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Novel Criteria for Deterministic Remote State Preparation via the Entangled Six-Qubit State

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Abstract: In this paper, our concern is to design some criteria for deterministic remote state preparation for preparing an arbitrary three-particle state via a genuinely entangled six-qubit state. First, we put forward two schemes in both the real and complex Hilbert space, respectively. Using an appropriate set of eight-qubit measurement basis, the remote three-qubit preparation is completed with unit success probability. Departing from previous research, our protocol has a salient feature in that the serviceable measurement basis only contains the initial coefficients and their conjugate values. By utilizing the permutation group, it is convenient to provide the permutation relationship between coefficients. Second, our ideas and methods can also be generalized to the situation of preparing an arbitrary N-particle state in complex case by taking advantage of Bell states as quantum resources. More importantly, criteria satisfied conditions for preparation with 100% success probability in complex Hilbert space is summarized. Third, the classical communication costs of our scheme are calculated to determine the classical recourses required. It is also worth mentioning that our protocol has higher efficiency and lower resource costs compared with the other papers.

Keywords: criteria; deterministic remote state preparation; entangled six-qubit state; permutation group

1. Introduction

With the rapid development of quantum information processing, many difficult tasks have been successfully solved by using entanglement resources. One of great discoveries is quantum teleportation (QT) [1] in this research filed. The QT, first proposed by Bennett, has enabled the transmission of quantum state between remote places without sending the state itself. In a typical QT protocol, the sender makes a collective measurement and informs the receiver through classical communications. The receiver can then perform proper unitary operations on his particle to reconstruct the initial quantum state. When the state is completely known to the sender previously, another economical scheme, called remote state preparation (RSP) [2–4]. The RSP, which was firstly introduced by Lo [5], also deals with the remote transmission of quantum state. Hence there is the trade-off of resources in RSP. Pati [6] showed that the preparation of the state of polar great circles or equatorial line needs only one classical bit, smaller than the cost in QT. However, with states of a generic set considered, the cost in RSP is equal to that in QT [5,7]. The research of RSP has been a popular direction in quantum communications, including joint remote state preparation (JRSP) protocols [8–10], controlled remote state preparation (CRSP) protocols [11–13] and deterministic remote state preparation (DRSP) protocols [14,15]. There have also been experimental studies of RSP [16,17], they can be applied to some encryption schemes [18–22].

Among all aspects of RSP, DRSP is starting to attract substantial attention. Many researchers have focused on this branch, finding a special set of states that can be remotely prepared with certainty. Pati [6] focused on the states chosen from equatorial or polar great circles on a Bloch sphere. Zeng et al. [23] generalized Pati's protocol to higher dimensions. They proved that deterministic RSP in real Hilbert space can only be implemented in two, four, or eight dimensions. They also studied the RSP protocol of equatorial state, where RSP can be generalized to arbitrary dimension case. Liu et al. [24] considered the arbitrary two and three-qubit RSP cases using nonmaximally entangled states, i.e., an remote preparation of arbitrary quantum pure states of two and three qubits with different success probabilities. By using a six-qubit maximally entangled state as the quantum channel, Zhang et al. [25] proposed a theoretical scheme for bidirectional remote state preparation. Yan et al. [26] constructed the complex orthogonal bases by using the two-particle state, under which the success probability of the RSP protocol is 1/2. They showed that RSP can be done deterministically if the coefficients are all real. Wang et al. [27] presented a protocol to prepare a class of three-qubit states

$$|u\rangle = \alpha\,|000\rangle + \beta\,|111\rangle + \gamma\,|001\rangle + \delta\,|110\rangle \tag{1}$$

Here, α is real, β, γ, δ are complex and satisfy $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. By using three Einstein–Podolsky–Rosen (EPR) pairs as the quantum channel, their protocol can be successfully realized with 1/4 probability.

There is an intriguing question: if the coefficients α, β, γ, δ are some special values, can the initial state $|u\rangle$ be prepared with a higher probability? Very recently, some researchers were interested in these coefficients which have an effect on the success probability. Wang et al. [27] classified them into four simple types, which they believed to be final optimal conditions. Similarly, Pan [28] drew the same conclusion by using the Bell state and asymmetric W state as the quantum channel. Sheng et al. [29] provided a protocol of deterministic entanglement purification, and completed nonlocal Bell-state analysis with hyper entanglement. Zhan [30] studied a deterministic RSP of preparing an arbitrary two-qubit pure state by using two bipartite partially entangled states as the quantum channel. Wang [31] presented a bipartite scheme for remotely preparing an arbitrary three-qubit state with a four-qubit cluster state and EPR state as the shared quantum channel. Ma et al. [32] proposed several RSP schemes of arbitrary two- and three-qubit states via the χ state as the entangled resource, and discussed the success probability of real and complex coefficients for the two cases.

The rest of our paper is organized as follows: Related literatures are overviewed in Section 2. In Section 3, we investigate two schemes to remotely prepare an arbitrary three-particle state in eight-dimensional Hilbert space by using the six-particle entangled state as the quantum channel. In Section 4, we generalize our research into remote state preparation of an arbitrary N-particle state which used Bell states as quantum resources. The classical communication cost (CCC) of the present scheme is calculated in Section 5, while discussion and conclusions are given in Sections 6 and 7, respectively.

2. Related Works

There exists a number of studies on genuinely entangled six-qubit state, whose basic settings are related to this paper. We provide a brief overview below.

In 2007, the usefulness of the genuinely entangled six-qubit state that had been introduced by Borras et al. [33] was investigated for the quantum teleportation of an arbitrary three-qubit state. The state is a genuinely entangled six-qubit state which is not decomposable into pairs of Bell states.

Choudhury et al. [34] reported that it is also difficult to disentangle this state by performing local operations, and entanglement still prevails after three local measurements. It has been shown that entanglement of $|\psi_6\rangle$ decays more slowly than that of the Greenberger–Horne–Zeilinger (GHZ) state. Further, it has been shown that the entanglement of $|\psi_6\rangle$ is robust against the depolarizing channel.

Recently, the entangled six-qubit state was widely used in the field of quantum cryptography. Zha et al. [35] presented two schemes for the remote preparation of a four-qubit W state and calculated the success probability of the RSP scheme in general. Li et al. [36] proposed a scheme for state sharing of an arbitrary single-qubit state by using this state as the quantum channel. Sun et al. [37] presented a scheme of bidirectional quantum controlled teleportation in which a six-qubit maximally entangled state quantum channel was initially shared by the Alice, Bob, and supervisor Charlie. Sun et al. [38] gave a multi-party quantum key agreement protocol utilized the state. However, to our best knowledge, up to now, there have been no protocols for how to generate DRSP of an arbitrary three-qubit state with this entangled six-qubit state as the shared quantum channel. These factors give us motivation to investigate this six-qubit entangled channel for the above mentioned DRSP.

So, the quantum channel by us can be expressed as follows:

$$|\psi_6\rangle = \frac{1}{2}[|F_0\rangle|\phi_-\rangle + |F_1\rangle|\phi_+\rangle + |F_2\rangle|\psi_-\rangle + |F_3\rangle|\psi_+\rangle]_{123456} \tag{2}$$

Here,

$$\begin{array}{l} |F_0\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle),\ |F_1\rangle = \frac{1}{\sqrt{2}}(|0011\rangle + |1100\rangle), \\ |F_2\rangle = \frac{1}{\sqrt{2}}(|0110\rangle + |1001\rangle),\ |F_3\rangle = \frac{1}{\sqrt{2}}(|0101\rangle + |1010\rangle). \end{array} \tag{3}$$

$$|\phi_\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle),|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle) \tag{4}$$

3. Remote State Preparation of an Arbitrary Three-Particle State

In this section, we focus on the remote state preparation in eight dimensions, i.e. the initial states with three particles. Suppose that Alice wants to help Bob prepare an arbitrary three-particle state remotely

$$|\Psi\rangle = \alpha_0|000\rangle + \alpha_1|001\rangle + \alpha_2|010\rangle + \alpha_3|011\rangle + \alpha_4|100\rangle + \alpha_5|101\rangle + \alpha_6|110\rangle + \alpha_7|111\rangle \tag{5}$$

Here, the coefficients satisfy the normalization condition $\sum_{i=0}^{i=7}|\alpha_i|^2 = 1$. Alice knows these coefficient completely, but Bob does not know them at all. We suppose Alice and Bob share a entangled six-particle state $|\psi_6\rangle$ which is shown in Equation (2) as the quantum channel. Here, Alice and Bob possess the particles 1, 2, 3, and 4, 5, 6, respectively.

