# The Impact of Return Shipping Insurance on a Retailer Based on Restricting Rights 

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#### Abstract

In e-commerce, retailers often use return shipping insurance (RSI) to solve consumer returns, leading to a high return rate. To reduce this negative effect, we consider restricting rights to restrict consumers from obtaining RSI or buying products. We examine the effect of RSI on retail pricing strategies and profits under restricting rights. We formulate a game-theoretical model which consists of one insurer, a retailer and two types of consumers (informed consumers and uninformed consumers). By solving the model, we find that even though the insurer has restricted uninformed consumers from obtaining RSI, the retailer further restricts them from buying the product when the salvage value is low. Second, when the retailer and insurer have no right to restrict uninformed consumers, purchasing RSI may hurt the retailer. Third, when the insurer restricts uninformed consumers and the product salvage is low, the retailer adopts the high-price strategy; otherwise, the retailer adopts the low-price strategy. Finally, when the product salvage is low, the retailer will prevent uninformed consumers from buying the product by adopting the high-price strategy or using the restricting right. In this case, the insurer will set a higher premium.


Keywords: online return; return shipping insurance; restricting rights; insurance premium

## 1. Introduction

Online shopping is growing fast due to its convenience and price advantage. In China, online sales reached RMB 13.1 trillion in 2021, an increase of $14.1 \%$ over 2020, accounting for $24.5 \%$ of total retail sales [1]. In the United States, online sales were USD 870 billion, a $14.2 \%$ increase over 2020, representing $13.2 \%$ of all retail sales in 2021 [2]. However, different from shopping in brick-and-mortar stores, where consumers can value products by trying them out, consumers shopping online often do not know whether products meet their needs before receiving product. This leads to a significantly high return rate. Invesp estimates that the return rate of online purchases is at least $30 \%$ as compared to $8.89 \%$ in brick-and-mortar stores [3].

These tremendous returns incur high return shipping costs. These costs are a heavy burden on both retailers and consumers. From retailers' perspective, retailers shouldering the costs of free return shipping significantly squeezes their margins. Not all retailers can afford these fees. ParcelLab survey showed that $43 \%$ of retailers charged for returns [4]. From the consumers' perspective, forcing them to pay the return shipping fee would lower their satisfaction or even drive them off. Consumers value a free return. If consumers have to pay for return shipping, $69 \%$ of them do not buy online [5]. How to deal with return shipping fee has become a challenging problem.

To solve this pain point, Taobao.com, the Chinese largest ecommerce platform, introduced the Huatai Insurance Group to launch special insurance "return shipping insurance" (RSI) for online transactions in 2010. When RSI is offered, retailers can pay a relatively low premium to the insurer for transactions. Once consumers return products, the insurer compensates consumers for the return shipping fee. This insurance has achieved
great success in China. Consequently, in addition to Taobao.com, many platforms such as JD.com, Pingduoduo and TikTok have introduced RSI. More than 15 billion policies were sold in 2019, accounting for more than $50 \%$ of non-auto insurance policies among online insurance [6].

However, RSI results in a very high return rate. If retailers buy RSI, consumers may not carefully learn about the characteristics of products. For example, when they buy products, they do not carefully read the product introduction and reviews on the website, or consult with friends who have used it. These behaviors lead to a sharp rise in the return rate and result in great loss to retailers and insurers. To solve this problem, platforms may give insurers and retailers the right to restrict consumers. If platforms give insurers the restricting right, insurers can restrict consumers with high return rates from obtaining RSI. For example, Taobao.com and JD.com give insurers this restricting right. If platforms give retailers the restricting right, retailers can restrict consumers with high return rates from buying products. For example, Taobao.com and Pingduoduo.com give retailers this restricting right. Thus, we seek to answer the following questions: How does the restricting right affect the retailer's decisions? Does RSI benefit the retailer? When will the retailer restrict consumers from buying the product? How does the restricting right affect the premium?

To answer these questions, we develop a model consisting of an insurer, a retailer and consumers. Consumers are divided into two segments: informed consumers and uninformed consumers. Informed customers are individuals who possess knowledge and understanding about products before making a purchasing decision. They may gather information about the product from online reviews, social media, and friends' opinions. Contrarily, uninformed consumers are individuals who lack knowledge or understanding about products, and make purchases without doing proper research or seeking advice from experts. The insurer and the retailer may have different restricting rights. The insurer decides the premium. The retailer decides the retail price and whether to restrict consumers. Consumers decide whether to buy the product and whether to return the product.

