can be expressed as the following [4]:

$$HUHPM = \begin{array}{c}\text{Deterministic Preferential Attachment } (DPA) \\ + \\ \text{Random Preferential Attachment } (RPA)\end{array}$$

This means that the *HUHPM* can rest on any type of network's original growth way and *RPA* pattern by adding the *DPA* pattern according to certain arrangement of the degree distribution to arrive an expected topological properties. This implementation combines the random connection with the deterministic connection by using the hybrid ratio *d/r* to request growth scale size of the networks. In the process of the network evolution, the hybrid ratio must maintain the same value by combining *RPA* and *DPA*. Actually, the kind of preferential attachments carried on at first is flexible. This means that one can use different order to make the two hybrids grow the network in turn, until the required scale size is achieved. Figure 1 illustrates diagrams for evolution of *HUHPM* growing networks with N=14 and m=m_0=3 under different hybrid ratio *d/r*=1/6, 1/1 and 6/1.



(a) (b) (c)

Figure 1. The network's evolution process according to different *d/r* in *HUHPM* model. And the number of nodes is N=14, m=m_0=3. Fig(a) for *d/r*=1/6, Fig(b) for *d/r*=1/1and Fig(c) for *d/r*=6/1.

Barabási and Albert (*BA*) [5] proposed an un-weighted network model using a famous preferential attachment mechanism. Because the weighted networks can more carefully portray the nodes connection and mutual interaction, not only reflect the topology of real network, but also reveal physical and dynamical characteristics for the real-world networks. Barrat, Barthelemy and Vespignani

(*BBV*) [6,7] proposed a weighted network model with a tunable perturbation parameter δ to rearrange the edges' weights locally using a random preferential attachment mechanism. *BBV* turns to *BA* when δ=0. *BBV* model can yield scale-free (*SF*) properties of the degree, strength and weight distributions controlled by weight associated parameter δ. However, both models, *BA* and *BBV*, only consider random preferential attachment (*RPA*) and do not consider deterministic preferential attachment (*DPA*) or connection effect. This is in contradiction to a fundamental observation, which has both deterministic and random factors extensively exist in the unifying real-world. Therefore, both of *RPA* and *DPA* in various complex network models should be paid attention to and be investigated carefully. Therefore, our group proposed the harmonious unifying hybrid preferential model (*HUHPM*), and applied it to the un-weighted *BA* and the weighted *BBV* models, so called *HUHPM-BA* and *HUHPM-BBV* models. Since *BA* and *BBV* can be described as different network statistical characterizations. In fact, the *HUHPM* can be applied to any un-weighted and weighted networks. The concrete growth process may refer to the related paper [4-7, 10-11, 18]. It is known that the *HUHPM* network has a series of topological property.

3. Entropy characteristic in the HUHPM-BA

In this section, we use the *BGS* to measure the relationship of entropy and *d/r*, as well as power exponent γ. In reference [10] we theoretically get the approximate degree distribution as follows, respectively. For un-weighted network *HUHPM-BA*, we have

$$P(k) \propto 2m^{\frac{1}{\beta}} k^{-\left(\frac{1}{\beta}+1\right)}$$

$$\gamma = \frac{1}{\beta} + 1 = \frac{A_1}{\exp\left[\left(\dfrac{d/r}{A_2}\right)^{A_3}\right]} + A_4 \tag{3}$$