We will first discuss the prepared states in the real Hilbert space in Section 3.1. More importantly, the initial state $|\Psi\rangle$ with complex coefficients is considered in Section 3.2.

3.1. The Coefficients Are Real

Suppose that α_i $(0 \leqslant i \leqslant 7)$ is an arbitrary real number. Using Equations (3) and (4), the entangled six-particle state $|\psi_6\rangle$ in Equation (2) can be rewritten

$$\begin{aligned} |\psi_6\rangle_{123456} = {} & \tfrac{1}{2\sqrt{2}}\cdot\tfrac{1}{\sqrt{2}}[(|0000\rangle + |1100\rangle)_{1234}|00\rangle_{56} + (|0110\rangle + |1010\rangle)_{1234}|01\rangle_{56} + (-|0110\rangle + |1010\rangle)_{1234}|10\rangle_{56} \\ & +(-|0000\rangle + |1100\rangle)_{1234}|11\rangle_{56} + (|1111\rangle + |0011\rangle)_{1234}|00\rangle_{56} + (|1001\rangle + |0101\rangle)_{1234}|01\rangle_{56} \\ & +(-|1001\rangle + |0101\rangle)_{1234}|10\rangle_{56} + (-|1111\rangle + |0011\rangle)_{1234}|11\rangle_{56}] \\ = {} & \tfrac{1}{2\sqrt{2}}\cdot\tfrac{1}{\sqrt{2}}[(|000\rangle + |110\rangle)_{123}|000\rangle_{456} + (|011\rangle + |101\rangle)_{123}|001\rangle_{456} + (-|011\rangle + |101\rangle)_{123}|010\rangle_{456} \\ & +(-|000\rangle + |110\rangle)_{123}|011\rangle_{456} + (|111\rangle + |001\rangle)_{123}|100\rangle_{456} + (|100\rangle + |010\rangle)_{123}|101\rangle_{456} \\ & +(-|100\rangle + |010\rangle)_{123}|110\rangle_{456} + (-|111\rangle + |001\rangle)_{123}|111\rangle_{456}] \end{aligned} \tag{6}$$

First, Alice makes a three-particle projective measurement on her particles 1, 2, 3. A set of mutually orthogonal basis $\{|\mu_1\rangle, |\mu_2\rangle, |\mu_3\rangle, |\mu_4\rangle, |\mu_5\rangle, |\mu_6\rangle, |\mu_7\rangle, |\mu_8\rangle\}$ with all real coefficients are given as

$$
\begin{aligned}
\begin{pmatrix} |\mu_1\rangle \\ |\mu_2\rangle \\ |\mu_3\rangle \\ |\mu_4\rangle \\ |\mu_5\rangle \\ |\mu_6\rangle \\ |\mu_7\rangle \\ |\mu_8\rangle \end{pmatrix}
&=
\begin{pmatrix}
\alpha_0 & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & \alpha_7 \\
\alpha_1 & -\alpha_0 & \alpha_3 & -\alpha_2 & \alpha_5 & -\alpha_4 & \alpha_7 & -\alpha_6 \\
\alpha_2 & -\alpha_3 & -\alpha_0 & \alpha_1 & \alpha_6 & -\alpha_7 & -\alpha_4 & \alpha_5 \\
\alpha_3 & \alpha_2 & -\alpha_1 & -\alpha_0 & -\alpha_7 & -\alpha_6 & \alpha_5 & \alpha_4 \\
\alpha_4 & -\alpha_5 & -\alpha_6 & \alpha_7 & -\alpha_0 & \alpha_1 & \alpha_2 & -\alpha_3 \\
\alpha_5 & \alpha_4 & \alpha_7 & \alpha_6 & -\alpha_1 & -\alpha_0 & -\alpha_3 & -\alpha_2 \\
\alpha_6 & -\alpha_7 & \alpha_4 & -\alpha_5 & -\alpha_2 & \alpha_3 & -\alpha_0 & \alpha_1 \\
\alpha_7 & \alpha_6 & -\alpha_5 & -\alpha_4 & \alpha_3 & \alpha_2 & -\alpha_1 & -\alpha_0
\end{pmatrix}
\begin{pmatrix}
\frac{|000\rangle+|110\rangle}{\sqrt{2}} \\
\frac{|011\rangle+|101\rangle}{\sqrt{2}} \\
\frac{-|011\rangle+|101\rangle}{\sqrt{2}} \\
\frac{-|000\rangle+|110\rangle}{\sqrt{2}} \\
\frac{|111\rangle+|001\rangle}{\sqrt{2}} \\
\frac{|100\rangle+|010\rangle}{\sqrt{2}} \\
\frac{-|100\rangle+|010\rangle}{\sqrt{2}} \\
\frac{-|111\rangle+|001\rangle}{\sqrt{2}}
\end{pmatrix} \\
&=
\begin{pmatrix}
\alpha_0 & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & \alpha_7 \\
\alpha_1 & -\alpha_0 & \alpha_3 & -\alpha_2 & \alpha_5 & -\alpha_4 & \alpha_7 & -\alpha_6 \\
\alpha_2 & -\alpha_3 & -\alpha_0 & \alpha_1 & \alpha_6 & -\alpha_7 & -\alpha_4 & \alpha_5 \\
\alpha_3 & \alpha_2 & -\alpha_1 & -\alpha_0 & -\alpha_7 & -\alpha_6 & \alpha_5 & \alpha_4 \\
\alpha_4 & -\alpha_5 & -\alpha_6 & \alpha_7 & -\alpha_0 & \alpha_1 & \alpha_2 & -\alpha_3 \\
\alpha_5 & \alpha_4 & \alpha_7 & \alpha_6 & -\alpha_1 & -\alpha_0 & -\alpha_3 & -\alpha_2 \\
\alpha_6 & -\alpha_7 & \alpha_4 & -\alpha_5 & -\alpha_2 & \alpha_3 & -\alpha_0 & \alpha_1 \\
\alpha_7 & \alpha_6 & -\alpha_5 & -\alpha_4 & \alpha_3 & \alpha_2 & -\alpha_1 & -\alpha_0
\end{pmatrix}
\cdot
\begin{pmatrix}
\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\
0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\
-\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\
0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\
0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\
0 & 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}}
\end{pmatrix}
\begin{pmatrix} |000\rangle \\ |001\rangle \\ |010\rangle \\ |011\rangle \\ |100\rangle \\ |101\rangle \\ |110\rangle \\ |111\rangle \end{pmatrix} \\
&=
\begin{pmatrix}
A_0 & A_7 & A_6 & A_1 & A_5 & A_2 & A_3 & A_4 \\
A_2 & A_5 & -A_4 & -A_3 & -A_7 & -A_0 & A_1 & A_6 \\
-A_1 & A_6 & -A_7 & A_0 & A_4 & -A_3 & A_2 & -A_5 \\
A_3 & A_4 & A_5 & A_2 & -A_6 & -A_1 & -A_0 & -A_7 \\
A_4 & -A_3 & A_2 & -A_5 & A_1 & -A_6 & A_7 & -A_0 \\
A_5 & -A_2 & -A_3 & A_4 & -A_0 & A_7 & A_6 & -A_1 \\
A_6 & A_1 & -A_0 & -A_7 & A_3 & A_4 & -A_5 & -A_2 \\
A_7 & -A_0 & -A_1 & A_6 & A_2 & -A_5 & -A_4 & A_3
\end{pmatrix}
\begin{pmatrix} |000\rangle \\ |001\rangle \\ |010\rangle \\ |011\rangle \\ |100\rangle \\ |101\rangle \\ |110\rangle \\ |111\rangle \end{pmatrix}
\end{aligned} \tag{7}
$$

where the real coefficients A_i $(0 \leqslant i \leqslant 7)$ are given as

$$
\begin{aligned}
A_0 = \frac{1}{\sqrt{2}}\left(\alpha_0 - \alpha_3\right), A_1 = \frac{1}{\sqrt{2}}\left(\alpha_1 - \alpha_2\right), A_2 = \frac{1}{\sqrt{2}}\left(\alpha_1 + \alpha_2\right), A_3 = \frac{1}{\sqrt{2}}\left(\alpha_0 + \alpha_3\right), \\
A_4 = \frac{1}{\sqrt{2}}\left(\alpha_4 - \alpha_7\right), A_5 = \frac{1}{\sqrt{2}}\left(\alpha_5 - \alpha_6\right), A_6 = \frac{1}{\sqrt{2}}\left(\alpha_5 + \alpha_6\right), A_7 = \frac{1}{\sqrt{2}}\left(\alpha_4 + \alpha_7\right).
\end{aligned} \tag{8}
$$

