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Accuracy-Risk Trade-Off due to Social Learning in Crowd-Sourced Financial Predictions

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A. Supplementary Material

A.1. External Validity of Data Collected

In Table S1, we summarize information about the asset prices predicted and the various measures of accuracy. Overall, our participants are collectively accurate — in agreement with past Wisdom of the Crowd studies [23,24] — indicating that their predictions are being thoughtfully elicited. This is evidenced by the fact that

- The crowd's mean prediction error is much less than the overall price change of the assets for the 3-week prediction period.
- The crowd is generally doing more than just linear extrapolation as their error is smaller than than from a linear extrapolation model (using a first order price momentum prediction).
- The crowd's collective prediction error over each round tracks (and sometimes outperforms) the futures of each asset being predicted (we calculate the futures error as the difference between the futures price and the asset price). Because futures prices are commonly used a measure of the global market's prediction of the price asset [115,116], the fact that the crowd's performance is on-par with the futures prices indicates that our dataset is externally valid.

Note that the higher relative errors in round 2 are an artifact of the fact that a few dollars' error on the lower price (about $45 per share) of WTI Oil seems like a higher error compared to same absolute error on the higher prices of the other assets ($1300 per gold share and $2100 per S&P 500 share).

Table S1: Summary of data collected. Our crowd is accurate, and sometimes even outperforms the futures underlying the asset.

	Prediction Round						
	1	**2**	**3**	**4**	**5**	**6**	**7**
Asset	**S&P 500**	**WTI Oil**	**Gold**	**S&P 500**	**S&P 500**	**S&P 500**	**S&P 500**
Grounth Truth	2037.41	45.95	1335.80	2153.74	2126.41	2191.95	2262.53
Num. Prediction Sets	284	207	134	1174	925	1441	469
Price Change (%)	4.01	11.03	3.63	1.77	1.75	2.24	3.56
Linear Extrapolation Error (%)	6.66	16.4	1.26	1.62	2.75	0.75	3.10
Crowd Mean Error (%)	2.22	4.95	0.46	0.84	0.58	3.20	2.40
Futures Mean Error (%)	2.03	3.05	0.94	0.38	0.40	0.48	1.50

Additionally, whenever we predict a final closing price, we only use participant prediction data up to the week before the day of prediction (i.e., we don't use any data during the last week of the round) so that our predictions are not too easy. We use each asset's futures price as a proxy for the prediction of the entire market to test the external validity of our crowd's predictions. To do so, we chose the start and end dates of each round so that the expiry dates of the asset's underlying *futures* would not affect the price of both the asset and its futures. Financial data (asset and futures prices) is obtained through Barchart.com's API.

A.2. Momentum Transformation of Price History

When using price history B_T, the time-series of prices needs to first be transformed into a cognitively-interpretable likelihood distribution [65]. A number of methodologies and approaches exist, and here we use a simple, interpretable, and theoretically-motivated approach from prior work [66,67] where it has been shown that people process time-series as a distribution of changes as opposed to a distribution of the quantity itself. In our case, this means that we cannot just use the prices from the price history as a histogram, and should instead create a histogram of daily changes (slopes) in prices. Therefore, for each day's price B_t, a daily rate, r_t, of asset price change is calculated for each day t during the 6-month interval that a participant is shown, $r_t = \frac{B_t - B_{t-1}}{B_t}$. Then these rates are used to create a histogram (similar to the histogram of social information) and this rate histogram is utilized for both the simple Gaussian models and the numerical models.

In the simple Gaussian models the mean of this histogram (which is the mean rate $\bar{r}_t$ over the 6 month period) is multiplied by the number of days between the pre-exposure prediction and the final day of the prediction round (for when the asset's price is being predicted) to obtain the post-exposure prediction, $B_{post}^{pred} = B_{pre} + B_{pre}.\bar{r}_t.n_{days}$. The same calculation is done for the numerical model, but for each bin in the rates histogram.