The remainder of this paper is organized as follows. Section 2 summarizes the relevant literature. Section 3 describes the problem and presents the model. Section 4 analyzes the equilibrium outcome. Section 5 discusses the paper's main results and management insights. All proofs are presented in Appendix A.

## 2. Literature Review

The management of product return policies has received significant interest. They mainly focus on whether retailers provide more lenient return policies. Though lenient return policies lead to costs to the retailer and "free-ride", they may still increase retailers' profits. For example, Davis et al. [7] find that a money-back guarantee (MBG) benefits the retailer when the retailer's salvage advantage is higher than the buyer's transaction cost of returns. Moorthy and Srinivasan [8] investigate the impact of the MBG on signaling product quality. They find that the MBG policy can be a very effective tool for signaling product quality, even if there are other signals, such as price and advertising. Others focus on the partial return policy. For example, Su [9] explores a model where the consumers' valuation for the product is uncertain and the demand is also uncertain. In this environment, the results show that the retailer's optimal return policy is partial return, and the optimal refund equals the salvage. Hsiao and Chen [10] investigate the impact of return policies on the manufacturer when the product quality is different. They show that a generous return policy may not signal a high product quality. Xu et al. [11] consider products with different life cycles and study how the refund and the return deadline affect the retailer's return strategies. They find that a full refund policy is not an ideal return policy for the retailer. The retailer should set the refund equal to the salvage. Altug and Aydinliyim [12] consider strategic consumers who are discount seeking. They find that adopting an appropriate return policy can reduce the adverse impact from strategic consumers. If the retailer can salvage the unsold and returned products for a high price,
the retailer should give consumers a positive refund for returns. Some other works study return policies in omnichannel operations [13-16]. In this work, we focus on the return shipping insurance, which is a new form of insurance that covers the return shipping fee for product returns.

Our work is also closely related to the return shipping insurance. Previous works consider the return insurance in various settings. For example, Geng et al. [17] consider online product reviews and the product fit uncertainty when they study the return shipping insurance. They show how online product reviews affect the retailer's and the insurer's decisions. They find that the product fit uncertainty is an important fact when the insurer decides the optimal insurance premium. Consumers can use the return shipping insurance to estimate the product fit uncertainty and the insurer can earn higher revenue by announcing a higher premium and compensation. Chen et al. [18] examine the impact of the return insurance under the reselling format and the agency selling format. They find that offering the return insurance may narrow the consumer market, whereas offering the return insurance always benefits the retailer when the per-unit return loss is low or high. When the fee that the platform charges the manufacturer is medium, adopting the agency selling format can benefit both the platform and the manufacturer. Li et al. [19] find that when the retailer adopts the partial-refund policy, the return insurance can benefit the retailer. The results also imply that the return shipping insurance is beneficial to social welfare when the retailer buys it. Zhang et al. [20] examine return shipping insurance as an informational tool. They assume that the product quality is private information of the retailer. However, the retail price, the purchase of return shipping insurance and the insurance premium can transmit quality information to a certain extent. They show that the insurer never offers the buyer insurance. Furthermore, the seller insurance can be an effective tool to signal high quality. Then they confirm their results by empirical evidence from JD.com that high-quality retailers are more likely to buy the seller insurance. Yang and Ji [21] compare buy-online-return-in-store (BORS) and return shipping insurance. They find that the retailer's profit decreases when return shipping insurance cannot attract new consumers and cross selling does not affect the retailer's purchase decision of return shipping insurance. Fan and Chen [22] study the impact of return shipping insurance on the supply chain when the manufacturer shares the premium. They find that return shipping insurance decreases the profits of supply chain members when the premium is high. Others address the insurance in other setting, such as omnichannel operations [23], different product quality [24], and consumer heterogeneity [25]. However, we consider that consumers have different fit probability. Second, we study the return insurance under different restricting rights.

Our paper is related to the literature on consumer search. Branco et al. [26] develop a model which considers consumer search. They show how the retailer's decision affect the consumers' search behavior. Ke et al. [27] consider the consumer's gradual search, finding that consumers only search for information on the product when the product has a high expected utility. Ding and Zhang [28] compare consumer search under pricedirected search and under random search. They show that when search costs decrease, price-directed search is more beneficial to retailers than random search. Kuksov [29], Rhodes and Zhou [30] study how consumer search affects product design. However, we examine the impact of consumers with different product information on return policies and restriction strategies.

Our paper departs from the previous literature in three major aspects. First, not only consumer valuation uncertainty but also different consumer return rates can significantly affect return policies. Thus, we consider these two facts, especially the heterogeneity of consumer returns, which is less considered in other papers. Second, we consider the insurer's right to restrict consumers from obtaining return shipping insurance. Third, we consider the retailer's right to restrict consumers from buying the product. In order to reduce the return loss from uninformed consumers, previous studies often consider using
pricing strategies to discourage uninformed consumers from buying products. However, we study the impact of these two restricting rights on return shipping insurance.