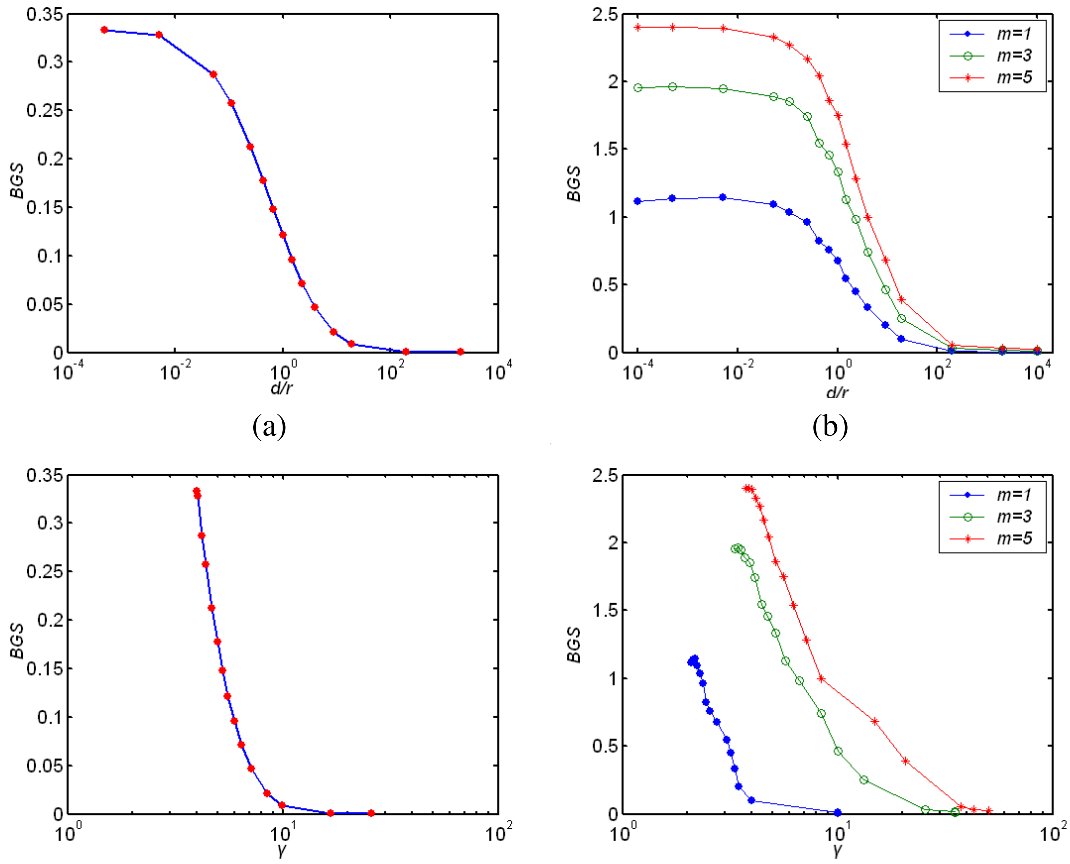
where k is the degree of network node and $P(k)$ denotes the probability distribution of nodes whose degree is k. The *d/r* is hybrid ratio and the γ is exponents of power-law. All the coefficients A_i above are taken from reference [10], i=1, 2, 3, 4. To compute the entropy, we consider approximately that β is a constant for different m and

$$P(k) = \frac{2m^{\frac{1}{\beta}} k^{-(\frac{1}{\beta}+1)}}{\sum_i 2m^{\frac{1}{\beta}} i^{-(\frac{1}{\beta}+1)}} = \frac{1}{\sum_i (\frac{k}{i})^{\frac{1}{\beta}+1}} \tag{4}$$

