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Low-Complexity Iterative Approximated Water-Filling Based Power Allocation in an Ultra-Dense Network

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Abstract: It is highly possible that future wireless communication systems will adopt ultra-dense deployment to cope with the increasing demand on spectrum efficiency and energy efficiency. The pivotal issue to achieve the potential benefits of the ultra-dense network is to deal with the complex inter-site interference. In this paper, in order to maximize the spectrum efficiency of the system, we first make a reasonable approximation on the inter-site interference to convert the problem into a convex optimization problem. Then, the Lagrangian Multiplier method is adopted to obtain the expression of the optimum power allocation, and the water filling algorithm, as one of the most classical algorithms in the information theory, can be applied to maximize the sum rate or spectrum efficiency of the system. Since the classical iteratively searching water filling algorithm needs many iterations to converge to the optimal solution, we develop a low-complexity iterative approximate water filling algorithm. Simulation results show that the developed algorithm can achieve very close performance to the classical iteratively searching water filling based power allocation with only a few iterations under different scenarios, which leads to a significant complexity reduction.

Keywords: ultra-dense network; water filling algorithm; power allocation; information theory; spectrum efficiency

1. Introduction

The rapid expansion of data traffic and mobile users has proposed some challenging requirements for the future 5G wireless network in terms of capacity, spectrum efficiency (SE), energy efficiency (EE), and so on [1]. The ultra-dense network (UDN) is expected to be an effective method to fulfill these requirements, along with some other technologies, such as massive multiple-input multiple-output (MIMO), device to device (D2D), and so on [2]. UDN is dedicated to solving ultra-high peak rate, ultra-high traffic density and ultra-high density requirements by deploying the low-power base stations (BSs) densely in the network. In some cases, the number of BSs is even larger than the number of mobile stations (MSs) [2,3]. Despite benefits brought by the dense deployment of the BSs, UDN will face many new technical challenges with the increase of cell density, like the more complex interference [4]. Therefore, more advanced cell virtualization techniques, interference management techniques and resource allocation techniques are needed to improve SE and EE under the serious and complex interference in UDN.

There has been much research of the UDN, mainly focused on resource management [5], especially on designing energy efficient and spectrum efficient power control mechanisms [6,7]. The effect of BS density on the network SE is analyzed in [7,8], and the authors analytically investigate the SE of densely deployed small cell networks in the downlink using tools from stochastic geometry, and give the optimal cell density and the corresponding optimal BS transmitting power to achieve a high SE. Other research concentrates on the interference management of UDN to improve SE of the system. Because of the dense deployment of the BS in UDN, the inter cell interference becomes serious, much research considers the BS cooperation techniques to alleviate the interference [9]. In [10], EE is defined as the cells throughput divided by the total power consumption. Zhou *et al*. [11] propose an energy-efficient matching algorithm based on the Gale–Shapley algorithm, which can achieve significant performance and satisfaction gains. Koudourdis *et al*. [12] draws attention to the area throughput and EE, and they conclude the area throughput and EE are relative to the BS transmitting power and the density of the BSs according to the theoretical analysis. Dudnikova *et al*. [13] studies the impact of idle mode capabilities and the density of MSs. Claussen *et al*. [14] gives a strategy to decide the number of BSs to switch off to maximize the energy saving, while maintaining coverage, capacity and quality of service. Cheng *et al*. [15] proposes the self-organized resource allocation scheme to adjust the BS transmitting power self-adaptively according to the number of its served MSs, and obtain an improved throughput with the same power. In [16], the authors propose a two-step joint clustering and scheduling scheme; that is, clustering based on load information using game theory and inter-cell resource allocation based a graph-coloring algorithm, which can significantly improve the cell average and cell edge throughput. However, the Coordinated Multi-Point transmission (CoMP) technique can't acquire the expected performance gains when applied in UDN, due to the limited backhaul resources.

This paper aims to maximize the capacity or SE of the system under the limited total power constraint in different scenarios, that is, the downlink BS transmitting power allocation and the subcarrier power allocation in UDN. Due to the dense deployment of low power BSs, there may exist several major interference sources, which can be hardly eliminated by the traditional interference management mechanisms and signal detection technologies. Hence, the sophisticated resource allocation in UDN has to be studied to avoid potential strong interference. The classical water-filling based power allocation is often used in Orthogonal Frequency Division Multiplexing (OFDM) systems [17]. As for subcarrier power allocation, it is also mainly based on Orthogonal Frequency Division Multiplexing Access (OFDMA) schemes [18,19], where the subcarriers are preassigned to MSs in a non-overlapping way, thus MSs (transmitting on different subcarriers) cause no interference to each other. However, in the UDN regime, the density of BS and MS are huge, leading to the complex inter-station interference. Fortunately, we can assume the interference from other stations to be the constant in UDN, just like many papers [20,21]. With this assumption, the optimization problem of the abovementioned two scenarios can be transformed into convex problems and can be solved with the low-complexity iterative water filling (LIAWF) algorithms.