## 3. Problem Description

Consider an insurer and an online retailer (referred to as "retailer") selling a product to consumers at price $p$. Consumers face uncertain valuation $V$, which is uniformly distributed on $[0,1]$. They do not know whether the product meets their needs before purchase. If the product fits consumers, they obtain valuation $v$; otherwise, they obtain 0 [18,31,32]. Similar to $[28,33,34]$, we assume that there are two consumer segments. A proportion $\theta$ of consumers will carefully read the product descriptions and reviews, search for information on the product, and consult customer service personnel or friends who used it. Thus, this part of consumers can buy more suitable products. We refer to these consumers as the informed consumers. Conversely, the remaining $1-\theta$ consumers do not search for information on products and will probably buy products that are not suitable for them. We refer to these consumers as the uninformed consumers. Let $\lambda_{h}\left(\lambda_{l}\right)$ denote the probability that an informed (uninformed) consumer will like the product, and $1-\lambda_{h}$ $\left(1-\lambda_{l}\right)$ denote that an informed (uninformed) consumer buys the product not meeting their needs, and $0<\lambda_{l}<\lambda_{h}<1$. Once they receive the mismatch product, they can return it for a full refund $p$. However, consumers need to bear the return shipping fee $h$.

The retailer procures the product at $c$ a unit, and faces a salvage $s$. We assume that $s<c[7,9,35]$. The insurer sets the insurance premium $m$. The retailer can decide whether to pay the insurance premium to buy RSI for each product sold if RSI is offered. When RSI is purchased, consumers obtain the compensation $h$ which covers the return shipping fee from the insurer if they return the product. Same as $[17,18,25,36]$, we focus on the seller insurance, which is the most common in practice. There are platforms which only introduce the seller insurance, such as Pinduoduo, TikTok and Suning.com.

Uninformed consumers are likely to return products when buying products, leading to significant loss to the insurer and the retailer. Thus, e-commerce platforms may give insurers the right to restrict consumers from obtaining RSI or give retailers the right to restrict consumers from purchasing products. For example, Taobao.com and Pingduoduo give insurers and retailers the right to restrict consumers. JD.com gives insurers the right to restrict consumers. Amazon gives retailers the right to restrict consumers. Thus, there are four possible scenarios.
(1) Scenario NN: Both the insurer and the retailer do not have the restricting right. In this case, the insurer decides the premium $m$. Then, the retailer decides the retail price $p$ and whether to purchase RSI. Finally, consumers make their purchase and return decisions.
(2) Scenario NR: The insurer does not have the restricting right, whereas the retailer has the restricting right. In this case, the insurer decides the premium $m$. Then, the retailer decides the retail price $p$, and whether to purchase RSI and restrict uninformed consumers from buying the product. Finally, consumers make their purchase and return decisions.
(3) Scenario RN: The insurer has the restricting right, whereas the retailer does not have the restricting right. In this case, the insurer restricts uninformed consumers from obtaining RSI and decides the premium $m$. Then, the retailer decides the retail price $p$ and whether to purchase RSI. Finally, consumers make their purchase and return decisions.
(4) Scenario RR: Both the insurer and the retailer have the restricting right. In this case, the insurer restricts uninformed consumers from obtaining RSI and decides the premium $m$. Then, the retailer decides the retail price $p$ and whether to purchase RSI and restrict uninformed consumers from buying the product. Finally, consumers make their purchase and return decisions.
Figure 1 depicts the sequence of events. In stage 1, the insurer announces the premium $m$. In stage 2 , the retailer determines the retail price $p$, whether to purchase RSI and
whether to restrict consumers. In stage 3, consumers arrive and decide whether to purchase the product. After purchasing the product, consumers learn the product's fit. Finally, consumers decide whether to return the product.


Figure 1. Sequence of events.

## 4. Equilibrium Analysis

We derive the subgame perfect equilibrium decisions following backward induction. According to the sequence of events, first, we analyze consumers' purchase decisions and return decisions. The retailer will take these decisions into account in the stage 2 . Next, we study the retailer's optimal decisions, including the retail price, RSI strategy and consumer restriction strategy. Finally, we analyze the insurer's decisions on the profit maximizing premium of stage 1.