Using Equation (8), the whole system will be rewritten in terms of this measurement basis as

$$
|\psi_6\rangle_{123456} = \frac{1}{2\sqrt{2}}
\begin{pmatrix} |\mu_1\rangle \\ |\mu_2\rangle \\ |\mu_3\rangle \\ |\mu_4\rangle \\ |\mu_5\rangle \\ |\mu_6\rangle \\ |\mu_7\rangle \\ |\mu_8\rangle \end{pmatrix}^T
\begin{pmatrix}
\alpha_0 & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & \alpha_7 \\
\alpha_1 & -\alpha_0 & \alpha_3 & -\alpha_2 & \alpha_5 & -\alpha_4 & \alpha_7 & -\alpha_6 \\
\alpha_2 & -\alpha_3 & -\alpha_0 & \alpha_1 & \alpha_6 & -\alpha_7 & -\alpha_4 & \alpha_5 \\
\alpha_3 & \alpha_2 & -\alpha_1 & -\alpha_0 & -\alpha_7 & -\alpha_6 & \alpha_5 & \alpha_4 \\
\alpha_4 & -\alpha_5 & -\alpha_6 & \alpha_7 & -\alpha_0 & \alpha_1 & \alpha_2 & -\alpha_3 \\
\alpha_5 & \alpha_4 & \alpha_7 & \alpha_6 & -\alpha_1 & -\alpha_0 & -\alpha_3 & -\alpha_2 \\
\alpha_6 & -\alpha_7 & \alpha_4 & -\alpha_5 & -\alpha_2 & \alpha_3 & -\alpha_0 & \alpha_1 \\
\alpha_7 & \alpha_6 & -\alpha_5 & -\alpha_4 & \alpha_3 & \alpha_2 & -\alpha_1 & -\alpha_0
\end{pmatrix}
\begin{pmatrix} |000\rangle \\ |001\rangle \\ |010\rangle \\ |011\rangle \\ |100\rangle \\ |101\rangle \\ |110\rangle \\ |111\rangle \end{pmatrix}_{456}
$$

$$
\begin{aligned}
= \frac{1}{2\sqrt{2}}[& |\mu_1\rangle\left(\alpha_0|000\rangle + \alpha_1|001\rangle + \alpha_2|010\rangle + \alpha_3|011\rangle + \alpha_4|100\rangle + \alpha_5|101\rangle + \alpha_6|110\rangle + \alpha_7|111\rangle\right)_{456} \\
& + |\mu_2\rangle\left(\alpha_1|000\rangle - \alpha_0|001\rangle + \alpha_3|010\rangle - \alpha_2|011\rangle + \alpha_5|100\rangle - \alpha_4|101\rangle + \alpha_7|110\rangle - \alpha_6|111\rangle\right)_{456} \\
& + |\mu_3\rangle\left(\alpha_2|000\rangle - \alpha_3|001\rangle - \alpha_0|010\rangle + \alpha_1|011\rangle + \alpha_6|100\rangle - \alpha_7|101\rangle - \alpha_4|110\rangle + \alpha_5|111\rangle\right)_{456} \\
& + |\mu_4\rangle\left(\alpha_3|000\rangle + \alpha_2|001\rangle - \alpha_1|010\rangle - \alpha_0|011\rangle - \alpha_7|100\rangle - \alpha_6|101\rangle + \alpha_5|110\rangle + \alpha_4|111\rangle\right)_{456} \\
& + |\mu_5\rangle\left(\alpha_4|000\rangle - \alpha_5|001\rangle - \alpha_6|010\rangle + \alpha_7|011\rangle - \alpha_0|100\rangle + \alpha_1|101\rangle + \alpha_2|110\rangle - \alpha_3|111\rangle\right)_{456} \\
& + |\mu_6\rangle\left(\alpha_5|000\rangle + \alpha_4|001\rangle + \alpha_7|010\rangle + \alpha_6|011\rangle - \alpha_1|100\rangle - \alpha_0|101\rangle - \alpha_3|110\rangle - \alpha_2|111\rangle\right)_{456} \\
& + |\mu_7\rangle\left(\alpha_6|000\rangle - \alpha_7|001\rangle + \alpha_4|010\rangle - \alpha_5|011\rangle - \alpha_2|100\rangle + \alpha_3|101\rangle - \alpha_0|110\rangle + \alpha_1|111\rangle\right)_{456} \\
& + |\mu_8\rangle\left(\alpha_7|000\rangle + \alpha_6|001\rangle - \alpha_5|010\rangle - \alpha_4|011\rangle + \alpha_3|100\rangle + \alpha_2|101\rangle - \alpha_1|110\rangle - \alpha_0|111\rangle\right)_{456}]
\end{aligned}
$$

From the above equation, it is transparent that a process of entanglement swapping happens after Alice's three-particle projective measurement on the particles 1, 2, 3, under the basis $\{|\mu_i\rangle\,(i = 1, 2, \cdots, 8)\}$, and a new entanglement is established among particles 4, 5, 6. Suppose three classical bit strings $m_1m_2m_3 = 000 \sim 111$ correspond to the measurement result $|\mu_i\rangle (i = 1, 2, ..., 8)$. Then Alice tells Bob the measurement result through a classical channel. Whatever Alice's measurement outcomes are, Bob can always obtain the prepared state on his particles by performing an appropriate unitary operation (shown in Table 1).

Table 1. The recovery operations for Bob. The measurement result of Alice on particles 1, 2, 3 is AMR_{123}, and the counterpart classical message is $m_1m_2m_3$. Bob's appropriate unitary operation is defined as BAUO.