A.3. Modeling Belief Update

Here we formally describe the derivation of the simple Gaussian models and the numerical models and how the posterior is estimated.

A.3.1. Approximate Gaussian Approach:

We describe `GaussianSocial` here. `GaussianPrice` follows the same derivation, substituting the social histogram B_H with the price history B_T.

Our notation here follows that of [117]. We assume that people's estimate of the future price before information exposure, B_{pre}, is being sampled from an internal prior distribution [80], and that the sample we obtain is indicative of the mean of the prior distribution following the results of [118].

We suppose that people think each asset has a true value, V^*, which people are trying to estimate to predict the future asset value, V (the ground truth); that prior beliefs about V^* follow a Normal (Gaussian) prior distribution, $V^* \sim Normal(\mu_{prior}, \sigma_{prior})$; and that evidence about V^* can be understood as being generated from a Normal distribution, $Normal(V^*, \sigma_{data})$. In this case the posterior beliefs people have follows a simple form. Letting information content be defined as the inverse of the Normal distribution's variance $I = \frac{1}{\sigma}$, we have that

$$\mu_{posterior} = \frac{\mu_{prior} \cdot I_{prior} + \mu_{data} \cdot I_{data}}{I_{prior} + I_{data}}. \tag{1}$$

Additionally, the social histogram is treated as representing the information content of data about V^*, then we have:

$$\mu_{posterior} = \frac{B_{pre} \cdot I_{prior} + \overline{B_H} \cdot I_{data}}{I_{prior} + I_{data}}. \tag{2}$$

The `GaussianSocial` rule therefore can be viewed as reflecting an assumption of a Normal distribution as a mental model, and assuming private information and social information have the same information content ($I_{prior} = I_{data}$), which gives:

$$\mu_{posterior} = \frac{B_{pre} + \overline{B_H}}{2}. \tag{3}$$

Although simple, this belief update approach has been shown to faithfully model people's belief update in a variety of domains from predicting movie grossing revenue to pharaoh reign lengths. [28,29]

A.3.2. Numerical Approach:

Here, we can use a numerical approach by binning the likelihood distributions to estimate the posterior distribution using Monte Carlo methods. Because we do not have access to the distribution of the prior belief of each participant (as we only have an individual point estimate for each prediction set), we still have to approximate the prior. We model the prior to be Gaussian, with the mean set as the pre-exposure prediction of a participant, B_{pre}, and the standard deviation set as the standard deviation of the social histogram B_H or the standard deviation of the price history B_T, depending on which likelihood distribution we are modeling with.

We estimate the posterior distribution $P_{posterior}(b)$ of a participant's post-exposure prediction b in the following way: let b_j be a unique value in $\mathbf{B_H}$, and $P_{B_H}(b_h)$ be the probability density of b_h in $\mathbf{B_H}$. Let $P_{prior}(b)$ be the density of b in the parametrized prior distribution. The posterior distribution for the numerical model is defined as $P_{posterior}(b) = \frac{P_{B_H}(b) \times P_{prior}(b)}{\sum_{b_j \in B_H} P_{B_H}(b_j) \times P_{prior}(b_j)}$ when using the social information B_H. The distributions is simple enough that sampling was not needed. After computing this posterior distribution, we use the mean of the distribution as the *modeled* updated belief of a participant.

A.3.3. Evaluating Model Error

For all models, we compute the residual error between the model's prediction of the posterior ($\mu_{\sim P_{posterior}(V)}$) and the actual post-exposure prediction (B_{post}) as: $(\mu_{\sim P_{posterior}(V)} - B_{post})/B_{post}$. For the approximate approach, $\mu_{\sim P_{posterior}(V)}$ is simply the mean of the normal distribution representing the posterior, while in the numerical approach, the mean is estimated through averaging over all bins of the empirical distribution (the distribution is small enough that sampling was not needed).