The contributions of this paper can be summarized in the following aspects:

- We derive the optimum power allocation in the UDN region with the limitation of total power by the Lagrangian Multiplier method, and the water filling based power allocation can be applied to maximum the area capacity and the SE.
- Since the traditional iteratively searching water filling needs many iterations, we developed the LIAWF algorithm to reduce the complexity without performance degradation. The proposed LIAWF can achieve the very close performance as the traditional iteratively searching water filling algorithm.

The remainder of the paper is organized as follows. Section 2 introduces the downlink BS transmitting power allocation, based on the LIAWF algorithm, including the signal model, the problem formulation, and the LIAWF algorithm. Section 3 clarifies the application of the LIAWF algorithm in the scenario of subcarrier and power allocation in UDN. Section 4 shows the simulation results and presents the comparison of the LIAWF algorithm with the average power allocation scheme and the traditional iteratively searching water filling algorithm. Section 5 discusses the benefits and the deeper research about the proposed LIAWF algorithm. Finally, the conclusion is obtained in Section 6.

2. Downlink BS Transmitting Power Allocation

In this section, we consider the downlink transmission in UDN, as shown in Figure 1. In this scenario, MSs and BSs are randomly distributed in the area, and the number of BSs is larger than the number of MSs. Each BS is equipped with a single antenna, as well as each MS. In our paper, we assume that each MS chooses the best BS as its serving BS according to the reference signal receiving power (RSRP); if the best BS has been chosen, then it chooses the next best BS. That is to say, each MS has only one serving BS and each BS serves only one MS.

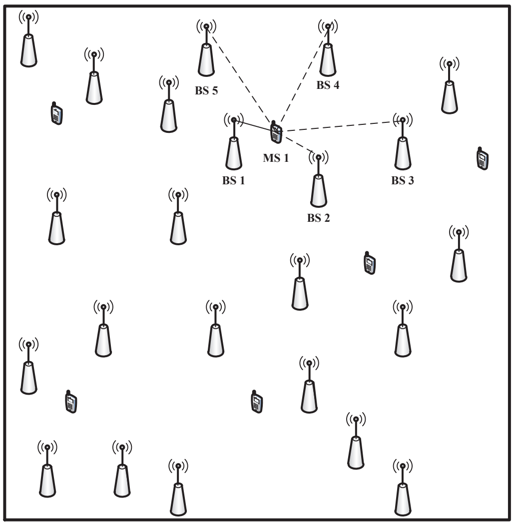

Figure 1. Ultra dense network with M BSs and K MSs $(M \gg K)$ are randomly distributed in the area. For MS 1, it chooses its closest BS (BS 1) as its serving BS. MS 1 receives not only the desired signal from BS 1, but also the interference signals from BS 2, BS 3, BS 4, and BS 5, the interference from the other BS can be neglected .

2.1. Signal Model and Problem Formulation

Suppose the system consists of M BSs and K MSs $(M > K)$. Each MS chooses the best serving BS according to the RSRP, if the best BS has been chosen, then chooses the next best BS, forming K transmitter-receiver pairs (TRPs). For simplicity, denote g_{kk} as the channel gain between the MS k and its serving BS. Let $g_{mk}(m \neq k)$ be the channel gain between the MS k and the transmission BS m, and $g_{mk} = \frac{|h_{mk}|^2}{PL}$, where, h_{mk} is the small scale fading coefficient of the BS m and the MS k, PL stands for the pass loss. Then, the received signal of MS k is

$$y_k = \sqrt{p_k g_{kk}} s_k + \sum_{i=1, i \neq k}^{K} \sqrt{p_i g_{ik}} s_i + n_k, \tag{1}$$

where $s_m (m = 1, 2, \cdots, M)$ is the transmitting signal from the BS m. p_m is the power allocated to BS m. n_k is the additional white Gaussian noise with distribution $\mathcal{CN}(0, \sigma^2)$.