AMR_{123}	$m_1m_2m_3$	**The State on the Particles 4, 5, 6**	**BAUO**
$\|\mu_1\rangle$	000	$\alpha_0 \|000\rangle + \alpha_1 \|001\rangle + \alpha_2 \|010\rangle + \alpha_3 \|011\rangle$ $+\alpha_4 \|100\rangle + \alpha_5 \|101\rangle + \alpha_6 \|110\rangle + \alpha_7 \|111\rangle$	$I_4 \otimes I_5 \otimes I_6$
$\|\mu_2\rangle$	001	$\alpha_1 \|000\rangle - \alpha_0 \|001\rangle + \alpha_3 \|010\rangle - \alpha_2 \|011\rangle$ $+\alpha_5 \|100\rangle - \alpha_4 \|101\rangle + \alpha_7 \|110\rangle - \alpha_6 \|111\rangle$	$I_4 \otimes I_5 \otimes \sigma_6^{xz}$
$\|\mu_3\rangle$	010	$\alpha_2 \|000\rangle - \alpha_3 \|001\rangle - \alpha_0 \|010\rangle + \alpha_1 \|011\rangle$ $+\alpha_6 \|100\rangle - \alpha_7 \|101\rangle - \alpha_4 \|110\rangle + \alpha_5 \|111\rangle$	$I_4 \otimes \sigma_5^{xz} \otimes \sigma_6^z$
$\|\mu_4\rangle$	011	$\alpha_3 \|000\rangle + \alpha_2 \|001\rangle - \alpha_1 \|010\rangle - \alpha_0 \|011\rangle$ $-\alpha_7 \|100\rangle - \alpha_6 \|101\rangle + \alpha_5 \|110\rangle + \alpha_4 \|111\rangle$	$\sigma_4{}^z \otimes \sigma_5^{xz} \otimes \sigma_6^x$
$\|\mu_5\rangle$	100	$\alpha_4 \|000\rangle - \alpha_5 \|001\rangle - \alpha_6 \|010\rangle + \alpha_7 \|011\rangle$ $-\alpha_0 \|100\rangle + \alpha_1 \|101\rangle + \alpha_2 \|110\rangle - \alpha_3 \|111\rangle$	$\sigma_4^{xz} \otimes \sigma_5^z \otimes \sigma_6^z$
$\|\mu_6\rangle$	101	$\alpha_5 \|000\rangle + \alpha_4 \|001\rangle + \alpha_7 \|010\rangle + \alpha_6 \|011\rangle$ $-\alpha_1 \|100\rangle - \alpha_0 \|101\rangle - \alpha_3 \|110\rangle - \alpha_2 \|111\rangle$	$\sigma_4^{xz} \otimes I \otimes \sigma_6^x$
$\|\mu_7\rangle$	110	$\alpha_6 \|000\rangle - \alpha_7 \|001\rangle + \alpha_4 \|010\rangle - \alpha_5 \|011\rangle$ $-\alpha_2 \|100\rangle + \alpha_3 \|101\rangle - \alpha_0 \|110\rangle + \alpha_1 \|111\rangle$	$\sigma_4^{xz} \otimes \sigma_5^x \otimes \sigma_6^z$
$\|\mu_8\rangle$	111	$\alpha_7 \|000\rangle + \alpha_6 \|001\rangle - \alpha_5 \|010\rangle - \alpha_4 \|011\rangle$ $+\alpha_3 \|100\rangle + \alpha_2 \|101\rangle - \alpha_1 \|110\rangle - \alpha_0 \|111\rangle$	$\sigma_4^x \otimes \sigma_5^{xz} \otimes \sigma_6^x$

In this scheme, we can see that Alice can help Bob remotely prepare the specified state with 100% success probability. For clearness, the schematic demonstration and quantum circuit diagram of our RSP scheme have been shown in Figures 1 and 2, respectively.

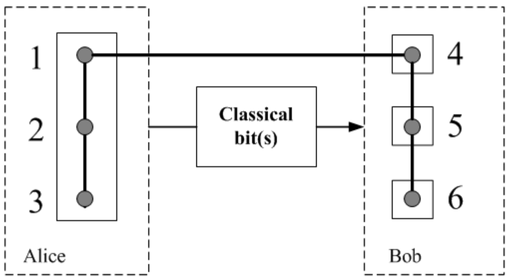

Figure 1. The schematic demonstration. Solid dots represent particles; a rectangle denotes a three-particle projective measurement; squares stand for single-particle unitary operations.

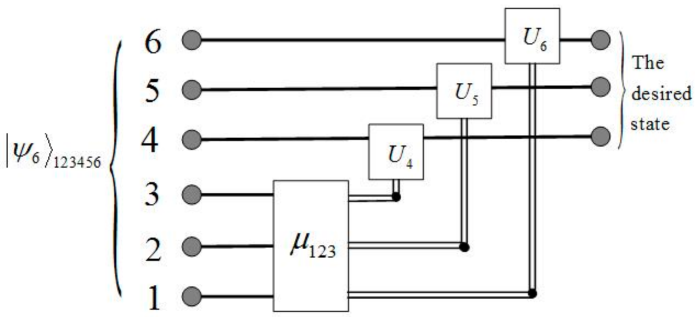

Figure 2. The quantum circuit diagram. In the circuit, the single lines are denoted as the quantum channels, while the double lines represent the classical channels. The element μ_{123} indicates making a three-particle projective measurement on particles 1, 2, 3; $U_i (i = 4, 5, 6)$ indicates performing an appropriate unitary operation on corresponding particles.

3.2. The Coefficients Are Complex

In this scenario, the new measurement basis is constructed in an eight-dimensional complex Hilbert space. They only contain the initial coefficients and their conjugate values, so it is convenient for us to further investigate the permutation relationship between coefficients. A criterion for deterministic RSP is given, to determine which states can be prepared with 100% success probability. For clarity, an example is given to show that such special set of states really exists.

Now we further consider the remote preparation of a more general arbitrary three-particle state

$$|\Psi\rangle = \alpha_0 |000\rangle + \alpha_1 |001\rangle + \alpha_2 |010\rangle + \alpha_3 |011\rangle + \alpha_4 |100\rangle + \alpha_5 |101\rangle + \alpha_6 |110\rangle + \alpha_7 |111\rangle \tag{9}$$

where the coefficients satisfy the normalization condition $\sum_{i=0}^{i=7} |\alpha_i|^2 = 1$. Here, α_i $(0 \leqslant i \leqslant 7)$ is an arbitrary complex number. We still assume Alice knows this state $|\Psi\rangle$, but Bob does not.

Likewise, Alice first performs a three-qubit projective measurement on her three particles 1, 2, 3, under eight mutual orthogonal measurement bases $|\eta_1\rangle$, $|\eta_2\rangle$, $|\eta_3\rangle$, $|\eta_4\rangle$, $|\eta_5\rangle$, $|\eta_6\rangle$, $|\eta_7\rangle$ and $|\eta_8\rangle$, which are given by

$$\begin{aligned}
\begin{pmatrix} |\eta_1\rangle \\ |\eta_2\rangle \\ |\eta_3\rangle \\ |\eta_4\rangle \\ |\eta_5\rangle \\ |\eta_6\rangle \\ |\eta_7\rangle \\ |\eta_8\rangle \end{pmatrix}
&= \begin{pmatrix}
\alpha_0^* & \alpha_1^* & \alpha_2^* & \alpha_3^* & \alpha_4^* & \alpha_5^* & \alpha_6^* & \alpha_7^* \\
\lambda\alpha_0^* & -\frac{\alpha_1^*}{\lambda} & -\frac{\alpha_2^*}{\lambda} & \lambda\alpha_3^* & \lambda\alpha_4^* & -\frac{\alpha_5^*}{\lambda} & -\frac{\alpha_6^*}{\lambda} & \lambda\alpha_7^* \\
\alpha_3 & \alpha_2 & -\alpha_1 & -\alpha_0 & \alpha_7 & \alpha_6 & -\alpha_5 & -\alpha_4 \\
\lambda\alpha_3 & -\frac{\alpha_2}{\lambda} & \frac{\alpha_1}{\lambda} & -\lambda\alpha_0 & \lambda\alpha_7 & -\frac{\alpha_6}{\lambda} & \frac{\alpha_5}{\lambda} & -\lambda\alpha_4 \\
\alpha_4^* & \alpha_5^* & \alpha_6^* & \alpha_7^* & \alpha_0^* & \alpha_1^* & \alpha_2^* & \alpha_3^* \\
\lambda\alpha_4^* & -\frac{\alpha_5^*}{\lambda} & -\frac{\alpha_6^*}{\lambda} & \lambda\alpha_7^* & \lambda\alpha_0^* & -\frac{\alpha_1^*}{\lambda} & -\frac{\alpha_2^*}{\lambda} & \lambda\alpha_3^* \\
\alpha_7 & \alpha_6 & -\alpha_5 & -\alpha_4 & \alpha_3 & \alpha_2 & -\alpha_1 & -\alpha_0 \\
\lambda\alpha_7 & -\frac{\alpha_6}{\lambda} & \frac{\alpha_5}{\lambda} & -\lambda\alpha_4 & \lambda\alpha_3 & -\frac{\alpha_2}{\lambda} & \frac{\alpha_1}{\lambda} & -\lambda\alpha_0
\end{pmatrix} \\
&\cdot \begin{pmatrix}
\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 \\
0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\
-\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\
0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\
0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\
0 & 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}}
\end{pmatrix}
\begin{pmatrix} |000\rangle \\ |001\rangle \\ |010\rangle \\ |011\rangle \\ |100\rangle \\ |101\rangle \\ |110\rangle \\ |111\rangle \end{pmatrix} \\
&= \begin{pmatrix}
A_0^* & A_7^* & A_6^* & A_1^* & A_5^* & A_2^* & A_3^* & A_4^* \\
\lambda A_0^* & \lambda A_7^* & -\frac{A_6^*}{\lambda} & -\frac{A_1^*}{\lambda} & -\frac{A_5^*}{\lambda} & -\frac{A_2^*}{\lambda} & \lambda A_3^* & \lambda A_4^* \\
A_3 & -A_4 & -A_5 & A_2 & A_6 & -A_1 & -A_0 & A_7 \\
\lambda A_3 & -\lambda A_4 & \frac{A_5}{\lambda} & -\frac{A_2}{\lambda} & -\frac{A_6}{\lambda} & \frac{A_1}{\lambda} & -\lambda A_0 & \lambda A_7 \\
A_4^* & A_3^* & A_2^* & A_5^* & A_1^* & A_6^* & A_7^* & A_0^* \\
\lambda A_4^* & \lambda A_3^* & -\frac{A_2^*}{\lambda} & -\frac{A_5^*}{\lambda} & -\frac{A_1^*}{\lambda} & -\frac{A_6^*}{\lambda} & \lambda A_7^* & \lambda A_0^* \\
A_7 & -A_0 & -A_1 & A_6 & A_2 & -A_5 & -A_4 & A_3 \\
\lambda A_7 & -\lambda A_0 & \frac{A_1}{\lambda} & -\frac{A_6}{\lambda} & -\frac{A_2}{\lambda} & \frac{A_5}{\lambda} & -\lambda A_4 & \lambda A_3
\end{pmatrix}
\begin{pmatrix} |000\rangle \\ |001\rangle \\ |010\rangle \\ |011\rangle \\ |100\rangle \\ |101\rangle \\ |110\rangle \\ |111\rangle \end{pmatrix}
\end{aligned} \tag{10}$$