For all models, the 95% confidence intervals are calculated as follows: we assume the data follows Student's t-distribution since the variance of the true distribution is unknown and, therefore, we estimate it from the sample data. Let s_e be the estimated standard error of the sample mean and t_e be the t-value for the 95% confidence interval desired, which can be computed via inverse t-distribution. The lower and upper limits for the 95% confidence interval are $[\mu_e - t_e s_e, \mu_e + t_e s_e]$, where μ_e is the estimated sample mean.

Table S2: Values of the residual for each round for all models. Numbers in parentheses show the 95% confidence interval.

MODEL	ROUND						
	1 (S&P 500)	**2 WTI Oil**	**3 Gold**	**7 (S&P 500)**	**8 (S&P 500)**	**9 (S&P 500)**	**12 (S&P 500)**
`GaussianSocial`	1.53 (0.19)	3.97 (0.48)	1.08 (0.13)	0.92 (0.04)	0.70 (0.04)	1.51 (0.07)	1.23 (0.13)
`GaussianSocialModes`	1.94 (0.20)	4.85 (0.54)	1.30 (0.19)	1.24 (0.05)	0.98 (0.04)	1.88 (0.08)	1.64 (0.13)
`NumericalSocial`	2.01 (0.23)	5.24 (0.61)	1.60 (0.25)	1.52 (0.08)	1.07 (0.06)	2.31 (0.10)	2.31 (0.22)
`NumericalPrice`	2.25 (0.23)	8.70 (0.87)	2.64 (0.19)	1.57 (0.08)	1.09 (0.06)	2.36 (0.10)	2.75 (0.23)
`GaussianPrice`	2.46 (0.24)	10.3 (0.92)	2.70 (0.22)	1.59 (0.07)	1.13 (0.06)	2.41 (0.10)	2.72 (0.22)
`DeGroot`	2.04 (0.22)	5.32 (0.60)	1.52 (0.13)	1.71 (0.07)	1.17 (0.06)	2.51 (0.09)	2.27 (0.21)

A.3.4. Boostrapping

Our reported value of improvement (the one in Fig. 4) is over 100 random bootstraps with replacement.

As defined in section 3.3.3 in the main text, the error of a subset S_{α_s} is

$$e_{i,S_{\alpha_s}} = \frac{|\sum_{j \in S_{\alpha_s}} [B_{post,j}] - V_i|}{V_i} \quad (4)$$

and the error for all predictions, the Wisdom of the Crowd is:

$$e_{i,S_{all}} = \frac{|\sum_{j \in S_{all}}[B_{post,j}] - V_i|}{V_i} \tag{5}$$

Which means that the improvement of a subset, within a bootstrap b for a particular round i is

$$I_{b,i}^{S_{\alpha_s}} = |e_{i,S_{\alpha_s}} - e_{i,S_{all}}| \tag{6}$$

In Fig 4, we report the average improvement in accuracy over rounds i and over bootstraps b, $\sum_b[\sum_i[I_{b,i}^{S_{\alpha_s}}]]$. Similarly, the risk of this subset S_{α_s} over rounds i is $\sqrt{\sum_i[(I_{b,i}^{S_{\alpha_s}} - \sum_i[I_{b,i}^{S_{\alpha_s}}])^2]}$. We use standard deviation instead of variance as it is the more popular measure of risk in practice [15]. For both the average improvement and the risk, we obtain the 95% confidence intervals over the bootstraps.

A.4. Table of Subset Improvement and Risk

Table S3: Improvements achieved by subsetting predictions via α_s for all rounds. 95% confidence intervals are calculated through 100 bootstraps.

