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Glassy States of Aging Social Networks

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Abstract: Individuals often develop reluctance to change their social relations, called "secondary homebody", even though their interactions with their environment evolve with time. Some memory effect is loosely present deforcing changes. In other words, in the presence of memory, relations do not change easily. In order to investigate some history or memory effect on social networks, we introduce a temporal kernel function into the Heider conventional balance theory, allowing for the "quality" of past relations to contribute to the evolution of the system. This memory effect is shown to lead to the emergence of aged networks, thereby perfectly describing—and what is more, measuring—the aging process of links ("social relations"). It is shown that such a memory does not change the dynamical attractors of the system, but does prolong the time necessary to reach the "balanced states". The general trend goes toward obtaining either global ("paradise" or "bipolar") or local ("jammed") balanced states, but is profoundly affected by aged relations. The resistance of elder links against changes decelerates the evolution of the system and traps it into so named *glassy states*. In contrast to balance configurations which live on stable states, such long-lived glassy states can survive in unstable states.

Keywords: glass state; social network; memory

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1. Introduction

"Yesterdays' friend (enemy) rarely become tomorrows' enemy (friend)."

Tension reduction is a predominant principle that contributes to the formation of human interactions [1]. This principle acts as a self-organizing process; it indicates that social communications are established based on the tendency towards balanced states [2–5]. Interesting questions that follow concern what "parameters" have a pivotal role in the social network dynamics. An appropriate answer seems to lie in the history of relationships. The ability of human beings to remember sequences of events (sometimes unconsciously) over time brings about social concepts such as commitment and allegiance that lead to the formation of cultural communities, sects, alliances, and political groups [6–8]. In psychological terms, the more potent the commitment is, the more probable the relations will remain unchanged over time. There are persons, such as family members, friends (or enemies), even business partners, with whom most people have no inclination to modify the nature of their relationships. This resistance to change can be explained by the history of relations and their importance, which can be referred to by two distinct parameters: "age of links" and "weight of links", respectively. Generally, depending on age or weight, breaking up or modifying the links among individuals gradually can become difficult.

In this study, the main intention is to explore the effect of relationship history following the hypothesis that newly-formed relations have more chance to change than older ones; thus, old relations are more resistant. Practically, we aim at mimicking the strength of relationship links through a model considering a combination of emotional intensity and life-time duration of relations, but keeping these "parameters" as independent of each other as proposed by Granovetter [9]. This leads to links which contain both aspects, depicted in Figure 1 through different thicknesses of lines (for the strengths) and different colors (for the ages).

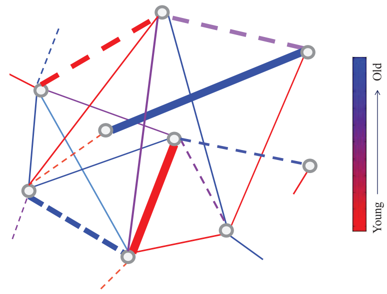

Figure 1. In terms of social networks dynamics, links can carry various information such as type, age, and strength of relations. This figure illustrates a network with two types of relations (for instance, friendship and animosity) which are denoted by solid and dashed lines, respectively. Here colors display the gradient of age from young (red) to old (blue); the weight (strength) of links are represented by the line thickness. Increasing the age or weight of links can lead to decreasing the tendency towards modifying relationships.

Therefore, in quantitative terms and in view of modeling the dynamics of social networks, it is useful to develop some study in order to comprehend how history (or memory) has global consequences on the evolution of a social system. Thanks to technological advances allowing the collection and analysis of empirical data about human interactions on social networks, it is currently easily revealed that social activities act as sequences of correlated events in which each event or decision depends on previous ones and on the "node environment". In this context, much effort has already been devoted to describing the evolution of in- and out-cluster relationships over time [10–14]. These endeavors have led to the finding that the heterogeneous dynamics are a consequence of memory; e.g., in cellphone users [15], face-to-face contacts [16], patterns of rumor-spreading processes [17,18], individuals' web browsing pattern [19], Boolean networks [20], or optimized strategy in Prisoner's Dilemma games [21,22].