Here, * denotes the conjugate of a matrix, the complex coefficients A_i $(0 \leqslant i \leqslant 7)$ are the same as Equation (8), and the coefficient λ is

$$\lambda = \frac{\sqrt{|A_1|^2 + |A_2|^2 + |A_5|^2 + |A_6|^2}}{\sqrt{|A_0|^2 + |A_3|^2 + |A_4|^2 + |A_7|^2}} = \frac{\sqrt{|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_5|^2 + |\alpha_6|^2}}{\sqrt{|\alpha_0|^2 + |\alpha_3|^2 + |\alpha_4|^2 + |\alpha_7|^2}} \tag{11}$$

Considering the orthogonality, the coefficients should satisfy the following conditions

$$\begin{aligned}
\alpha_0\alpha_4{}^* + \alpha_4\alpha_0{}^* + \alpha_3\alpha_7{}^* + \alpha_7\alpha_3{}^* &= 0 \\
\alpha_1\alpha_5{}^* + \alpha_5\alpha_1{}^* + \alpha_2\alpha_6{}^* + \alpha_6\alpha_2{}^* &= 0
\end{aligned} \tag{12}$$

Using Equation (8), the whole system will be rewritten in terms of this measurement basis as

$$|\psi_6\rangle_{123456}=\frac{1}{2\sqrt{2}}\begin{pmatrix}|\eta_1\rangle\\|\eta_2\rangle\\|\eta_3\rangle\\|\eta_4\rangle\\|\eta_5\rangle\\|\eta_6\rangle\\|\eta_7\rangle\\|\eta_8\rangle\end{pmatrix}^T\begin{pmatrix}\alpha_0^* & \alpha_1^* & \alpha_2^* & \alpha_3^* & \alpha_4^* & \alpha_5^* & \alpha_6^* & \alpha_7^*\\ \lambda\alpha_0^* & -\frac{\alpha_1^*}{\lambda} & -\frac{\alpha_2^*}{\lambda} & \lambda\alpha_3^* & \lambda\alpha_4^* & -\frac{\alpha_5^*}{\lambda} & -\frac{\alpha_6^*}{\lambda} & \lambda\alpha_7^*\\ \alpha_3 & \alpha_2 & -\alpha_1 & -\alpha_0 & \alpha_7 & \alpha_6 & -\alpha_5 & -\alpha_4\\ \lambda\alpha_3 & -\frac{\alpha_2}{\lambda} & \frac{\alpha_1}{\lambda} & -\lambda\alpha_0 & \lambda\alpha_7 & -\frac{\alpha_6}{\lambda} & \frac{\alpha_5}{\lambda} & -\lambda\alpha_4\\ \alpha_4^* & \alpha_5^* & \alpha_6^* & \alpha_7^* & \alpha_0^* & \alpha_1^* & \alpha_2^* & \alpha_3^*\\ \lambda\alpha_4^* & -\frac{\alpha_5^*}{\lambda} & -\frac{\alpha_6^*}{\lambda} & \lambda\alpha_7^* & \lambda\alpha_0^* & -\frac{\alpha_1^*}{\lambda} & -\frac{\alpha_2^*}{\lambda} & \lambda\alpha_3^*\\ \alpha_7 & \alpha_6 & -\alpha_5 & -\alpha_4 & \alpha_3 & \alpha_2 & -\alpha_1 & -\alpha_0\\ \lambda\alpha_7 & -\frac{\alpha_6}{\lambda} & \frac{\alpha_5}{\lambda} & -\lambda\alpha_4 & \lambda\alpha_3 & -\frac{\alpha_2}{\lambda} & \frac{\alpha_1}{\lambda} & -\lambda\alpha_0\end{pmatrix}^*\begin{pmatrix}|000\rangle\\|001\rangle\\|010\rangle\\|011\rangle\\|100\rangle\\|101\rangle\\|110\rangle\\|111\rangle\end{pmatrix}_{456}$$

$$\begin{aligned}
=\frac{1}{2\sqrt{2}}[&|\eta_1\rangle(\alpha_0|000\rangle+\alpha_1|001\rangle+\alpha_2|010\rangle+\alpha_3|011\rangle+\alpha_4|100\rangle+\alpha_5|101\rangle+\alpha_6|110\rangle+\alpha_7|111\rangle)_{456}\\
&+|\eta_2\rangle(\lambda\alpha_0|000\rangle-\tfrac{\alpha_1}{\lambda}|001\rangle-\tfrac{\alpha_2}{\lambda}|010\rangle+\lambda\alpha_3|011\rangle+\lambda\alpha_4|100\rangle-\tfrac{\alpha_5}{\lambda}|101\rangle-\tfrac{\alpha_6}{\lambda}|110\rangle+\lambda\alpha_7|111\rangle)_{456}\\
&+|\eta_3\rangle(\alpha_3{}^*|000\rangle+\alpha_2{}^*|001\rangle-\alpha_1{}^*|010\rangle-\alpha_0{}^*|011\rangle+\alpha_7{}^*|100\rangle+\alpha_6{}^*|101\rangle-\alpha_5{}^*|110\rangle-\alpha_4{}^*|111\rangle)_{456}\\
&+|\eta_4\rangle(\lambda\alpha_3{}^*|000\rangle-\tfrac{\alpha_2{}^*}{\lambda}|001\rangle+\tfrac{\alpha_1{}^*}{\lambda}|010\rangle-\lambda\alpha_0{}^*|011\rangle+\lambda\alpha_7{}^*|100\rangle-\tfrac{\alpha_6{}^*}{\lambda}|101\rangle+\tfrac{\alpha_5{}^*}{\lambda}|110\rangle-\lambda\alpha_4{}^*|111\rangle)_{456}\\
&+|\eta_5\rangle(\alpha_4|000\rangle+\alpha_5|001\rangle+\alpha_6|010\rangle+\alpha_7|011\rangle+\alpha_0|100\rangle+\alpha_1|101\rangle+\alpha_2|110\rangle+\alpha_3|111\rangle)_{456}\\
&+|\eta_6\rangle(\lambda\alpha_4|000\rangle-\tfrac{\alpha_5}{\lambda}|001\rangle-\tfrac{\alpha_6}{\lambda}|010\rangle+\lambda\alpha_7|011\rangle+\lambda\alpha_0|100\rangle-\tfrac{\alpha_1}{\lambda}|101\rangle-\tfrac{\alpha_2}{\lambda}|110\rangle+\lambda\alpha_3|111\rangle)_{456}\\
&+|\eta_7\rangle(\alpha_7{}^*|000\rangle+\alpha_6{}^*|001\rangle-\alpha_5{}^*|010\rangle-\alpha_4{}^*|011\rangle+\alpha_3{}^*|100\rangle+\alpha_2{}^*|101\rangle-\alpha_1{}^*|110\rangle-\alpha_0{}^*|111\rangle)_{456}\\
&+|\eta_8\rangle(\lambda\alpha_7{}^*|000\rangle-\tfrac{\alpha_6{}^*}{\lambda}|001\rangle+\tfrac{\alpha_5{}^*}{\lambda}|010\rangle-\lambda\alpha_4{}^*|011\rangle+\lambda\alpha_3{}^*|100\rangle-\tfrac{\alpha_2{}^*}{\lambda}|101\rangle+\tfrac{\alpha_1{}^*}{\lambda}|110\rangle-\lambda\alpha_0{}^*|111\rangle)_{456}]
\end{aligned}$$