### 4.1. No Insurance-Benchmark Case (Scenario N)

In the benchmark case, there is no RSI. Consumers pay the return shipping fee $h$ if they return the product. First, we study consumers' decisions. Clearly, consumers willl return the product if the product does not fit them. A consumer's expected utility is

$$
\begin{align*}
& u_{n h}=v \lambda_{h}+\left(1-\lambda_{h}\right)(p-h)-p .  \tag{1}\\
& u_{n l}=v \lambda_{l}+\left(1-\lambda_{l}\right)(p-h)-p . \tag{2}
\end{align*}
$$

where $u_{n h}\left(u_{n l}\right)$ represents an informed (uninformed) consumer's expected utility. Thus, informed (uninformed) consumers buy the product if $u_{n h}>0\left(u_{n l}>0\right)$. Then, we obtain consumers' decisions. The informed consumer whose valuation satisfies $v>p+h\left(1 / \lambda_{h}-\right.$ $1)=v_{n h}$ would buy the product. The uninformed consumer whose valuation satisfies $v>p+h\left(1 / \lambda_{l}-1\right)=v_{n l}$ will buy the product.

Based on consumers' decisions, we derive the expected profit of the retailer:

$$
\begin{equation*}
\pi_{R}^{N}=\theta\left(1-v_{n h}\right)\left(p-c-\left(1-\lambda_{h}\right)(p-s)\right)+(1-\theta)\left(1-v_{n l}\right)\left(p-c-\left(1-\lambda_{l}\right)(p-s)\right) \tag{3}
\end{equation*}
$$

The first term is the profit from informed consumers' purchases; the second term is the profit from uninformed consumers' purchases. The retailer sets the price $p$ to maximize its profit, leading to the following result.

Proposition 1. When there is no RSI, the optimal price is $p_{N}^{*}=\frac{-c+h \theta \lambda_{l}-h \theta \lambda_{h}-h \lambda_{l}+\theta \lambda_{l}-\theta \lambda_{h}+h-\lambda_{l}+\theta s \lambda_{l}-\theta s \lambda_{h}-s \lambda_{l}+s}{2\left(\theta \lambda_{l}-\theta \lambda_{h}-\lambda_{l}\right)}$.
(i) The optimal price $p_{N}^{*}$ decreases in the salvage value s.
(ii) The optimal price $p_{N}^{*}$ decreases in the return shipping fee $h$.

If there is no RSI, from the previous analysis, we derive that $1-v_{n h}$ of informed consumers would buy the product and $1-v_{n l}$ of uninformed consumers would buy the product. Thus, we derive the retailer's profit as Equation (3) shows. By maximizing the retailer's profit, we have the optimal price as Proposition 1 states.

Then, we analyze how the price changes in the salvage value $s$ and the return shipping fee $h$. From Proposition 1 (i), we obtain that the retailer sets a lower price when the salvage $s$ increases. When the salvage value increases, any return represents less loss. Setting a lower price increases sales but also results in more returns. The higher salvage $s$ mitigates the negative effect of returns, which allows the retailer to make a higher profit by setting a lower price. On the other hand, we obtain that the price also decreases in the return shipping fee from Proposition 1 (ii). Intuitively, a higher return shipping fee diminishes consumers' utility. Thus, to ensure consumers' purchase, the retailer lowers the price.

### 4.2. Both the Insurer and the Retailer Do Not Have the Restricted Right (Scenario NN)

In this scenario, the insurer offers RSI to the retailer. However, the insurer has no right to restrict consumers from obtaining RSI, that is, if the retailer purchases RSI, all consumers buying the product can obtain RSI. The retailers also have no right to restrict uninformed consumers from buying the product. First, we study consumers' decisions. If RSI is purchased, the consumers' expected utility is

$$
\begin{gather*}
u_{i h}=v \lambda_{h}+\left(1-\lambda_{h}\right) p-p .  \tag{4}\\
u_{i l}=v \lambda_{l}+\left(1-\lambda_{l}\right) p-p . \tag{5}
\end{gather*}
$$

where $u_{i h}\left(u_{i l}\right)$ represents an informed (uninformed) consumer's expected utility when the retailer buys RSI. Thus, informed (uninformed) consumers buy the product if $u_{i h}>0$ $\left(u_{i l}>0\right)$. Then, we obtain the consumers' decisions. Obviously, consumers whose valuation is higher than the price $p$ will buy the product and return the product when it does not fit. Thus, $1-p$ consumers buy the product.

