Supporting Data Information

"Is Cetacean Intelligence special? New Perspectives on the Debate"

The data provided contains a Wolfram Mathematica notebook (file formulae.nb) with the equations of the average entropy $S(\mu_k, \mu_m)$, the average free energy $F(\mu_k, \mu_m)$ (see Supplementary figure 1), and the logarithm of average number of brain representations $R(\mu_k, \mu_m)$ (function $R_2(\mu_k, \mu_m) \equiv Log(R(\mu_k, \mu_m))$ in the mathematica notebook) for generating the data.

The necessary data for plotting the graphs of the entropy $S(\mu_k, \mu_m)$, the free energy $F(\mu_k, \mu_m)$, and brain representations $R(\mu_k, \mu_m)$ according to the research presented in the manuscript is also provided in text files together with a Matlab workspace file (IntCetENTROPY.mat) containing these data for ease of use. More specifically, the data provided is for values of the average number of information-processing levels μ_k in the integer set {2,3,8,9,12} and within the integer interval [2..50] for the model parameter μ_m (i.e., average connectivity between brain representations).

The notation used to denote the attached text files is as follows: letter "s" to denote entropy data, letter "f" to denote free energy data, and letter "r" to denote brain representations data.

- Files s2,s3,s8,s9 and s12 contains the raw data values of the function $S(\mu_k, \mu_m)$ within the range of μ_k and μ_m commented above.

- Files f2,f3,f8,f9, and f12 are the files containing the raw data values of the function $F(\mu_k, \mu_m)$. Similarly, the files f2n,f3n,f8n,f9n, and f12n contains the data of $F(\mu_k, \mu_m)$ scaled between [0,-1] for visualization purposes.

- Files r2,r3,r8,r9 and r12 contains the data of $Log(R(\mu_k, \mu_m))$, specifically the natural logarithm of the values provided by the function $R(\mu_k, \mu_m)$ within the range of μ_k and μ_m commented above. The logarithm of the function $R(\mu_k, \mu_m)$ was used for better visualization purposes due to the hyperexponential growth of this function.

Please note that the graphs of the entropy function $S(\mu_k, \mu_m)$, free energy function $F(\mu_k, \mu_m)$ and brain representations numbers function $R(\mu_k, \mu_m)$ that appear in the manuscript were plotted as follow:

- The raw data of the entropy function $S(\mu_k, \mu_m)$ (i.e., s2,s3,s8,s9, and s12) were plotted using a logarithmic scale.

- The scaled data of the free energy function $F(\mu_k, \mu_m)$ (i.e., f2n,f3n,f8n,f9n, and f12n) were plotted using a linear scale.

- The raw data of the brain representations function (i.e., r2,r3,r8,r9, and r12) were plotted using a linear scale (remember that these files contain the values of $Log\left(R(\mu_k,\mu_m)\right)$). The graphs corresponding to this function that appear in Figures 1 and 2 of the manuscript were plotted using different zooming factors for better visualization purposes.

$$F(\beta,\mu_k,\mu_m)=\frac{1-{\mu_m}^{\mu_k}}{1-\mu_m}\Bigg\{1-$$

$$-\frac{1}{\beta}\log\left(1+\sum_{l\geq 2}^{\mu_m}\binom{\mu_m}{l}e^{-\beta(l-1)}A_l^l\right)\Bigg\}$$

$$S(\beta,\mu_k,\mu_m)=\beta\frac{1-{\mu_m}^{\mu_k}}{1-\mu_m}\Bigg\{\frac{\sum_{l\geq 2}^{\mu_m}\binom{\mu_m}{l}(l-1)e^{-\beta(l-1)}A_l^l}{1+\sum_{l\geq 2}^{\mu_m}\binom{\mu_m}{l}e^{-\beta(l-1)}A_l^l}+$$

$$+\frac{1}{\beta}\log\left(1+\sum_{l\geq 2}^{\mu_m}\binom{\mu_m}{l}e^{-\beta(l-1)}A_l^l\right)\Bigg\}$$

Supplementary Figure 1. Equations of the average Entropy and the average Free Energy. This equations are coded in the attached wolfram mathematica notebook (formulae.nb) for a value of $\beta=1$(see [9] for further details) together with the function $Log\left(R(\mu_k,\mu_m)\right)$ for generating the data.