From the above equation, suppose three classical bit strings $m_1m_2m_3 = 000 \sim 111$ correspond to the measurement result $|\eta_i\rangle\,(i = 1, 2, ..., 8)$. After the measurement, Alice sends the classical bits to Bob. For the states with arbitrary complex coefficients, Bob can recover the prepared state only when the measurement result is 000 (100). The recovery operations performed by him is $I_4 \otimes I_5 \otimes I_6(\sigma_4^x \otimes I_5 \otimes I_6)$, respectively. Otherwise, he cannot find a recovery operation independent of the coefficients. So, in this case the success probability is 2/8 = 25%.

However, if the coefficients are of some special values, Bob may reconstruct the initial state successfully with higher probability. To achieve this, we suppose the coefficient $\lambda = 1$. This may set restrictions to the coefficients of the initial state, because:

$$\lambda = 1 \rightleftarrows |\alpha_0|^2 + |\alpha_3|^2 + |\alpha_4|^2 + |\alpha_7|^2 = |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_5|^2 + |\alpha_6|^2 \tag{13}$$

Now, Bob can recover the prepared initial state successfully when the measurement result of Alice is 001 (101). It is easy to see that the success probability will be raised to 4/8=50%. The recovery operations performed by Bob are $I_4 \otimes \sigma_5^z \otimes \sigma_6^z$ and $\sigma_4^x \otimes \sigma_5^z \otimes \sigma_6^z$, respectively.

Furthermore, if the coefficients meet certain relationships between $\{\alpha_0, \alpha_1, \cdots, \alpha_7\}$ and $\{\alpha_0^*, \alpha_1^*, \cdots, \alpha_7^*\}$, we can further increase the success probability.

We now introduce the permutation group S_8, which contains the set of 8! permutations for the eight coefficients $\{\alpha_0, \alpha_1, \cdots, \alpha_7\}$.

$$S_8 = \{(1), (12), (13), ..., (1234)(5678), ..., (123456)(78), ..., (12345678), ...\} \tag{14}$$

In Appendix, we partially give some permutations in S_8 and the corresponding recovery operations, not the whole outcomes.

Criterion 1. The arbitrary three-particle state Equation (9) can be remotely prepared with 100% success probability if the coefficients $\{\alpha_0, \alpha_1, \cdots, \alpha_7\}$ satisfy

(i) $\alpha_0\alpha_4{}^* + \alpha_4\alpha_0{}^* + \alpha_3\alpha_7{}^* + \alpha_7\alpha_3{}^* = 0$,
(ii) $\alpha_1\alpha_5{}^* + \alpha_5\alpha_1{}^* + \alpha_2\alpha_6{}^* + \alpha_6\alpha_2{}^* = 0$,
(iii) $|\alpha_0|^2 + |\alpha_3|^2 + |\alpha_4|^2 + |\alpha_7|^2 = |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_5|^2 + |\alpha_6|^2$,
(iv) $(\alpha_0^*, \alpha_1^*, \cdots, \alpha_7^*) = \lambda_g g(\alpha_0, \pm\alpha_1, \cdots, \pm\alpha_7)$.

Here, λ_g is a global constant that depends on the permutation $g \in S_8$.

A natural question would be: Is there a quantum state that satisfies all the above conditions? To show its existence, we now give an example

$$\begin{gathered}\alpha_0=\frac{1}{4\sqrt{2}},\ \alpha_1=\frac{\sqrt{3}}{4\sqrt{2}}e^{i\frac{3}{4}\pi},\ \alpha_2=\frac{1}{4\sqrt{2}}e^{i\pi},\ \alpha_3=\frac{\sqrt{3}}{4\sqrt{2}}e^{i\frac{1}{4}\pi},\\ \alpha_4=\frac{1}{4}e^{i\pi},\ \alpha_5=\frac{1}{4}e^{i\frac{7}{4}\pi},\ \alpha_6=\frac{1}{4},\ \alpha_7=\frac{1}{4}e^{i\frac{5}{4}\pi}.\end{gathered} \tag{15}$$

It is easy to verify that the above coefficients satisfy the conditions (i), (ii), (iii). The permutation relationships are

$$\frac{\alpha_0{}^*}{\alpha_2}=\frac{\alpha_1{}^*}{\alpha_3}=\frac{\alpha_2{}^*}{\alpha_0}=\frac{\alpha_3{}^*}{\alpha_1}=\frac{\alpha_4{}^*}{\alpha_6}=\frac{\alpha_5{}^*}{\alpha_7}=\frac{\alpha_6{}^*}{\alpha_4}=\frac{\alpha_7{}^*}{\alpha_5}=e^{i\pi} \tag{16}$$

No matter what measurement result Alice gets, Bob can always find the recovery operations to reconstruct the initial state. The detailed recovery operations are summarized in Table 2.

Table 2. The recovery operations for Bob. The measurement result of Alice on particles 1, 2, 3 is AMR_{123}, and the counterpart classical message is $m_1m_2m_3$. Bob's appropriate unitary operation is defined as BAUO.