If RSI is purchased, consumers can return the product for free because the compensation covers the return shipping fee. This policy increases consumer satisfaction, reduces consumers' perceived risk and signals higher quality, which leads to additional value to the retailer $[8,20,37,38]$. Similar to [35], we use $\beta(\geq 1)$ to indicate the increase in additional value when the retailer purchases RSI. On the contrary, if the retailer prohibits uninformed consumers from buying the product, it will cause dissatisfaction among such consumers and affect the retailer's reputation, thus bringing negative effects to the retailer. We use $k(1-\theta)$ to denote the retailer's loss if the retailer restricts uninformed consumers from buying the product. $(1-\theta)$ is the number of uninformed consumers, and $k(\geq 0)$ represents the loss coefficient. Based on consumers' decisions, the retailer's expected profit is

$$
\begin{equation*}
\pi_{R}^{N N}=\beta(1-p)\left(\theta\left(p-c-\left(1-\lambda_{h}\right)(p-s)\right)+(1-\theta)\left(p-c-\left(1-\lambda_{l}\right)(p-s)\right)-m\right) \tag{6}
\end{equation*}
$$

The insurer's profit equals premium income minus the return shipping fee compensation:

$$
\begin{equation*}
\pi_{I}^{N N}=(1-p)\left(m-\theta h\left(1-\lambda_{h}\right)-(1-\theta) h\left(1-\lambda_{l}\right)\right) . \tag{7}
\end{equation*}
$$

Using backward induction, the retailer sets the price to maximize the expected profit. Then, the insurer chooses the premium to maximize the expected profit. Thus, we derive the following result:

Proposition 2. In scenario NN,
$p_{N N}^{*}=\frac{c+h \theta \lambda_{l}-h \theta \lambda_{h}-h \lambda_{l}-3 \theta \lambda_{l}+3 \theta \lambda_{h}+h+3 \lambda_{l}-\theta \lambda_{l} s+\theta \lambda_{h} s+\lambda_{l} s-s}{-4 \theta \lambda_{l}+4 \theta \lambda_{h}+4 \lambda_{l}}$;
$m_{N N}^{*}=\frac{1}{2}\left(-c+h\left(\theta \lambda_{l}-\theta \lambda_{h}-\lambda_{l}+1\right)-\theta \lambda_{l}+\theta \lambda_{h}+\lambda_{l}+\theta \lambda_{l} s-\theta \lambda_{h} s-\lambda_{l} s+s\right)$.
(i) The optimal price $p_{N N}^{*}$ increases in the salvage value $h$.
(ii) The optimal premium $m_{N N}^{*}$ increases in the return shipping fee $h$.

If RSI is offered and the insurer and the retailer have no restricting right, from the previous analysis, we derive that $1-p$ of consumers would buy the product. Thus, we
derive the retailer's profit as Equation (6) shows. By maximizing the retailer's profit, we have the optimal price with respect to $m$. Substituting the price into the insurer's profit (Equation (7)) and maximizing the insurer's profit, we have the optimal premium $m_{N N}^{*}$. Substituting the premium $m_{N N}^{*}$ into the price with respect to $m$, we derive the price $p_{N N}^{*}$ as Proposition 2 shows.

Then, we analyze how the premium and the price change in the return shipping fee $h$. From Proposition 2 (i), we obtain that the insurer announces a higher premium when the return shipping fee $h$ increases. A higher return shipping fee means higher compensation to consumers when they return the product. Thus, the insurer increases the premium. In contrast to Proposition 1, in which the optimal price deceases in the return shipping fee, Proposition 2 (ii) shows that the optimal price increases in the return shipping fee in scenario NN. This is because the insurer announces a higher premium when the return shipping fee increases, which makes the cost of purchasing RSI higher for the retailer. Therefore, the retailer charges a higher price.

What is the impact of RSI on the retailer? The following proposition states the result.
Proposition 3. Compared with scenario $N$,
(i) $p_{N N}^{*}>p_{N}^{*}$.
(ii) $\pi_{R}^{N N}<\pi_{R}^{N}$, if $s<s_{1}$; otherwise $\pi_{R}^{N N} \geq \pi_{R}^{N}$, if $s \geq s_{1}$.

Here, $s_{1}$ is the larger non-negative real root of the equation: $\pi_{R}^{N N}-\pi_{R}^{N}=0$.
Proposition 3 (i) shows that when RSI is purchased, the retailer charges a higher price $\left(p_{N N}^{*}>p_{N}^{*}\right)$. This result is consistent with the findings of $[19,22]$. When RSI is purchased, consumers can return the product for free, leading to higher consumers' utility. This motivates the retailer to set a higher price to earn a higher profit. However, compared with no RSI, purchasing RSI may hurt the retailer. When the retailer purchases RSI, consumers can return the product for free, which leads to more returns. This disadvantage to the retailer is aggregated when the salvage value is low. Therefore, when the salvage is sufficiently low ( $s<s_{1}$ ), the retailer earns a lower profit compared with no RSI ( $\pi_{R}^{N N}<\pi_{R}^{N}$ ). By contrast, when the salvage is sufficiently high ( $s \geq s_{1}$ ), this disadvantage to the retailer is mitigated, which allows the retailer to earn a higher profit by purchasing RSI ( $\pi_{R}^{N N} \geq \pi_{R}^{N}$ ).