In order to introduce the influence of memory on social networks, we have followed the Heider balance theory [1], which has provided a fundamental platform for tackling many sociological problems ranging from social systems [23–26] to political ones [27], and online networks [28,29] (see [30] and references therein). In this theory, relations between agents are considered as positive or negative links: the positive sign for a link indicates, for instance, friendship [28,31], attractiveness [32], profit [33], or tolerance [23], whereas a negative sign would indicate the opposite. This theory proposes a model based on triadic configurations in which relations evolve in order to reduce the number of unbalanced triads and in view of attaining "minimum tension states". Assigning a potential energy to these systems allows a more quantitative view of the landscape of the network's dynamic over time [34].

Let us denote by s_{ij} the link sign, corresponding to a feeling of friendship or of animosity between the nodes i and j. Let the social network potential energy be calculated by summing all products of links in all triads, normalized by the total number of triangles. From this energy point of view, minimum tension states can be distributed between global minima (namely, "balanced states") and local minima (named "jammed states") [34–36]. In the presence of memory, agent relations (friendships and animosities) build up and can become vigorous over some long period of time. This age gain for links decreases the probability of relationship changes [37]. Gallos et al. [38] explored the effect of nodes' attributes (e.g., age, genders) on the formation of specific configurations in real social networks. They observed that younger individuals have less triadic closure in their relations (friends of friends tie), while raising age increases this propensity. In the spirit of scaling ideas, based on the nature of memory features in real systems [39–41], we mimic this aging process through a kernel function which emphasizes that the strength of the relationships increases like a power law, from past to present. In so doing, we generalize the order of the derivative in the conventional continuous equation of balance theory [42–45] in physics to a non-integer order. The results presented below reveal the emergence of some novel states, called *glassy states* that refer to conditions in which for a reasonably long time interval, there is no propensity to reduce the amount of stress within the system in favor of equilibrium.

It can be informative to compare the notions of jammed and glassy states. Although both of these states are not global minimum states, they have a distinct characteristic that make them distinguishable. According to the tension reduction condition, complex networks tend to move toward lower energy levels and reach the global minimum states (paradise of bipolar). During this evolution, a system may be trapped in some local minimum states (namely, jammed states) for a while. In such a situation, there is no possibility to reduce tension (energy) within a system by altering links. It has been shown that jammed states are rare events in networks dynamics and occur with negative energies. In contrast, in glassy states, agents resist modification of the quality of their relationships in order to reduce the stress which preserves the system in unstable states. Glassy states are established by a combination of aged and young links. Here, aged links refer to those that have not changed for a long time, while young links change quickly without resistance. Although there exist some links in glassy states which if changed can guide the system toward lower energy states, thanks to their age, the probability of change is low. It will be shown that the energy of glassy states can be negative or even positive; due to the memory intensity, they can be the only final states of the network's dynamical evolution.

It will be further explained that the present study considers an endogenous condition for a link switch. Exogenous shocks, or threshold of awareness [46], are outside the scope of the present study. So, we imply a difference between simultaneous or sequential updating [47]. Glassy states in condensed matter are indeed subject to frustration considerations and specific phase transition patterns.

2. The Evolving Network

The dynamics of social networks is a sort of self-organized process in which relationships are modified based upon agents' benefits. These modifications occur at a microscopic level; they impose a collective behavior that guides the network along particular structural evolution paths. In this context, Heider [1] proposed a "balance theory" based on triadic relations among two persons and their "attitude" towards an issue (as the third node), where connections are rearranged to become stable [48]. According to this description, relations among individuals can be categorized into two classes based on balanced and unbalanced triadic relations. In a unbalanced triad, tension stimulates agents to modify their current relations to reach some balance. Later, by putting the concept of "attitude" in the background and substituting an individual as the third node, Cartwright et al. [49] generalized Heider's approach in terms of "sign graphs", in which the links among members can be ± 1 (without any weight). Intuitively, a positive value denotes friendship, cooperation, or tolerance (to name but a few), while a negative value depicts animosity, rivalry, or intolerance in social groups [28,31–33]. According to the balance theory, a triadic relation among any three agents is balanced (unbalanced) only if the product of signs assigned to the links is positive (negative). Obviously, a network is "balanced" if all triangles are balanced.