It is obvious that the three parts on the right side of the Equation (1) are the desired signal, interference signals and the noise of the MS k, respectively. Then, we can easily obtain the received signal to interference plus noise ratio (SINR) of MS k:

$$\mathrm{SINR}_k = \frac{g_{kk} p_k}{\sum_{i \neq k}^{K} g_{ik} p_i + \sigma_k^2}. \tag{2}$$

According to Shannon equation [22], the achievable rate of MS k can be expressed as:

$$\begin{aligned} R_k &= B_k \log_2 (1 + \mathrm{SINR}_k) \\ &= B_k \log_2 \frac{\sum_{i=1}^{K} g_{ik} p_i + \sigma_k^2}{\sum_{i \neq k}^{K} g_{ik} p_i + \sigma_k^2}, \end{aligned} \tag{3}$$

where B_k is the allocated bandwidth of the serving BS of MS k . In this section, we assume that each BS is allocated the equal bandwidth, denoted as $B_k = B$ $(k = 1, 2 \cdots, K)$. The downlink sum rate of the whole MS can be denoted as:

$$R = B \sum_{k=1}^{K} \log_2 \frac{\sum_{i=1}^{K} g_{ki} p_i + \sigma_k^2}{\sum_{i \neq k}^{K} g_{ik} p_i + \sigma_k^2}. \tag{4}$$

This section aims to maximize the downlink sum rate of the whole MSs under the limited total power constraint. Since each BS allocates the equal bandwidth, we can omit the bandwidth B and easily denote the objective function as the following:

$$\begin{aligned} &\max \sum_{k=1}^{K} \log_2 \frac{\sum_{i=1}^{K} g_{ki} p_i + \sigma_k^2}{\sum_{i \neq k}^{K} g_{ik} p_i + \sigma_k^2}, \\ &st. \quad \sum_{k=1}^{K} p_k \leq P_{max} \end{aligned}, \tag{5}$$

where P_{max} is the total available power.

2.2. LIAWF Based Power Allocation

Obviously, the objective function is non-convex. However, the problem can be much easier when regarding the interferences of the MS as constants, and then the problem is transformed into a convex constrained optimization [20,21]. Applying our assumption into the problem to get the convex optimization problem, the Lagrangian Multiplier method can be adopted to solve the optimum power allocation [21]. Define the Lagrangian function as:

$$L(p_k, \lambda) = \sum_{k=1}^{K} \log_2 \frac{\sum_{i=1}^{K} g_{ik} p_i + \sigma_k^2}{\sum_{i \neq k}^{K} g_{ik} p_i + \sigma_k^2} + \lambda (P_{max} - \sum_{k=1}^{K} p_k), \tag{6}$$

where λ is the Lagrangian multiplier for the total power constraint. After taking the partial derivative with respect to p_k, we can obtain

$$\frac{\partial L}{\partial p_k} = \frac{1}{\ln 2} \frac{g_{kk}}{g_{kk} p_k + \sum_{i=1, i \neq k}^{K} g_{ki} p_i + \sigma_k^2} - \sum_{j=1, j \neq k}^{K} \frac{\gamma_j p_j g_{ik}}{\sum_{i=1, i \neq j}^{K} g_{ij} p_i + \sigma_k^2} - \lambda. \tag{7}$$

According to the Karush–Kuhn–Tucker (KKT) condition, set it to zero, and then the expression of p_k, is obtained as:

$$p_k = [\frac{1}{\lambda \ln 2 + \sum_{j=1, j \neq k}^{K} \frac{\gamma_j p_j g_{jk}}{\sum_{i=1, i \neq j}^{K} g_{ij} p_i + \sigma_j^2}} - \frac{1}{\gamma_k}]^+, \tag{8}$$

where $[x]^+ = \max\{x, 0\}$, $\gamma_k = \frac{g_{kk}}{\sum_{i \neq k}^{K} g_{ik} p_i}$. From the form of p_k, it is easy to find that the allocated power is relative to its channel quality γ_k. This kind of problem can be easily solved by the water filling algorithm to get the optimum power allocation if we regard the first part of the expression, which is about the interference, as constant.

This subsection details the water filling based power allocation. Based on the information theory, when the channel fade level is known at both the transmitter and the receiver level, the fading channel capacity with channel side information at both the transmitter and receiver is achieved when the transmitter adapts its power to the channel variation [23] . The water filling algorithm, as one of the classical algorithms in the information theory, is often used in OFDM systems to allocate power to acquire the maximum throughput. The intuition behind water filling is to take advantage of good channel conditions: when channel conditions are good, more power and a higher data rate is sent over the channel. As channel quality degrades, less power and rate are sent over the channel. If the instantaneous channel signal to noise ratio (SNR) falls below the cutoff value, the channel does not use more power, and a higher data rate is sent over the channel. In the following sections, we first give the traditional iteratively searching water filling algorithm, that is, to obtain the optimum water filling level by the iterative formula according to a certain step length. Since it needs many iterations to converge to the optimum solution, we present the LIAWF algorithm to reduce the complexity.