where $P(k)$ satisfying normalization condition

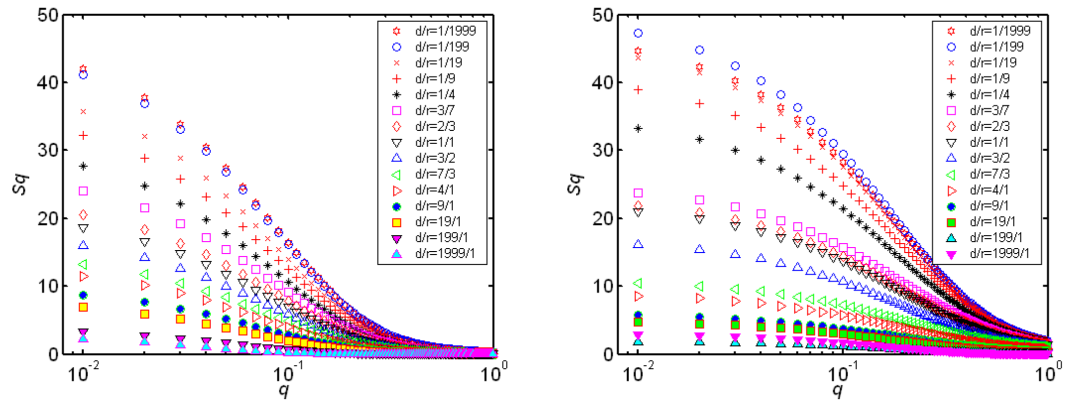
$$\sum P(k) = 1$$

Using formula (1), (3) and (4), we draw the theoretical curves *BGS* VS *d/r* and *BGS* VS γ, as shown in figure 2(a) and (c). The corresponding results of numerical simulation are shown in figure 2(b) and (d).



(a) (b) (c) (d)

Figure 2. *BGS* versus *d/r* and *γ* with network size *N*=2000, *m*=1, 3, 5, where (a), (c) are theoretical results for *BGS* and (b), (d) are numerical simulation results respectively.



(a) (b)

Figure 3. The relationship between S_q and q under different *d/r*, *N*=2000. The *d/r* of these plots spans from 1/1999 to 1999/1, where (a) is theoretical result and (b) is numerical simulation result.

According to the formula (2), figure 3 gives the relationship between S_q and q under different *d/r*. It is seen from figure 3 that for the fixed q value S_q reduces along with the *d/r* increase. When q = 1 it turns to *BGS*. Clearly, the tendency of change for two kinds of entropy above with the hybrid ratio *d/r* are similar each other. This means that two kinds of entropy are in consistent with each other basically when q is getting bigger. As mentioned above, however, because parameter q *only* describes summarily all effects [16, 17]. A few differences of two results for two kinds of entropy may not avoid.

4. Entropy characteristic in the weighted *HUHPM-BBV*

HUHPM-BBV model generates weighted graphs exhibiting the statistical properties observed in several real world systems. Considering a network grown by *HUHPM-BBV* model, its degree distribution is shown below.

$$P(k) \propto \frac{1}{m^{1-\frac{1}{\beta}} \beta k^{1+\frac{1}{\beta}}}$$

$$\gamma = 1 + \frac{1}{\beta} = \frac{4\delta + \dfrac{A_1}{\exp\left[\left(\dfrac{d/r}{A_2}\right)^{A_3}\right]} + A_4}{2\delta + 1} \tag{5}$$

where k is the degree of network node and $P(k)$ denotes the probability distribution of nodes whose degree is k. The d/r is total hybrid ratio and the γ is exponents of power-law. All the coefficients A_i above are taken from reference [10], i=1, 2, 3, 4. δ is the weighted parameter associated with perturbation of model.

To compute the entropy, we consider approximately that β is a constant for different m and

$$P(k) = \frac{1}{m^{1-\frac{1}{\beta}} \beta k^{1+\frac{1}{\beta}} \sum_i \dfrac{1}{m^{1-\frac{1}{\beta}} \beta i^{1+\frac{1}{\beta}}}} = \frac{1}{\sum_i \left(\dfrac{k}{i}\right)^{1+\frac{1}{\beta}}} \tag{6}$$

where $P(k)$ satisfying normalization condition

$$\sum P(k) = 1 .$$

Using formula (1), (5) and (6), we obtain the curves between *BGS* and *d/r*, *BGS* versus γ, as shown in figure 4(a) and (c). The corresponding results of numerical simulation are shown in figure 4(b) and (d). And the results of m=5 in figure 3 are the states of δ=0 corresponding to figure 4.