Study on the dynamics of such sign networks, from the view point of balance theory, can be traced back to Antal et al. [35]. They proposed a model based on the evolution of links for fully-connected networks—namely, Constrained Triadic Dynamics (CTD), which could describe how an initially imbalanced society attains a balanced state [35,50].

$$\frac{d}{dt}X_{ki}(t) \equiv \sum_{j=1} X_{kj}(t)X_{ji}(t). \tag{1}$$

Having this CTD model in hand, to investigate the memory effects on the evolution of the system, we have considered the following assumption: although changes in a relationship (link) would guide the system toward a minimum tension state, agents have less tendency to alter the nature of their connections due to some memory of pertinent relations. This approach results in the concept of aged networks, where the history of links is associated with their ages. In previous studies, there was no discrepancy between age (duration) and strength of links in terms of weighted networks [9,51–53].

Many studies (e.g., [54–57]) have considered fractional integrals or derivatives as a generalization of ordinary differential-integral operators to non-integer ones, in view of describing the effects of past events on the present one. Such a fractional calculus approach can also be taken for the conventional balance differential equation in order to explore these effects on social interactions.

$$ {}_{t_0}^{c}D_t^{\alpha} X_{ki}(t) \equiv \sum_{j=1} X_{kj}(t)X_{ji}(t). \tag{2}$$

The left-hand side of Equation (2) is the Caputo [58] fractional differential operator of order α, where $0 < \alpha < 1$. This α "the fractional order of derivation" denotes the significance of the memory in the interaction mechanisms; i.e., $\alpha = 1$ refers to the balance theory master equation with no memory [42–45].

Equation (2) can be rewritten in the form of its equivalent Volterra integral [59,60],

$$X_{ki} = X_{ki0} + \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} dt'(t-t')^{(\alpha-1)} \left[\sum_{l=1} X_{kl}(t')X_{li}(t') \right]. \tag{3}$$

This functional form with a scale-invariant time-dependent kernel makes it possible to consider historical effects. Indeed, due to the "non-local" (more exactly, "time lag-dependent") nature of these operators, past events play a "non-Markovian" role on the system dynamics. This mathematical technique practically means that those past events which are "further away" from the present time are those which are less likely to contribute to the current state, unless those incidents hold high levels of importance.

To solve the integral of Equation (3) numerically, the predictor–corrector algorithm is employed [60–62]. In this way, the product rectangle rule [60] is used, in which the time domain is divided in an equispaced grid $t_j = t_0 + hj$ with equal space h. The right-hand side of Equation (3) is approximated on this grid, such that Equation (3) becomes

$$X_{ki} = X_{ki0} + h^{\alpha} \sum_{j=0}^{n-1} b_{n-j-1} \left[\sum_{l=1} (X_{kl}X_{li})_j \right]. \tag{4}$$

In so doing, the $b_n = \frac{(n+1)^{\alpha} - (n)^{\alpha}}{\Gamma(\alpha+1)}$ coefficients are the essential terms which indicate and control the role of the past events in the model.

For the simulation part of this study, a fully-connected network including N nodes where all agents are acquainted with each other is first considered. Here a link between two agents i and j is represented by s_{ij}, with initial value $+1$ (-1) which denotes friendship (animosity). We may start from a fully antagonistic network, where a link between any two agents has -1 for initial value. In order to check the evolution of the network at each time step, any link must fulfill *two* conditions. The $first$ condition: a link is selected randomly; switching the sign of the link is made permissible only if the total number of unbalanced triangles is reduced. This description is in accordance with the reduction of the total energy of the system [34],

$$U = \frac{-1}{\binom{N}{3}} \sum_{i,j,k} s_{ij} s_{jk} s_{ki}, \tag{5}$$

where the sum is over all triadic relations. U can accept values from 1 (antagonistic configuration) to -1 (balance configuration).

The *second* condition: a competition occurs between the tendency of a link to reduce energy and the insistence on maintaining the past relation due to the memory effect. This competitive process identifies whether the selected link (which should fulfill the condition of the $first$ step) tends to switch into the opposite sign. In this respect, a random number is generated from a normal distribution over $(0, 1)$: when this number is less than $A_{ij}^{(\alpha-1)}$, a switch into the opposite sign (± 1) is made for the selected link (s_{ij}). The magnitude of A_{ij} is equivalent to somewhat measured by the age of the link, and controls the lapse of time ("resting time") during which agents do not alter their relations. Here, α exhibits the rate of memory effects on relations and its value changes over $(0, 1)$. Obviously, the lower the magnitude of A_{ij} or the higher the value of α is, the more probable it is that a link will change sign.