2.2.1. Iteratively Searching Water Filling Algorithm

From Equation (8), the allocated power of the serving BS of the MS k is proportional to γ_k. According to the idea of a water filling algorithm, when the MS suffers little interference from other BSs, its serving BS should allocate more power to achieve greater sum rate gains. In addition, for the MSs which from suffer interference, their serving BS should allocate less and even no power. Considering the interference generated by the BS as the constant [21], then we can set

$$\beta = \frac{1}{\lambda \ln 2 + \sum_{j=1, j \neq k}^{K} \frac{\gamma_j p_j g_{jk}}{i=1, \sum_{i \neq j}^{K} g_{ij} p_i + \sigma_j^2}}, \tag{9}$$

where β represents the water-filling level, then Equation (8) can be rewritten as:

$$p_k = [\beta - \frac{1}{\gamma_k}]^+. \tag{10}$$

We can not get the optimum power allocation directly according to Equation (10), since the optimum water filling level can not be determined directly. The water filling level β can be obtained by an iterative method. First, given the initial value of β as

$$\beta = \frac{1}{K}[P_{max} + \sum_{k=1}^{K} \frac{1}{\gamma_k}]. \tag{11}$$

According to the initial water filling level, the initial power allocation can be obtained. Then, update the water filling level iteratively. The renew rule obeys the following expression:

$$\beta \leftarrow \beta + \mu \frac{1}{N_{on}} (P_{max} - \sum_{k=1}^{K} p_k), \tag{12}$$

where $0 < \mu < 1$ is the adjustment step size. N_{on} represents the number of the indeed turn-on BSs (except for the BS which did not allocate the power). Renew the value of β until it converges, the convergency value is the optimum water filling level. According to Equation (10), the optimum power allocation is obtained.

In the iteratively searching water filling algorithm, $2K$ times of add operations and $K + 2$ times of multiply operations are needed in one iteration. Therefore, in the massive BS deployed scenario, some modified algorithms should be researched to reduce the iteration times.

2.2.2. LIAWF Based Power Allocation

Because the iteratively searching water filling algorithm needs many iterations, the LIAWF algorithm based power allocation is detailed as follows.

First, give the initial power allocation like the initial β in the iteratively searching water filling algorithm,

$$p_k = \frac{1}{K}(P_{max} + \sum_{i=1}^{K} \frac{1}{\gamma_i}) - \frac{1}{\gamma_k}. \tag{13}$$

Because of the difference of the channel gains, the allocated power of the terrible-quality channels may be negative. Divide them into two sets: $A = \{p_k | p_k > 0\}$ and $B = \{p_k | p_k \leq 0\}$. Compute the mean value,

$$\Delta = \frac{\sum_{p_i \in B} p_i}{|A|}. \tag{14}$$

Δ is negative. Set the elements in B to zero, in order to fulfill the total power constraint, the elements in A should be decreased, that is

$$p'_k = p_k(p_k \in A) + \Delta. \tag{15}$$

After that, negative allocated power may emerge again in A. Remove the negative elements from A to B, and repeat the above two steps until all the elements in A is positive. Then, the optimum power allocation is obtained.

Theoretically, in the proposed LIAWF algorithm, the elements in set A minus Δ, since all the indeed turn-on BSs have the equal water filling level, that is, the water filling level declines by Δ. In a word, the water filling level can be adjusted more quickly than the iterative searching. Thus, the LIAWF algorithm can return the optimum power allocation with less iterations, compared with the traditional iteratively searching water filling algorithm. In addition, it is proved by the simulations in Section 4 that the LIAWF algorithm can achieve nearly the same performance as the iteratively searching water filling algorithm.

3. Joint Subcarrier Power Allocation

In this section, we extend the LIAWF algorithm to the joint sub-carrier power allocation, and it can also achieve good performance in terms of system SE. In this scenario, each MS is served by one BS and one BS only serves one MS, forming K TRPs, and the total bandwidth B is divided into N sub-carriers, each with a bandwidth of $\Delta f = B/N$.