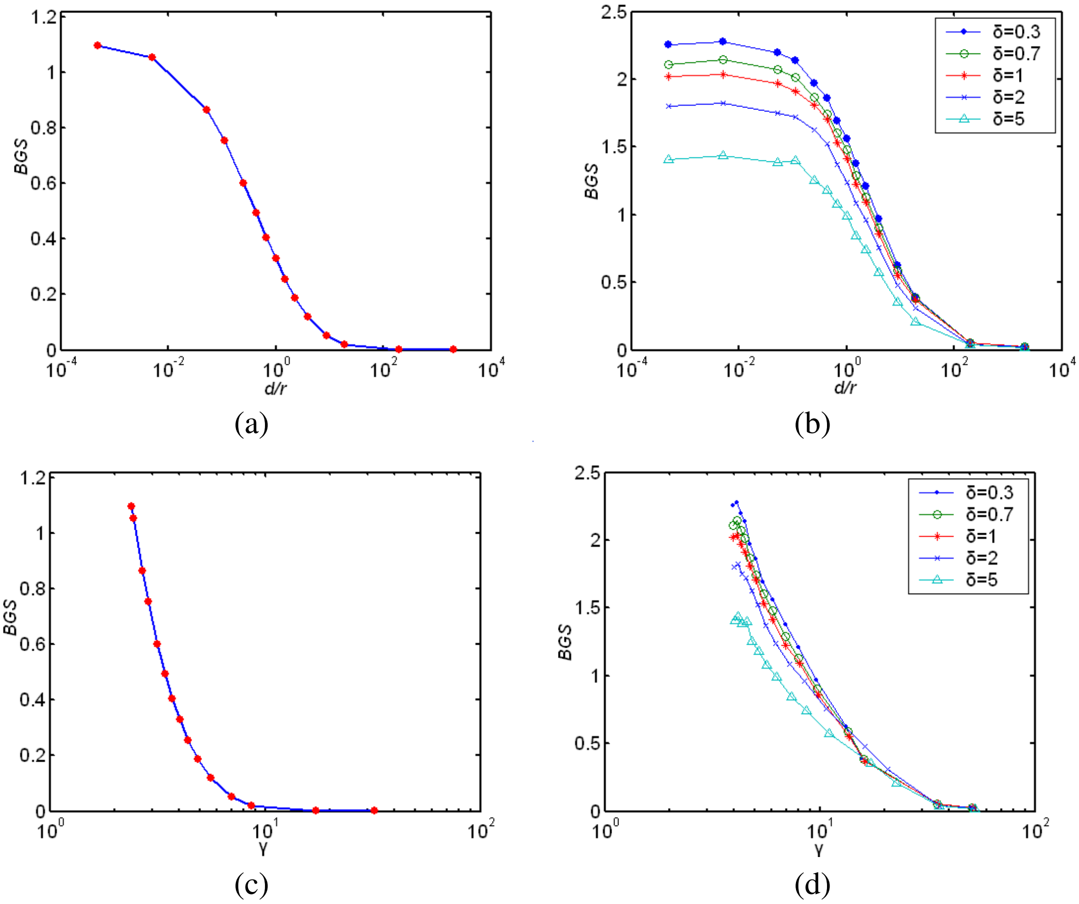


Figure 4. *BGS* versus *d/r* and γ with *N*=2000, *m*=5, where (a), (c) are theoretical results for *BGS* with δ=2 and (b), (d) are numerical simulation results respectively. δ is parameter in *BBV* model.

According to the formula (2), figure 5 gives the relationship between S_q and q under different *d/r*. If we consider the fixed q value, S_q reduces along with the *d/r* increasing, and does not associate with *m*. When q = 1 it turn to *BGS*.