In the aging process, for any N (the number of nodes in the system) steps, the age (A_{ij}) of links which do not change sign increases by one unit. According to this dynamical evolution, a system in various paths towards minimum tension states remains unchanged within some time interval. When the system remains so static for time intervals of the order of the number of links (i.e., in order to give the same probability for each link to be chosen), we call those states *glassy states*.

3. Results and Discussion

In Heider balance theory, the dynamics of a network is a sort of Markovian process in which there is no evidence of the past incidents (memory) in link rearrangements, whence the evolution of relationships is entirely stochastic. Thus, in such a basic theory, when a person decides to modify a connection (apart from what happened in the past), she or he only checks the quality of a specific relation at the present moment. In terms of social networks, links carry various information such as type, age, and strength of links. Age denotes the duration of a relation and the strength of a link is described by its weight. Figure 1 display type, age, and weight of links through solid (dashed) lines, colored lines, and thick (thin) lines, respectively, for nine nodes which are part of a larger network. Since age of links indicates the effect of past relationships on current decisions, it can be interpreted that this parameter (age) is responsible for the memory of relationships. According to recent studies [10–13], memory imposes a scaling (increasing power law) behavior onto a system in such a way that the older the relations get, the more significant a role they perform for the network destiny. Converting the conventional continuous time equation of balance theory [63,64] to the fractional form (Equation (2)) appropriately allows us to include and describe a memory effect over time. Thus, the interactions among individuals become time-dependent and the system experiences a non-Markovian process in an endogenous way. Note that considering the fractional space only rescales the evolution time without any impact on the phase space or the dynamics of the system. However, changes are slowing down; in other words, time intervals between changes appear to become longer; the system finally attains a balanced state later than if it is a memory-less system.

In the investigation of networks dynamics based on the strict CTD model, Antal et al. [35] demonstrated that systems may follow various paths before obtaining the network (global) minimum tension in a finite time interval. These systems mostly move towards final balanced states (either paradise or bipolar), where all triangles are balanced. A "paradise state" refers to a system in which all members are friendly with each other, while a "bipolar state" refers to a system which consists of *two* main clusters, in which the members of each group are friendly with each other, but not friendly with any member of the other group. Antal et al. [35] also showed that there are a few "rare" states named as "jammed states" (local minima) in which a system is divided into *several* ($\neq$2) communities.

According to the present model (i.e., when introducing memory effects into the *CTD* model), one even describes the formation of aged (and aging) networks. Hence, over the course of time, depending on the value of α (which indicates the significance of relations), links can gain age with probability $A_{ij}^{(\alpha-1)}$. It can be concluded that the larger α is, the less relevant are effects of past events on the individual's decision. For $\alpha = 1$, the memory effect completely disappears and the current model is reduced to the *CTD* model. In the presence of memory, even though the least tension principle is enforced, the network—before obtaining either a global or a local equilibrium—can be trapped into intermediate states during several time intervals. This practically means that people do not forget their long-lasting friendships or animosities readily; in other words, the system can resist changes over time. Figure 2 illustrates the most prolonged periods in various paths that the system remains unchanged before reaching its balanced state, in the case of a network with 21 nodes for $\alpha = 0.7$, after 10,000 realizations. These time intervals are found to follow a Poisson distribution function $p(x) = (1/k!)\, e^{-\lambda}\lambda^k$, where λ controls the expected frequency with which an event can occur. Such a Poisson distribution demonstrates that long time intervals become probable; this deviation from the normal distribution leads to the emergence of inhomogeneity in the time intervals, during which the system remains unchanged. Here it is found that λ has values 83.640 and 13.157 for $\alpha = 0.5$ and 0.7, respectively. For larger values of α, λ tends to zero. In such a situation, as α decreases, the time intervals in which the system shows no inclination towards changes increases.