3.1. Signal Model

Let $\mathcal{K} = \{1, 2, \ldots K\}$ and $\mathcal{N} = \{1, 2, \ldots N\}$ denote the sets of active MSs and all subcarriers, respectively. Since there are K TRPs in the system, we can assume that MS k is served by the BS with the same index k. Hence, the channels between all the MSs and BSs on all subcarriers can be expressed by $H \in \mathbb{C}^{K \times K \times N}$, the small scale fading of which is assumed to be i.i.d zero-mean circularly symmetric complex Gaussian. For any $k \in \mathcal{K}$ and $n \in \mathcal{N}$, suppose s_k^n to be the desired symbol that BS k sends to MS k on subcarrier n, then the received signal on subcarrier n for MS k can be expressed as follows:

$$\tilde{s}_k^n = h_{k,k}^n s_k^n + \sum_{j=1, j \neq k}^{K} h_{k,j}^n s_j^n + z_k^n, \tag{16}$$

where $h_{k,j}^n$ is the $(j, k, n)^{th}$ element of H and denotes the channel coefficient from BS j to MS k on sub-carrier n, and z_k^n is the additive white Gaussian noise (AWGN) with distribution

$\mathcal{CN}(0,\sigma^2)$. Denote the transmitting power of BS k on subcarrier n as p_k^n, we can write the signal-to interference-plus-noise-ratio (SINR) of MS k on subcarrier n as:

$$\mathrm{SINR}_k^n = \frac{g_{k,k}^n p_k^n}{\sigma^2 + \sum\limits_{j=1, j\neq k}^{K} g_{k,j}^n p_j^n}, \tag{17}$$

where $g_{k,j}^n = \frac{\left|h_{k,j}^n\right|^2}{PL}$ stands for the channel gain from BS j to MS k on sub-carrier n, and PL stands for path loss. Then, the achievable rate of MS k on subcarrier n is calculated as:

$$R_k = \sum_{n=1}^{N} \Delta f \log_2(1 + \mathrm{SINR}_k^n), \tag{18}$$

and the channel capacity of the whole system is the sum of the achievable rate of the whole MSs on all subcarriers,

$$C = \sum_{k=1}^{K} \sum_{n=1}^{N} \Delta f \log_2(1 + \mathrm{SINR}_k^n). \tag{19}$$

The cell SE is estimated using the Shannon capacity bound and averaged within cell across all MSs in [24]. Similarly, we use the Shannon capacity to bound the SE of the UDN system. Dividing Equation (19) by the total bandwidth gives an upper bound of the system SE, denoted as η:

$$\eta = \frac{C}{B} = \frac{\sum\limits_{k=1}^{K} \sum\limits_{n=1}^{N} \Delta f \log_2(1 + \mathrm{SINR}_k^n)}{N}. \tag{20}$$

This paper aims to achieve high SE by optimizing subcarrier and power allocation in UDN.

With the decision variables p_k^n, an optimization problem can be formulated as maximizing the spectrum efficiency of all users and subcarriers

$$\max \frac{\sum\limits_{k=1}^{K} \sum\limits_{n=1}^{N} \Delta f \log_2(1 + \mathrm{SINR}_k^n)}{N}, \tag{21}$$

subject to

$$s.t. \sum_{n=1}^{N} p_k^n \leq P \qquad k \in \mathcal{K}, \tag{22}$$

$$p_k^n \geq 0 \qquad k \in \mathcal{K}, n \in \mathcal{N}, \tag{23}$$

where Equation (22) is the constraint on power of k^{th} user for all subcarriers, the total allocated power can not exceed P, which is the total power of all the BSs.

As the number of BSs is much larger than the number of MSs in a typical UDN system, we can assume that the inter-cell interference converges to a subcarrier specific constant when the network is dense enough. Thus, the optimized objective Equation (21) could be rewriten as:

$$\max \frac{\sum\limits_{k=1}^{K} \sum\limits_{n=1}^{N} \Delta f \log_2(1 + \frac{g_{k,k}^n p_k^n}{\sigma^2 + J_k^n})}{N}, \tag{24}$$

where J_k^n is the constant interference received by the k^{th} MSs on subcarrier n. In fact, this assumption is reasonable because of the huge amount of major interference source in UDN. The denser the network is, the flatter the J_k^ns are. With this assumption in mind, the problem can be easily transformed into a convex optimization problem, whose solution is straightforward.