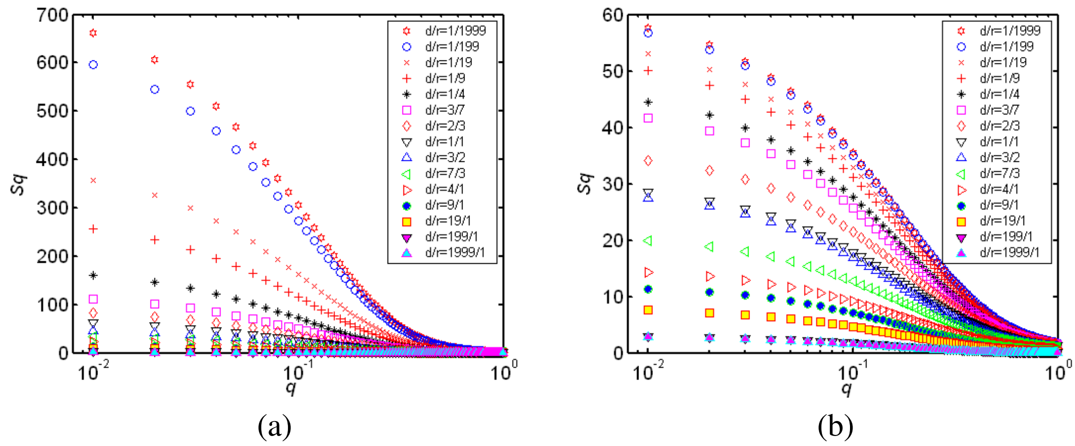


Figure 5. The relationship between Sq and q under different d/r for *N*=2000, *m*=5, δ=2. The *d/r* of these plots spans from 1/1999 to 1999/1, where (a) theoretical results and (b) numerical simulation results.

5. Conclusion and potential of application

Based on the networks grown by *HUHPM-BA* model and *HUHPM-BBV* model, we derive and compute the general relation of the power exponent of the degree distribution with the entropy using the *BG* entropy (*BGS*) and the Tsallis entropy (S_q). It is found that the *BGS* decreases as *d/r* increases and the current of the *BGS* along with hybrid ratio d/r or exponent γ of power-law is in consistent. The relationship between the S_q and q under different *d/r* is also given. And the S_q approaches to the *BGS* when $q \rightarrow 1$. These results can provide a better understanding for evolution characteristic in growing complex networks and further applications in network engineering are of prospective potential.

It is found that the *BGS* decreases as *d/r* increases and the current of the *BGS* along with hybrid ratio *d/r* or exponent *γ* of power-law is in consistent according to numerical simulation and theoretical analysis in both models. The entropy of a network provides an average measure of network's heterogeneity, since it measure the diversity of the degree distribution. In theoretical analysis, $P(k)$ is power exponent functions and it cut off the long tails(diversity) observed in simulations. So *BGS* in the simulations takes higher values than in the theory for both models. Also as exponent γ increases, the network becomes less heterogeneous and as a result lower entropy is observed. These results are good consistent with reference [19]. On the other hand, there still have some factors unconsidered while the values of *BGS* and S_q existing errors drawing from figure 2 – figure 5. One of reasons produced error is that because parameter *q only* describes total all effects in global and ignored detail factor, thus a few theoretical error for two kinds of entropy may be understood. Therefore, the form of $P(k)$ and exact entropy theory are still needed to improve deeply. So this subject is still open and challenge in network science for us.

Self-organization is an evolution process from disorder to order. In *HUHPM* networks, *BGS* and S_q decrease as *d/r* increases, which mean that states of disorder tend to order. This situation is easily understood that, since the total hybrid ratio $d/r \geq 1$ implies that determinism takes up the dominance and much more order is taken place within *HUHPM* networks. So the decrease of entropy for a complex system implies the self-organization of *HUHPM* networks is enhanced with total hybrid ratio *d/r* increases.

However, we believe that the *HUHPM* networks reveal one of essential mechanisms to the actual network to produce the *SF* and *SW* effects as well as entropy characteristic, therefore further applications in more widespread types of network are possible. For example, because the exponents of the three power law for the *HUHPM* networks have high sensitivity to the hybrid ratio *d/r* change, this may make a corresponding encryption method that applies to the cryptology and privacy communication domain. That means that based on track to initiate sensitive disposition, further experiments can be explored combining corresponding network theory. On the other hand, due to the fact that the power exponent, entropy, average path length and average clustering coefficient can adjust by the hybrid ratio *d/r*, one may design the required network architecture to satisfy different special requirements in network engineering.

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