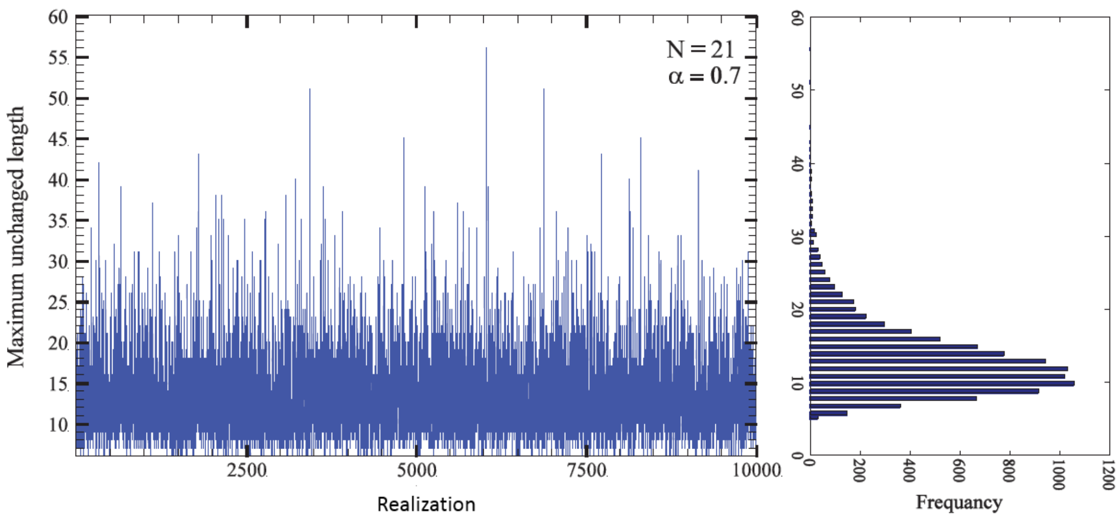

Figure 2. The maximum time interval during which a network resists against any change (long-lived states). The left panel depicts this time interval for networks with 21 nodes in various paths (10,000 realizations). The right panel illustrates that the maximum time intervals follow a Poisson distribution.

Figure 3 illustrates such a situation: the energy of final states, Equation (5), versus the mean value of positive and negative links for a 45 node network at different α values, with 10,000 realizations. It can be seen that for $\alpha = 0.3$ (left top panel) for all realizations, the system is trapped in glassy states before it reaches its balanced states. Interestingly, however, balanced and jammed states always happen in negative energies; however, here some glassy states occur at positive energies, which implies an intrinsic instability within the system. In such cases, the system "tolerates" some high-tension condition. However, no agent tends to alter the quality of her or his relationships. For larger $\alpha = 0.5$ (right top panel) or $= 0.65$ (left bottom panel in Figure 3), the system exhibits more "flexibility", moving through the phase space; consequently, biased relationships are later-forming, implying that the system has much opportunity to reach several lower energy states with lesser tensions. In fact, α is an indication of a time correlation within the system; the lesser the value of α, the more the system becomes correlated to past events. The bottom-right panel in Figure 3 displays the total number of glassy states versus the total number of realizations for different α values, for networks with 21 and 45 nodes. Accordingly, it can be seen that when the network's members flexibility is so promoted, the chance of occupying lower energy states increases; so does the probability of obtaining balanced states. It can be practically concluded that a society containing very biased (or stubborn) people has no way to reach balanced states, but become stuck in intermediate states—namely, the long-lived (*glassy*) states. On the other hand, when the intensity of commitment and/or bias in relationships decreases, a society is enabled to move towards balanced states in a "short" time. The bottom-right panel in Figure 3 shows a sudden change in the percentage (density) of glassy states to balanced states about $\alpha \sim 0.6$.