3.2. LIAWF Based Power Allocation

Considering that the problem formulated in Equation (24) is convex, the traditional water filling based method [13] can be applied straightforwardly. To find the solution p_k^n of this convex optimization problem, we write the Lagrangian formulation as:

$$L(p_k^n, \lambda) = \sum_{n=1}^{N} \log_2(1 + \frac{g_{k,k}^n p_k^n}{\sigma^2 + J_k^n} p_k^n) - \lambda(\sum_{n=1}^{N} p_k^n - P), \tag{25}$$

where λ is Lagrangian multiplier. For simplicity, we define the channel gain to interference plus noise ratio (GINR) on subcarrier n at MS k as:

$$m_k^n = \frac{g_{k,k}^n}{\sigma^2 + J_k^n}, \tag{26}$$

we can get the following equations by differentiating Equation (25) with respect to p_k^n

$$\frac{\partial L(p_k^n, \lambda)}{\partial p_k^n} = \frac{1}{\ln 2} \frac{m_k^n}{1 + m_k^n p_k^n} - \lambda. \tag{27}$$

Letting Equation (27) equal zero for all k and n, we get the power allocation on each subcarrier for each MS in the form of water filling,

$$p_k^n = \left[\frac{1}{\beta} - \frac{1}{m_k^n}\right]^+, \tag{28}$$

where

$$\beta = \frac{m_k^n}{1 + m_k^n p_k^n}. \tag{29}$$

Obviously, Equation (28) is similar to Equation (10), then the water filling algorithm can be used to get the optimum power allocation, like the abovementioned iteratively searching water filling algorithm, Equations (11) and (12). This iteratively searching water filling algorithm needs many times of iterations for the parameter β to converge. The complexity of this iterative water filling algorithm is $\mathcal{O}(\xi KN)$, where ξ is the iteration times. Water filling algorithm allocates power based on the state of channel, *i.e.*, the good state of the channel is obviously allocated to more power. Thus, if we could determine the subcarriers which have not allocated power, we could obtain good performance based on the rest of subcarriers as well as complexity reduction. The main idea of the method is just like the formerly mentioned LIAWF algorithm. From Equation (29), we can get:

$$\frac{m_k^n}{1 + m_k^n p_k^n} = \frac{m_k^b}{1 + m_k^b p_k^b}, \tag{30}$$

where $n, b \in \{1, 2, \cdots, N\}$, this reveals the relationship between the powers allocated on different subcarriers:

$$p_k^n = p_k^b + \frac{1}{m_k^b} - \frac{1}{m_k^n}. \tag{31}$$

According to Equation (31), we can obtain the power allocated on any other subcarriers as long as we know one of the subcarriers' power. Having this in mind, the sum power of all the subcarriers can be calculated as follows:

$$P_T = \sum_{n=1}^{N} p_k^n = N(p_k^b + \frac{1}{m_k^b}) - \sum_{n=1}^{N} \frac{1}{m_k^n}. \tag{32}$$

With the total transmitting power consumption constraint, we can get

$$p_k^b \leq \frac{1}{N}(P - \frac{N}{m_k^b} + \sum_{n=1}^{N} \frac{1}{m_k^n}), \tag{33}$$

then we can get the power of each subcarrier. However, the power we obtained may not satisfy the constraint conditions of and Equations (22) and (23). As a result, we can let $p_k^n = 0$ when we calculate $p_k^n < 0$, then $P_T > P$, which also can not meet the constraint condition of Equation (22). Thus, we need to select the appropriate water line through multiple iterative operations. From Equation (31), we know that if we know the ordered version of $m_k = \{m_k^1, m_k^2, \cdots, m_k^N\}$, then we can get the ordered version of p_k^n. Hence, we sort the GINR on the subcarriers of MS k in ascending order so that $m_k^1 \leq m_k^2 \leq \cdots \leq m_k^n$, then $p_k^1 \leq p_k^2 \leq \cdots \leq p_k^n$. Thus, we can directly turn off the subcarriers whose power is less or equal to zero. From Equation (33), we can get

$$p_k^1 = \frac{1}{N}(P - \frac{N}{m_k^1} + \sum_{n=1}^{N} \frac{1}{m_k^n}). \tag{34}$$

If $p_k^1 \leq 0$, make $p_k^1 = 0$ and remove this subcarrier, then calculate the allocated power of the rest subcarriers

$$p_k^i = \frac{1}{N}(P - \frac{N-i+1}{m_k^i} + \sum_{n=i}^{N} \frac{1}{m_k^n}), \tag{35}$$

if $p_k^i \leq 0$, make $p_k^i = 0$ and remove this subcarrier until finding $p_k^i > 0$, where $i \in \{2, \cdots, N\}$. Then, we can get other subcarrier power from Equation (31). We have finished one iteration here. In fact, as long as the power of one subcarrier of any of the MSs changes, the GINR changes. This may require a reordering of the GINR. Nevertheless, we ignore these minor changes and assume that we can delay the reordering of the GINRs to the next iteration without affecting the SE performance of the algorithm. This assumption greatly reduced the times of ordering thus reduced the computation complexity significantly.