$\mathbf{AMR_{123}}$	$m_1m_2m_3$	The State on the Particles 4, 5, 6	BAUO
$\vert\eta_1\rangle$	000	$\alpha_0\vert000\rangle+\alpha_1\vert001\rangle+\alpha_2\vert010\rangle+\alpha_3\vert011\rangle$ $+\alpha_4\vert100\rangle+\alpha_5\vert101\rangle+\alpha_6\vert110\rangle+\alpha_7\vert111\rangle$	$I_4\otimes I_5\otimes I_6$
$\vert\eta_2\rangle$	001	$\lambda\alpha_0\vert000\rangle-(\alpha_1/\lambda)\vert001\rangle-(\alpha_2/\lambda)\vert010\rangle+\lambda\alpha_3\vert011\rangle$ $+\lambda\alpha_4\vert100\rangle-(\alpha_5/\lambda)\vert101\rangle-(\alpha_6/\lambda)\vert110\rangle+\lambda\alpha_7\vert111\rangle$	$I_4\otimes\sigma_5^z\otimes\sigma_6^z$
$\vert\eta_3\rangle$	010	$\alpha_3{}^*\vert000\rangle+\alpha_2{}^*\vert001\rangle-\alpha_1{}^*\vert010\rangle-\alpha_0{}^*\vert011\rangle$ $+\alpha_7{}^*\vert100\rangle+\alpha_6{}^*\vert101\rangle-\alpha_5{}^*\vert110\rangle-\alpha_4{}^*\vert111\rangle$	$I_4\otimes\sigma_5^z\otimes\sigma_6^x$
$\vert\eta_4\rangle$	011	$\lambda\alpha_3{}^*\vert000\rangle-(\alpha_2{}^*/\lambda)\vert001\rangle+(\alpha_1{}^*/\lambda)\vert010\rangle-\lambda\alpha_0{}^*\vert011\rangle$ $+\lambda\alpha_7{}^*\vert100\rangle-(\alpha_6{}^*/\lambda)\vert101\rangle+(\alpha_5{}^*/\lambda)\vert110\rangle-\lambda\alpha_4{}^*\vert111\rangle$	$I_4\otimes\sigma_5^z\otimes\sigma_6^{xz}$
$\vert\eta_5\rangle$	100	$\alpha_4\vert000\rangle+\alpha_5\vert001\rangle+\alpha_6\vert010\rangle+\alpha_7\vert011\rangle$ $+\alpha_0\vert100\rangle+\alpha_1\vert101\rangle+\alpha_2\vert110\rangle+\alpha_3\vert111\rangle$	$\sigma_4^x\otimes I_5\otimes I_6$
$\vert\eta_6\rangle$	101	$\lambda\alpha_4\vert000\rangle-(\alpha_5/\lambda)\vert001\rangle-(\alpha_6/\lambda)\vert010\rangle+\lambda\alpha_7\vert011\rangle$ $+\lambda\alpha_0\vert100\rangle-(\alpha_1/\lambda)\vert101\rangle-(\alpha_2/\lambda)\vert110\rangle+\lambda\alpha_3\vert111\rangle$	$\sigma_4^x\otimes\sigma_5^z\otimes\sigma_6^z$
$\vert\eta_7\rangle$	110	$\alpha_7{}^*\vert000\rangle+\alpha_6{}^*\vert001\rangle-\alpha_5{}^*\vert010\rangle-\alpha_4{}^*\vert011\rangle$ $+\alpha_3{}^*\vert100\rangle+\alpha_2{}^*\vert101\rangle-\alpha_1{}^*\vert110\rangle-\alpha_0{}^*\vert111\rangle$	$\sigma_4^x\otimes\sigma_5^z\otimes\sigma_6^x$
$\vert\eta_8\rangle$	111	$\lambda\alpha_7{}^*\vert000\rangle-(\alpha_6{}^*/\lambda)\vert001\rangle+(\alpha_5{}^*/\lambda)\vert010\rangle-\lambda\alpha_4{}^*\vert011\rangle$ $+\lambda\alpha_3{}^*\vert100\rangle-(\alpha_2{}^*/\lambda)\vert101\rangle+(\alpha_1{}^*/\lambda)\vert110\rangle-\lambda\alpha_0{}^*\vert111\rangle$	$\sigma_4^x\otimes\sigma_5^z\otimes\sigma_6^{xz}$

4. Remote State Preparation of an Arbitrary *N*-Particle State

In the previous section, we have discussed the deterministic RSP of an arbitrary three-particle state using the entangled six-particle state $|\psi_6\rangle$ as the quantum channel. We first consider the preparation of states with all real coefficients. Second, a criterion is proposed for the RSP in complex Hilbert space with 100% success probability. Different from previous literatures, we introduce the permutation group S_8, which generates the complete set of states for deterministic RSP. It is natural to ask whether our criterion can be generalized to higher dimensions. One should note that deterministic RSP cannot be realized in real Hilbert space when the dimension is larger than eight. As for complex coefficients, the answer is "yes", as long as a proper set of orthogonal bases is constructed.

Alice wants to help Bob prepare an arbitrary N-particle state

$$|\Psi\rangle=\alpha_0|00\cdots0\rangle+\alpha_1|00\cdots1\rangle+\cdots+\alpha_{M-1}|11\cdots1\rangle \tag{17}$$

where $M=2^N$. The generalized Bell states are used as quantum resources, which are shared between Alice and Bob

$$|\psi_{Bell}\rangle=\frac{1}{\sqrt{M}}\left(\sum_{i=0}^{M-1}|i\rangle\otimes|i\rangle\right) \tag{18}$$

Next, a set of orthogonal basis should be constructed containing the initial coefficients and their conjugate values

$$\begin{array}{l} |X_1\rangle = \alpha_0^* |00\cdots 0\rangle + \alpha_1^* |00\cdots 1\rangle + \cdots + \alpha_{M-1}^* |11\cdots 1\rangle \\ |X_2\rangle = \alpha_{M-1} |00\cdots 0\rangle + \alpha_{M-2} |00\cdots 1\rangle - \alpha_{M-3} |00\cdots 10\rangle - \alpha_{M-4} |00\cdots 11\rangle + \cdots \\ +(-1)^{M/2-1}\alpha_1 |11\cdots 0\rangle + (-1)^{M/2-1}\alpha_0 |11\cdots 1\rangle \\ |X_3\rangle = \alpha_0^* |00\cdots 0\rangle - \alpha_1^* |00\cdots 1\rangle - \alpha_2^* |00\cdots 10\rangle + \alpha_3^* |00\cdots 11\rangle \cdots \\ +\alpha_{M-4}^* |11\cdots 00\rangle - \alpha_{M-3}^* |11\cdots 01\rangle - \alpha_{M-2}^* |11\cdots 10\rangle + \alpha_{M-1}^* |11\cdots 1\rangle \\ |X_4\rangle = \alpha_{M-1} |00\cdots 0\rangle - \alpha_{M-2} |00\cdots 1\rangle + \alpha_{M-3} |00\cdots 10\rangle - \alpha_{M-4} |00\cdots 11\rangle + \cdots \\ +(-1)^{M-2}\alpha_1 |11\cdots 0\rangle + (-1)^{M-1}\alpha_0 |11\cdots 1\rangle \\ \cdots\cdots \end{array} \tag{19}$$

Here, the other M-4 bases are omitted.

Now, the criterion for deterministic RSP in higher dimensions is naturally drawn. Similar to the previous section, we introduce the permutation group S_M. So, we generalize Criterion 1 to

Criterion 2. The N-particle state Equation (17) can be remotely prepared with 100% success probability if the coefficients $\{\alpha_0, \alpha_1, ..., \alpha_{M-1}\}$ satisfy

(1) The normalization condition,

(2) $(\alpha_0^*, \alpha_1^*, ..., \alpha_{M-1}^*) = \lambda_g g(\alpha_0, \pm\alpha_1, \ldots, \pm\alpha_{M-1})$.

Here, λ_g is a global constant that depends on the permutation $g \in S_M$.

If the coefficients satisfy the conditions in Criterion 2, the proper recovery operations can always be found for the receiver Bob. Thus, the deterministic RSP of N-particle state can be realized.

5. Classical Communication Cost

In this section, we calculate the CCC which is very important to reflect the classical resources required in our RSP.

Consider our scheme in Section 3, in fact, the total measurement result is $|\mu_i\rangle\,(i = 1, 2, ..., 8)$, and each of them can be obtained with the probability $\frac{1}{8}$. Therefore, the CCC is

$$8 \times \frac{1}{8}\log_2 8 = 3\text{cbits}$$

6. Discussion

In this section, in order to better illustrate the current research work, we summarize the comparison for two cases. One is to use the same quantum channel to achieve DRSP, the other is use the different quantum channels to realize an arbitrary three-particle state. Clear lists are presented in Tables 3 and 4.

Table 3. The comparison for using the same quantum channel to achieve DRSP.

	Zha's Protocol [35]	**Our Protocol**
Entanglement resource	Entangled six-qubit state $\|\psi_6\rangle$	Entangled six-qubit state $\|\psi_6\rangle$
Prepared state	Four-particle W state	Arbitrary three-particle state
The number of parameters	4	8
Qubits	6	6
Cbits	2	3
Qubit efficiency	100%	100%
Success probability	100% (Non-deterministic)	100% (Deterministic)
Recovery operation	U	Pauli operations

Table 4. The comparison for using the different quantum channels to realize an arbitrary three-particle state for DRSP.