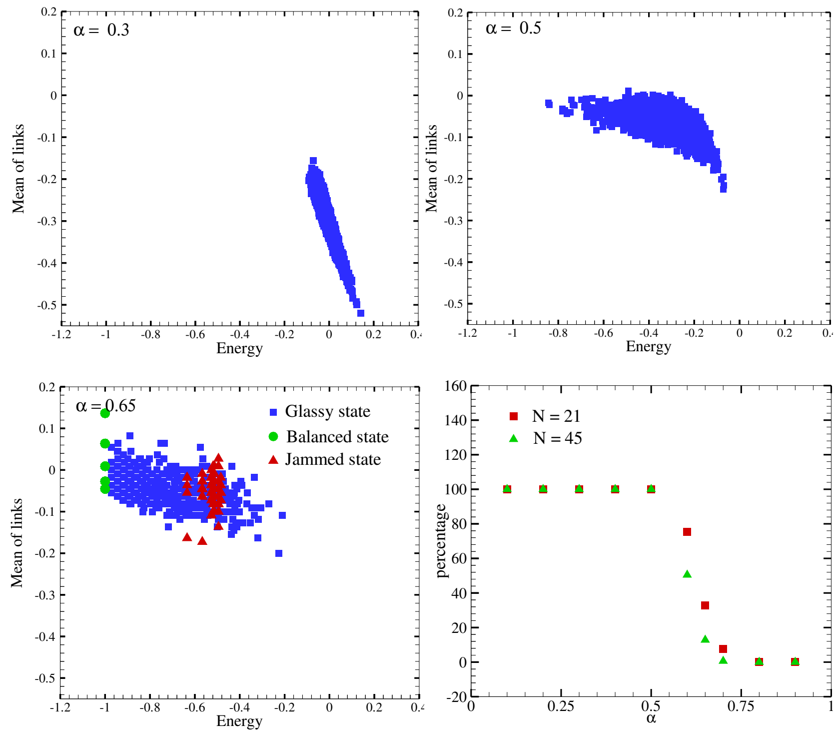

Figure 3. Energy of final states (Glassy, Balance and Jammed states) versus mean of positive and negative links. We show these final states for networks with 45 nodes at different α (strength of memory) values. With increasing α values, the system becomes more flexible against changes and the probability of reaching balance and jammed states increases. The right bottom panel shows a sudden "phase transition" in the percentage (density) of glassy states to balanced states about $\alpha \sim 0.6$.

4. Conclusions

The memory footprint is evident in most complex systems where life exits, from biological to economic and social systems. Thanks to the presence of memory, when we have friendship (animosity), this relationship can become vigorous over time, and the propensity for change may diminish; in other words, the breaking-up of aged friendship (animosity) becomes difficult. Here, introducing a kernel function into the differential equation of balance theory, we provide a better insight toward considering the contribution of past events on the dynamics of a social system. We have mimicked the aging process through a kernel function which emphasizes that the disinclination of relationships to change increases as a power law from past to present. This function leads to a fractional differential equation of order α, where $\alpha - 1$ indicates the strength of the memory effect. In other words, at high α, links have a high probability to change and the system is more flexible toward social tension reduction.

We have investigated the feasible paths and final states that a (fully-connected) network can experience, using the Constrained Triadic Dynamics (CTD) model with memory. We have employed a probability of links' disinclination to change as a $A_{ij}^{(\alpha-1)}$ function. We have found out that memory is a key factor indeed, which sometimes withstands the quick evolution of the network and eventually preserves the system in unstable but long-lived states—namely, glassy states. Under such circumstances, for various time intervals, the system has no tendency to evolve towards global or local minima. Evidence, depending on the value of the strength parameter α, indicates that a system is enabled to reach a lower tension and energy, in non-trivial pathways, visiting glassy states. In such situations, individuals become dormant or accustomed to the current state of the network. In contrast to jammed states (i.e., local minimum states), in which systems only experience negative energies, glassy states can occur in positive energy states, thereby imposing instability to and keeping stress in the system.

In conclusion, the natural concept of aging of agent-based networks provides a powerful tool to investigate the dynamical behavior of memory-prone social agents, but is also surely valid for other realistic complex systems. In real technological networks, several links necessarily have a temporary lifetime and are fleeting; they can develop into new relations (or functions) due to some feedback, automatic or external control, or more often reinforcement learning by memory effect over time. Therefore, the concept of aged links might be a suitable language to study a wide variety of temporal networks; e.g., besides strictly psychological aspects, including trading between companies, friendly and hostile relationships, scientific collaborations, political influences through cooperation or competition, and tweeting news in social media.

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