### 4.3. Only the Retailer Has the Restricted Right (Scenario NR)

In this scenario, the insurer offers RSI to the retailer and has no right to restrict consumers from obtaining RSI. The retailer has the right to restrict uninformed consumers from buying the product. First, we study consumers' decisions. If the retailer restricts uninformed consumers from buying the product, uninformed consumers cannot buy the product. Informed consumers' expected utility equals $u_{i h}$, and their decisions are the same as those in scenario NN. Based on consumers' decisions, we obtain that the retailer's expected profit is

$$
\begin{equation*}
\pi_{R}^{N R}=\beta(1-p) \theta\left(p-c-\left(1-\lambda_{h}\right)(p-s)-m\right)+k(1-\theta) . \tag{8}
\end{equation*}
$$

Then, in scenario NR, we derive the optimal decision of the retailer as summarized in Proposition 4.

Proposition 4. In scenario $N R$, we have:
(i) For a given premium $m$, the optimal price is as follows:
(i.a) If $m \leq m_{1}, p_{N R}^{*}=p_{N N}^{*}=\frac{c+h \theta \lambda_{l}-h \theta \lambda_{h}-h \lambda_{l}-3 \theta \lambda_{l}+3 \theta \lambda_{h}+h+3 \lambda_{l}-\theta \lambda_{l} s+\theta \lambda_{h} s+\lambda_{l} s-s}{-4 \theta \lambda_{l}+4 \theta \lambda_{h}+4 \lambda_{l}}$, the retailer does not restrict uninformed consumers.
(i.b) If $m>m_{1}, p_{N R}^{*}=\frac{c+\lambda_{h}+m+\lambda_{h} s-s}{2 \lambda_{h}}$, the retailer restricts uninformed consumers.
(ii) As the premium $m$ changes such that the retailer's optimal strategy switches from not restricting uninformed consumers to restricting uninformed consumers, $p_{N R}^{*}$ decreases discontinuously. Here, $m_{1}$ is the larger non-negative real root of the equation: $\pi_{R}^{N R}-\pi_{R}^{N N}=0$.

Figure 2 numerically illustrates Proposition 4. First, Proposition 4 (i) shows that the retailer restricts uninformed consumers from buying the product when the premium is high $\left(m>m_{1}\right)$. The high premium means that the retailer has to pay a high cost to purchase RSI. In addition, uninformed consumers have a high probability of returning the product. Preventing uninformed consumers' purchases can reduce the costs of premium and losses from uninformed consumers' returns. Thus, in this case, the retailer restricts uninformed consumers from buying the product.


Figure 2. Seller's optimal decision in scenario NR ( $c=0.4, \theta=0.6, \lambda_{l}=0.4, \lambda_{h}=0.5, \beta=2$, $h=0.048, s=0.1, k=0.0002$ ); (a) retailer's restricting decision in scenario $\mathrm{NR} ;(\mathbf{b})$ the optimal price changes in scenario NR.

Second, an increase in the premium increases the retailer's costs, which may encourage the retailer to charge a higher price. Interestingly, Proposition 4 (ii) shows a different result. When the premium is high $\left(m>m_{1}\right)$, the retailer sets a low price. This is because in this case, uninformed consumers are restricted and cannot cause loss by return. Thus, the retailer sets a lower price to increase the demand of informed consumers. Consequently, the price decreases discontinuously when the premium changes such that the retailer's decision switches from not restricting uninformed consumers to restricting uninformed consumers.

When the retailer restricts uninformed consumers, the insurer's profit is only from informed consumers' purchase. So the insurer's expected profit is

$$
\begin{equation*}
\pi_{I}^{N R}=(1-p) \theta\left(m-h\left(1-\lambda_{h}\right)\right) . \tag{9}
\end{equation*}
$$

The insurer maximizes its expected profit, leading to the following proposition.
Proposition 5. In scenario $N R$, the optimal premium is
$m_{N R}^{*}= \begin{cases}\frac{1}{2}\left(-c-h \lambda_{h}+h+\lambda_{h}-s \lambda_{h}+s\right) & \text { if } s \leq s_{2} \\ m_{1} & \text { if } s_{2}<s \leq s_{3} \\ m_{N N}^{*} & \text { if } s>s_{3}\end{cases}$
Here, $s_{2}$ is the smaller non-negative real root of the equation: $\pi_{I}^{N R}\left(\frac{c+\lambda_{h}+m+\lambda_{h} s-s}{2 \lambda_{h}}, \frac{1}{2}\left(-c-h \lambda_{h}+\right.\right.$ $\left.\left.h+\lambda_{h}-s \lambda_{h}+s\right)\right)=\pi_{I}^{N N}\left(p_{N N}^{*}, m_{1}\right) . s_{3}$ is the larger non-negative real root of the equation: $m_{N N}^{*}-m_{1}=0$.