	Entanglement Resource	Qubits	Cbits (General Case)	Success Probability
Zhan's protocol [30]	Three GHZ states	9	6	100%
Wang's protocol [31]	Four-qubit cluster state + EPR pair	6	3	50%
Ma's protocol [32]	χ state	6	4	50%
Our protocol	Entangled Six-qubit state $\vert\psi_6\rangle$	6	3	100%

Table 3 presents the comparison for using the same quantum channel $|\psi_6\rangle$ to achieve DRSP. Comparing to Zha's protocol in Reference [35], we have more advantages in the following three aspects despite using the same quantum. First, there are eight parameters in the prepared state, which need to construct a 8×8 matrix in eight-dimensional Hilbert space. More parameters means more complexity and difficulty of computing. Second, our success probability is deterministic and can reach 100% if the coefficients meet certain relationships. Third, Bob's recovery operations are easier than Zha's scheme. We just use simple Pauli operations instead of complex unitary transformation.

Table 4 shows the comparison for using the different quantum channels to realize an arbitrary three-particle state for DRSP. On the one hand, it is worth noting that six quantum resources are all consumed in our scheme and References [31,32], but the initial state can only be prepared with the probability 50% by theirs, whereas our scheme can achieve 100%. On the other hand, comparing to Reference [30], although the success of probability can reach 100% in this protocol, it sacrifices more quantum and classical resources to achieve recovery of the quantum state.

7. Conclusions

In summary, we have proposed two new criteria for DRSP. Using the entangled six-particle state $|\psi_6\rangle$, we investigate the DRSP that deals with real and complex coefficients in eight-dimensional Hilbert space. In the first case, we focus on the coefficients as real numbers. With the result of Alice's three-particle projective measurement, Bob can obtain the initial state with 100% success probability deterministically. For the latter, we make a discussion and find that Bob also can recover the initial state with a certain probability or even 1 according to our criterion if the coefficients satisfy some constraints.

Compared with previous studies, the feature of our protocol has the following distinct advantages: First, the initial state to be remotely prepared is an arbitrary three-qubit state, which is neither single-qubit state nor two-qubit state. So the projective measurement performed by Alice is in Hilbert spaces of eight dimensions (neither two nor four). Second, the serviceable measurement basis contains only the initial coefficients and their conjugate values. By utilizing the permutation group, it is convenient to provide the permutation relationship between coefficients. The scenarios of the state and corresponding recovery operations performed by the receiver are only partially listed in Appendix 7 because the number of permutations is enormous. Third, it is important that we present two perfect criteria to determine which states can be prepared with 100% success probability. Furthermore, our ideas and methods can also be generalized to the higher dimension state by taking advantage of the related operations of the permutation group and Bell states as quantum resources.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix

Table A1. Partial permutations of S_8 in complex coefficient case.

SN	Special Type	Permutation	Norm, Phase Factor	BAUO	Example
1	$\frac{\alpha_0^*}{\alpha_0} = \frac{\alpha_1^*}{\alpha_1} = \frac{\alpha_2^*}{\alpha_2} = \frac{\alpha_3^*}{\alpha_3}$ $= \frac{\alpha_4^*}{\alpha_4} = \frac{\alpha_5^*}{\alpha_5} = \frac{\alpha_6^*}{\alpha_6} = \frac{\alpha_7^*}{\alpha_7}$	(1)	$\theta_0 = \theta_1 = \theta_2 = \theta_3 =$ $\theta_4 = \theta_5 = \theta_6 = \theta_7 = 0$	I	$\alpha_0 = 1/6, \quad \alpha_1 = \sqrt{2}/6,$ $\alpha_2 = \sqrt{3}/6, \alpha_3 = 2/3,$ $\alpha_4 = \sqrt{5}/6, \alpha_5 = \sqrt{6}/6,$ $\alpha_6 = \sqrt{7}/6, \alpha_7 = \sqrt{2}/3.$
2	$\frac{\alpha_0^*}{\alpha_1} = \frac{\alpha_1^*}{\alpha_2} = \frac{\alpha_2^*}{\alpha_3} = \frac{\alpha_3^*}{\alpha_0}$ $= \frac{\alpha_4^*}{\alpha_5} = \frac{\alpha_5^*}{\alpha_6} = \frac{\alpha_6^*}{\alpha_7} = \frac{\alpha_7^*}{\alpha_4}$	(1234)(5678)	$\lvert\alpha_0\rvert = \lvert\alpha_1\rvert = \lvert\alpha_2\rvert = \lvert\alpha_3\rvert,$ $\lvert\alpha_4\rvert = \lvert\alpha_5\rvert = \lvert\alpha_6\rvert = \lvert\alpha_7\rvert$ $\theta_0 = \theta_2 = 0, \theta_4 = \theta_6$ $\theta_1 = \theta_3 = \theta_4 + \theta_5, \theta_5 = \theta_7$	$P_{1234}P_{5678}$	$\alpha_0 = \frac{1}{4}, \quad \alpha_1 = \frac{1}{4}e^{i\frac{\pi}{2}},$ $\alpha_2 = \frac{1}{4}, \quad \alpha_3 = \frac{1}{4}e^{i\frac{\pi}{2}},$ $\alpha_4 = \frac{\sqrt{3}}{4}e^{i\frac{3\pi}{8}}, \quad \alpha_5 = \frac{\sqrt{3}}{4}e^{i\frac{\pi}{8}},$ $\alpha_6 = \frac{\sqrt{3}}{4}e^{i\frac{3\pi}{8}}, \quad \alpha_7 = \frac{\sqrt{3}}{4}e^{i\frac{\pi}{8}}.$
3	$\frac{\alpha_0^*}{\alpha_1} = \frac{\alpha_1^*}{\alpha_2} = \frac{\alpha_2^*}{\alpha_3} = \frac{\alpha_3^*}{\alpha_4}$ $= \frac{\alpha_4^*}{\alpha_5} = \frac{\alpha_5^*}{\alpha_0} = \frac{\alpha_6^*}{\alpha_7} = \frac{\alpha_7^*}{\alpha_6}$	(123456)(78)	$\lvert\alpha_0\rvert = \lvert\alpha_1\rvert = \lvert\alpha_2\rvert = \lvert\alpha_3\rvert$ $= \lvert\alpha_4\rvert = \lvert\alpha_5\rvert, \lvert\alpha_6\rvert = \lvert\alpha_7\rvert$ $\theta_0 = \theta_2 = \theta_4 = 0,$ $\theta_1 = \theta_3 = \theta_5 = \theta_6 + \theta_7$	$P_{123456}P_{78}$	$\alpha_0 = \alpha_2 = \alpha_4 = \frac{1}{3},$ $\alpha_1 = \alpha_3 = \alpha_5 = \frac{1}{3}e^{i\frac{\pi}{4}},$ $\alpha_6 = \frac{\sqrt{6}}{6}e^{i\frac{\pi}{16}}, \quad \alpha_7 = \frac{\sqrt{6}}{6}e^{i\frac{3\pi}{16}}.$
4	$\frac{\alpha_0^*}{\alpha_1} = \frac{\alpha_1^*}{\alpha_2} = \frac{\alpha_2^*}{\alpha_3} = \frac{\alpha_3^*}{\alpha_4}$ $= \frac{\alpha_4^*}{\alpha_5} = \frac{\alpha_5^*}{\alpha_6} = \frac{\alpha_6^*}{\alpha_7} = \frac{\alpha_7^*}{\alpha_0}$	(12345678)	$\lvert\alpha_0\rvert = \lvert\alpha_1\rvert = \lvert\alpha_2\rvert = \lvert\alpha_3\rvert = \lvert\alpha_4\rvert$ $= \lvert\alpha_5\rvert = \lvert\alpha_6\rvert = \lvert\alpha_7\rvert = 1/2\sqrt{2},$ $\theta_0 = \theta_2 = \theta_4 = \theta_6 = 0,$ $\theta_1 = \theta_3 = \theta_5 = \theta_7$	$P_{12345678}$	$\alpha_0 = \alpha_2 = \alpha_4 = \alpha_6 = \frac{1}{2\sqrt{2}},$ $\alpha_1 = \alpha_3 = \alpha_5 = \alpha_7 = \frac{1}{2\sqrt{2}}e^{i\frac{\pi}{4}}.$

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