This proposed LIAWF can avoid the iterative power allocation step, which must revise the water filling level β after get p_k^n of each subcarrier. The computational complexity of LIAWF includes two parts, that is, the complexity of GINR ordering and the complexity of the power determination. In fact, the complexity of GINR ordering is $\mathcal{O}(KN\log_2 N)$, and the complexity of power determination is $\mathcal{O}(KN)$. The proposed LIAWF can essentially approach the SE performance of the traditional iteratively searching water filling scheme described in section III B within only a few iterations. Besides, when one iteration finishes, only a few subcarriers are turned off, so there is no need to sort all the GINRs for on MS. We just have to find the specific subcarriers with the smallest GINRs, this further reduces the complexity of LIAWF. To sum up, the computational complexity of LIAWF is approximately $\mathcal{O}(\theta KN)$, where θ is the iteration times of LIAWF, with a typical value that varies from two to five.

4. Simulation Results

In this section, we evaluate the performance gain of the proposed LIAWF based power allocation, in the abovementioned two scenarios, by simulations. In the first scenario, 100 BSs are uniformly and randomly deployed in the $300 \times 300\ m^2$ area, and 10 MSs are distributed uniformly and randomly in the area. In the second scenario, the number of MSs and BSs are $M = 250$ and $K = 100$, respectively, while the number of available subcarrier at each BS is set to $N = 20$. The channels are the randomly generated unit variance Rayleigh fading channels in our simulations. In addition, the pass loss model is used in both simulations [25], which can be expressed as follows:

$$PL = \begin{cases} 16.9\log_{10}(d) + 32.8 + 20\log_{10}(f_c), d < 10\ m, \\ 43.3\log_{10}(d) + 11.5 + 20\log_{10}(f_c), d \geq 10\ m, \end{cases} \tag{36}$$

where d is the distance between the user and the base station, and f_c is the center frequency, which is 2 GHz in the simulations. Here, we make an assumption that the MS whose link distance is less than 10 m have line-of-sight (LOS) path with probability 1, while the MS whose link distances are larger than 10 m and have non-line-of-sight (NLOS) paths with probability 1. Each subcarrier signal of the user undergoes Rayleigh fading independently. We give the sum rate *versus* the average power in the first scenario and the SE *versus* SNR, and SE difference between LIAWF and the traditional iterative water-filling algorithm at different LIAWF iteration times, in the second scenario. The following simulation results are drawn from the average of 10,000 times of Monte Carlo simulations.

Figure 2 shows the relationship between the sum rate and the average allocated power in the scenario of downlink BS transmitting power allocation, based on the traditional iteratively searching water filling algorithm and the LIAWF algorithm. From it, both the iteratively searching water filling based power allocation and the LIAWF based power allocation can achieve great improvements of the sum rate, and the greatest gain can reach and even exceed 100 percent. Figure 3 shows SE *versus* the SNR for the different power allocation methods, including traditional iteratively searching water filling based power allocation, the proposed LIAWF based power allocation and the average power allocation. It is obvious that the SE of the system increases with the increase of SNR for all the power allocation algorithms. We can also observe that there exists obvious SE gain when the traditional iterative water filling algorithm and the proposed LIAWF are used instead of the average power allocation scheme. Besides, the SE performance obtained by LIAWF is approaching that of traditional iteratively searching water filling algorithm as the iteration time increases. In fact, the proposed LIAWF can almost acquire the same SE as traditional iteratively searching water filling algorithm within five iteration times, while traditional iteratively searching water filling algorithm needs about 20 iteration times to converge. Thus, LIAWF outperforms the traditional iteratively searching water filling algorithm with respect to computational complexity while attaining almost the same SE. Figure 4 shows the SE difference between LIAWF and traditional iteratively searching water filling algorithm at different LIAWF iteration times at different SNR. We can see from Figure 4 that the SE difference between LIAWF and traditional iteratively searching water filling algorithm is less than 0.1 when the iteration times of the LIAWF algorithm is about five at high SNR regions. From the above three figures, we can conclude that the proposed LIAWF algorithm based power allocation can achieve nearly the same performance as the traditional iteratively searching water filling algorithm, especially in high SNR regions, and both algorithms can improve the sum rate or SE of the system a lot, compared with the average power allocation. In addition, the proposed LIAWF based power allocation can reduce the complexity significantly, compared with the traditional iteratively searching water filling based power allocation.