First, Figure 3 shows that the premium decreases discontinuously when the retailer's decision switches from restricting uninformed consumers to not restricting uninformed consumers. When the retailer restricts uninformed consumers, the insurer only collects the premium from informed consumers, whereas the insurer collects the premium from both informed and uninformed consumers otherwise. Thus, a lower premium increases more insurance sales when the retailer does not restrict uninformed consumers. So the premium decreases discontinuously when the retailer's decision switches from restricting consumers to not restricting consumers.


Figure 3. The optimal premium changes in scenario NR ( $c=0.4, \theta=0.6, \lambda_{l}=0.4, \lambda_{h}=0.5$, $\beta=2, h=0.048, k=0.0002$ ).

Second, the premium increases in the salvage $s$ when the retailer either restricts uninformed consumers salvage ( $s \leq s_{2}$ ) or does not restrict uninformed consumers ( $s>s_{2}$ ). This is because an increase in the salvage mitigates the retailer's return loss, which encourages the insurer to set a high premium.

### 4.4. Only the Insurer Has the Restricted Right (Scenario RN)

In this scenario, the insurer offers RSI to the retailer and restricts uninformed consumers from obtaining RSI. So even if the retailer purchases RSI, uninformed consumers cannot obtain RSI. Uninformed consumers pay the return shipping fee by themselves if they return the product. The retailer has no right to restrict uninformed consumers from buying the product. First, we study consumers' decisions. Uninformed consumers' expected utility is $u_{n l}$ and informed consumers' expected utility is $u_{i h}$ if they buy the product. From $u_{n l}>0$, we derive that the uninformed consumer whose valuation satisfies $v>p+h\left(1 / \lambda_{l}-1\right)=v_{n l}$ would buy the product. Thus, if $v_{n l} \geq 1$, that is, $p \geq 1+h-h / \lambda_{l}=p_{1}$, all uninformed consumers would not buy the product. From $u_{i h}>0$, we derive that the informed consumer whose valuation satisfies $v>p$ would buy the product. As a result, the retailer has two price strategies: low-price strategy ( $p<p_{1}$ ), where informed and uninformed consumers buy the product; high-price strategy ( $p \geq p_{1}$ ), where only informed consumers buy the product. Based on consumers' decisions, we derive the retailer's expected profit as follows:

$$
\begin{align*}
\pi_{R}^{R N-L}=\beta(1-p) \theta\left(p-c-\left(1-\lambda_{h}\right)(p-s)-m\right)+(1-\theta)\left(1-v_{n l}\right)\left(p-c-\left(1-\lambda_{l}\right)(p-s)\right) .  \tag{10}\\
\pi_{R}^{R N-H}=\beta(1-p) \theta\left(p-c-\left(1-\lambda_{h}\right)(p-s)-m\right) . \tag{11}
\end{align*}
$$

where $\pi_{R}^{R N-L}$ and $\pi_{R}^{R N-H}$ denote the retailer's expected profit under the low-price strategy and the high-price strategy, respectively.

The retailer maximizes its expected profit, leading to the following proposition.
Proposition 6. In scenario RN, for a given premium $m$, the optimal price is
(i) $p_{R N}^{*}=\frac{\beta \theta \lambda_{h}+(\beta-1) c \theta+c+h\left(-\theta \lambda_{l}+\theta+\lambda_{l}-1\right)-\theta \lambda_{l}+\lambda_{l}+\beta \theta m+\beta \theta \lambda_{h} s-\beta \theta s-\theta \lambda_{l} s+\theta s+\lambda_{l} s-s}{2\left(\beta \theta \lambda_{h}-\theta \lambda_{l}+\lambda_{l}\right)}$ if $m \leq m_{2}$.

The retailer adopts the low-price strategy.
(ii) $p_{R N}^{*}=p_{1}$ if $m_{2}<m \leq \max \left(m_{2}, m_{3}\right)$. The retailer adopts the high-price strategy.
(iii) $p_{R N}^{*}=\frac{c+\lambda_{h}+m+\lambda_{h} s-s}{2 \lambda_{h}}$ if $m>\max \left(m_{2}, m_{3}\right)$. The retailer adopts the high-price strategy.