α_s	**Improvement**	**Improvement CI**	**Risk**	**Risk CI**
-1.00	0.526	0.015	0.914	0.019
-0.92	0.866	0.013	1.584	0.021
-0.84	0.839	0.018	1.463	0.029
-0.76	0.882	0.014	1.552	0.025
-0.68	0.880	0.017	1.467	0.028
-0.60	0.798	0.019	1.408	0.038
-0.52	0.734	0.031	1.254	0.060
-0.44	0.682	0.038	1.186	0.069
-0.36	0.607	0.040	1.060	0.081
-0.28	0.344	0.067	0.941	0.073
-0.20	0.435	0.041	1.744	0.039
-0.12	-0.382	0.079	1.344	0.160
-0.04	0.032	0.044	0.498	0.035
0.04	-0.161	0.093	0.952	0.138
0.12	0.292	0.052	0.882	0.068
0.20	0.274	0.037	1.043	0.067
0.28	0.145	0.041	0.923	0.073
0.36	0.044	0.036	0.891	0.059
0.44	-0.073	0.039	0.793	0.060
0.52	-0.095	0.035	0.858	0.073
0.60	-0.238	0.034	0.605	0.051
0.68	-0.432	0.031	0.493	0.029
0.76	-0.460	0.023	0.606	0.032
0.84	-0.620	0.018	0.692	0.017
0.92	-0.885	0.011	0.742	0.014
1.00	-1.021	0.009	0.963	0.012

Table S4: Improvements achieved by subsetting predictions via α_s only for predictions the week before Brexit. Confidence intervals are calculated through 100 bootstraps.

α_s	**Improvement (%)**	**CI**
-1.0	-3.14	0.65
-0.2	-0.61	0.15
0.2	-0.66	0.09
0.6	-1.02	0.07
1.0	-1.03	0.05

A.5. Brexit Data

We deployed one of our experiments right before the Brexit vote during which there was a lot of market uncertainty [31]: the prediction round starting on June 1st 2016 ended on June 24th 2016, the day of the Brexit vote, and participants were predicting the price of the S&P 500, an asset sensitive to global events [119,120].

We collected 284 prediction sets during the first 2 weeks of the round, and 52 sets in the last week during which the global financial market first overestimated then underestimated the final price of the S&P 500 asset leading to a 3.7% crash, as shown in the candlestick plot in Fig. S1.

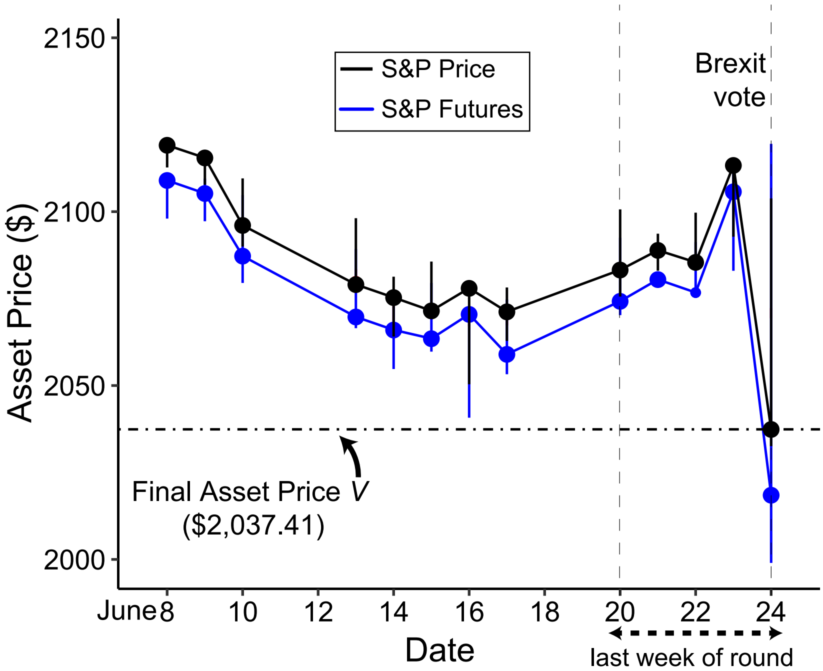

Figure S1. The close, low, and high price of the asset and its underlying futures are shown as candlestick plots. The asset and futures overestimated the price and then crashed during the last week.

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