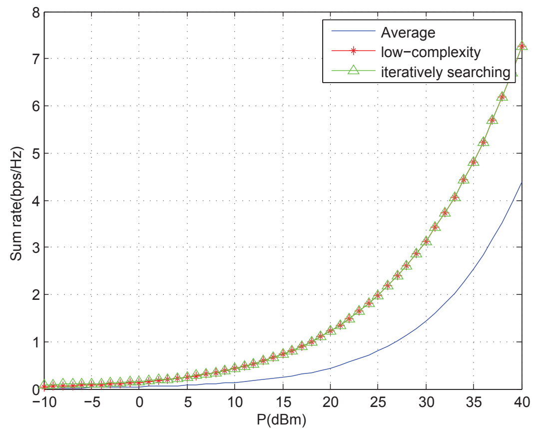

Figure 2. The achievable sum rates of the TIWF (iteratively searching) algorithm power allocation, the LIAWF (low-complexity) algorithm power allocation and the average power allocation.

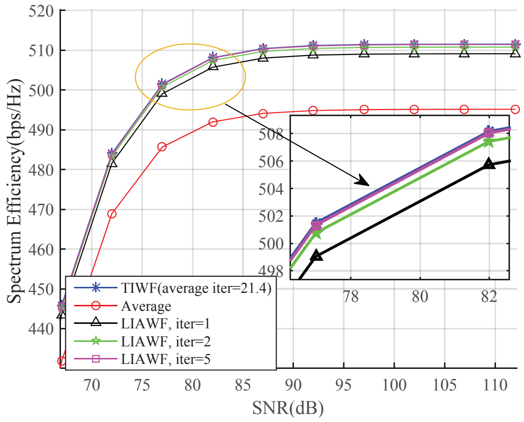

Figure 3. SE *versus* the SNR for the three different subcarrier and power allocation schemes. Particularly, SE are calculated for the proposed LIAWF at different iteration times one, two and five to verify how fast it can approach the traditional iteratively searching water filling (TIWF) scheme.

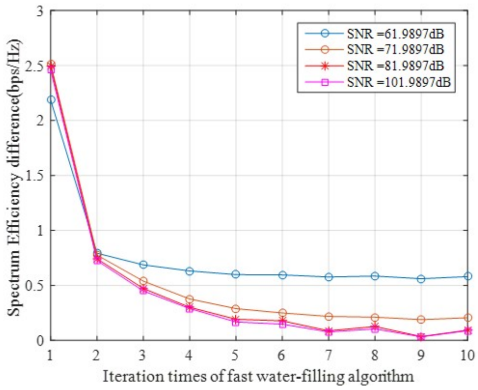

Figure 4. SE difference between LIAWF and traditional iteratively searching water filling algorithm at different LIAWF iteration times at different SNRs.

5. Discussion

The water filling algorithm is one of the classical algorithms in information theory to improve the throughput by power allocation. In this paper, we use it in UDN. We aim to maximize the sum rate or SE of the system by adjusting the transmitting power. We first give the objective functions, and make a reasonable assumption on the UDN inter-site interference, so that the optimization problem can be transformed into the convex problems. Then, the water filling based power allocation method can be used to solve the problem in the two different scenarios in UDN. However, the traditional iteratively searching water filling algorithm needs many iterations to get the optimum power allocation and leads to a very high complexity. We developed the LIAWF algorithm based power allocation method to reduce the times of iterations. From the theoretical analysis and the simulation results, we can see that the proposed LIAWF algorithm is simple but powerful. On one hand, it can be based on many different scenarios and achieve great performance (even the same as the traditional iteratively searching water filling based power allocation, especially in high SNR regions), such as the sum rate or SE of the system; on the other hand, LIAWF needs much fewer iterations to converge to the optimum solution, and can lower the computational complexity significantly, compared with the traditional iteratively searching water filling based power allocation.

Considering the works based on the cooperative game theory, like [26], it can achieve a trade-off between capacity and fairness. However, the information exchange between the BSs is needed, which is difficult in the practical system. Furthermore, some other water filling algorithms have been proposed to optimize the system capacity, like the multi-dimensional bisection search algorithm in [21], which is very high-complexity. As a result, the proposed LIAWF can be more practical to be applied to UDN to improve the system performance.

6. Conclusions

This paper presented a power allocation optimization algorithm, which aimed to maximize the capacity or SE of the network in different scenarios of UDN. We first made a reasonable assumption and transformed the optimization problem into a convex one. Then, the proposed LIAWF algorithm could be applied to maximize the capacity or SE in different scenarios, the downlink BS transmitting power allocation and the joint subcarrier and power allocation. Simulation results proved that the LIAWF based power allocation could achieve a very similar performance to the traditional iteratively searching water filling algorithm. Both power allocation methods improved the capacity or SE a lot, especially in relatively high SNR regions, compared with the average power allocation. Furthermore, the proposed LIAWF algorithm could converge to the optimum solution by less iterations, leading to a significant complexity reduction.

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