$R(\mu_k,\mu_m)$ ≡ Average number of brain representations ≡ **Proxy for neuron numbers**
$S(\mu_k,\mu_m)$ ≡ Average Entropy ≡ **Proxy for intelligence**
$\left\|F(\mu_k,\mu_m)\right\|$ ≡ Absolute value of the average Free Energy ≡ **Proxy for metabolic costs**

Tables 1,2,3, and 4 contain data obtained when evaluating the functions $R(\mu_k,\mu_m)$, $S(\mu_k,\mu_m)$, and $\left|F(\mu_k,\mu_m)\right|$ for a set values of the model parameters to illustrate the principal results of the study. Several examples (denoted as (a), (b) and (c)) are provided using these data. Please note that the values that are shown in the aforementioned tables are the result of applying logarithms to the corresponding values of the functions $R(\mu_k,\mu_m)$ (average number of representations), $S(\mu_k,\mu_m)$(average entropy), and $\left|F(\mu_k,\mu_m)\right|$ (average free energy).

(a) $S(12,18) \cong \frac{S(9,45)+S(9,46)}{2} = 35.4312$ and $|F(12,18)| < \frac{|F(9,45)|+|F(9,46)|}{2} = 35.0321$ but also $R(12,18) > \frac{R(9,45)+R(9,46)}{2} = 2.4725x10^{15}$

(b) $S(12,3) \cong S(5,12)$ and $|F(12,3)| < |F(5,12)|$ but also $R(12,3) > R(5,12)$

(c) $S(12,3) \cong S(3,52)$ and $|F(12,3)| < |F(3,52)|$ but also $R(12,3) > R(3,52)$

Table 1: Average Number of information-processing levels equal to 12 ($\mu_k = 12$)

	$\mu_m = 2$	$\mu_m = 3$	$\mu_m = 4$	$\mu_m = 5$	$\mu_m = 6$	...	$\mu_m = 15$	$\mu_m = 16$	$\mu_m = 17$	$\mu_m = 18$	$\mu_m = 19$
$R(\mu_k, \mu_m)$	2838.4	**476106**	$1.7x10^7$	$2.9x10^8$	$1.96x10^{10}$	...	$2.5855x10^{14}$	$5.7555x10^{14}$	$1.22x10^{15}$	**$2.4766x10^{15}$**	$4.8373x10^{15}$
$S(\mu_k, \mu_m)$	7.77	**12.93**	16.55	19.35	21.6483	...	33.1496	33.9532	34.7074	**35.4178**	36.0894
$\|F(\mu_k, \mu_m)\|$	7.94	**10.48**	14.94	18.22	20.69	...	32.5398	33.3597	34.1287	**34.8527**	35.5366

Table 2: Average Number of information-processing levels equal to 9 ($\mu_k = 9$)

	$\mu_m = 40$	$\mu_m = 41$	$\mu_m = 42$	$\mu_m = 43$	$\mu_m = 44$	$\mu_m = 45$	$\mu_m = 46$
$R(\mu_k, \mu_m)$	7.4153×10^{14}	9.3331×10^{14}	1.1681×10^{15}	1.4543×10^{15}	1.8014×10^{15}	**2.2205×10^{15}**	**2.7246×10^{15}**
$S(\mu_k, \mu_m)$	34.2306	34.461	34.6857	34.9051	35.1193	**35.3288**	**35.5336**
$\lvert F(\mu_k, \mu_m) \rvert$	33.8136	34.0475	34.2756	34.4983	34.7158	**34.9283**	**35.136**

Table 3: Average Number of information-processing levels equal to 5 ($\mu_k = 5$)

	$\mu_m = 2$	$\mu_m = 3$	$\mu_m = 4$	$\mu_m = 5$	$\mu_m = 6$	$\mu_m = 7$	$\mu_m = 8$	$\mu_m = 9$	$\mu_m = 10$	$\mu_m = 11$	$\mu_m = 12$
$R(\mu_k, \mu_m)$	21.487	216.8	1083	3729	10230	23879	49640	94490	167825	281875	**452131**
$S(\mu_k, \mu_m)$	2.89	5.23	6.84	8.08	9.1	9.96	10.71	11.37	11.96	12.49	**12.97**
$\lvert F(\mu_k, \mu_m) \rvert$	3.05	2.79	5.24	6.95	8.15	9.1	9.91	10.61	11.23	11.79	**12.29**

Table 4: Average Number of information-processing levels equal to 3 ($\mu_k = 3$)

	$\mu_m = 50$	$\mu_m = 51$	$\mu_m = 52$	$\mu_m = 53$	$\mu_m = 54$	$\mu_m = 55$
$R(\mu_k, \mu_m)$	378767	404343	**431087**	459028	488195	518617
$S(\mu_k, \mu_m)$	12.8379	12.9034	**12.9676**	13.0306	13.099	13.153
$\lvert F(\mu_k, \mu_m) \rvert$	12.4514	12.5194	**12.5861**	12.6514	12.7155	12.7783