Figure 4 shows that the retailer adopts the high-price strategy when the premium is high $\left(m>m_{2}\right)$; otherwise, he adopts the low-price strategy. Informed consumers return the product with low probability, so their buying the product benefits the retailer. On the contrary, uninformed consumers buying the product may hurt the retailer due to their high return probability. Thus, the retailer aims to increase the demand of informed consumers and reduce the demand of uninformed consumers. Adopting the low-price strategy can improve both consumer demands at the same time. When the premium is low ( $m \leq m_{2}$ ), the cost of purchasing RSI is low. Therefore, in this case, the retailer adopts the low-price strategy. On the contrary, when the premium is high $\left(m>m_{2}\right)$, the cost of purchasing RSI is high. If the retailer still adopts the low-price strategy, the insurance premium and the loss from uninformed consumers' purchases decreases the retailer's profit. Thus, in this case, the retailer adopts the high-price strategy.


Figure 4. Seller's pricing strategy in scenario $\mathrm{RN}\left(c=0.4, \theta=0.6, \lambda_{l}=0.4, \lambda_{h}=0.6, \beta=2, h=0.1\right)$.
Whether the retailer adopts the high-price strategy or the low-price strategy, the insurer only collects premiums from informed consumers. Thus, the insurer's expected profit is

$$
\begin{equation*}
\pi_{I}^{R N}=(1-p) \theta\left(m-h\left(1-\lambda_{h}\right)\right) . \tag{12}
\end{equation*}
$$

The insurer maximizes its expected profit, leading to the following proposition.
Proposition 7. In scenario $R N$, the optimal premium is
$m_{R N}^{*}= \begin{cases}\frac{1}{2}\left(-c-h \lambda_{h}+h+\lambda_{h}-s \lambda_{h}+s\right) & \text { if } s \leq s_{4} \\ m_{3} & \text { if } s_{4}<s \leq s_{5} \\ \overline{m_{3}} & \text { if } s_{5}<s \leq \max \left(s_{4}, s_{5}\right) \\ m_{R N}^{1} & \text { if } s>\max \left(s_{4}, s_{5}\right)\end{cases}$
Here, $m_{R N}^{1}=\frac{\beta \theta \lambda_{h}+c(-\beta \theta+\theta-1)+\theta h\left(-\beta \lambda_{h}+\beta+\lambda_{l}-1\right)-h \lambda_{l}-\theta \lambda_{l}+h+\lambda_{l}-\beta \theta \lambda_{h} s+\beta \theta s+\theta \lambda_{l} s-\theta s-\lambda_{l} s+s}{2 \beta \theta}$
Figure 5 shows that the optimal premium increases in the salvage in each pricing strategy. A higher salvage means low return loss for the retailer, which allows the retailer to make a higher profit. This incentivizes the insurer to set a higher premium to earn more profits. Figure 5 also depicts that when the pricing strategy switches from the high-price strategy to the low-price strategy, the optimal premium decreases discontinuously. When the retailer adopts the high-price strategy, fewer consumers buy the product, meaning fewer insurance purchases. Thus, in this case, the insurer sets a high premium. On the contrary, when the retailer adopts the low-price strategy, the insurer sets a low premium. Therefore,
when the retailer's pricing strategy switches to the low-price strategy, the optimal premium decreases discontinuously.


Figure 5. The premium changes in $s$ in scenario $R N\left(c=0.4, \theta=0.6, \lambda_{l}=0.4, \lambda_{h}=0.6, \beta=2\right.$, $h=0.048$ ).

### 4.5. Both the Insurer and the Retailer Do Not Have the Restricted Right (Scenario RR)

In this scenario, the insurer offers RSI to the retailer and restricts uninformed consumers from obtaining RSI. The retailer also has the right to restrict uninformed consumers from buying the product. Uninformed consumers pay the return shipping fee by themselves if they return the product. When the retailer does not restrict uninformed consumers, uninformed consumers' and informed consumers' expected utility, and the retailer' and the insurer's decisions are the same as in scenario RN. If the retailer also restricts uninformed consumers, only informed consumers may buy the product. The informed consumer whose valuation is higher than the retail price would buy the product. We derive the retailer's expected profit as

$$
\begin{equation*}
\pi_{R}^{R R}=\beta(1-p) \theta\left(p-c-\left(1-\lambda_{h}\right)(p-s)-m\right)+k(1-\theta) . \tag{13}
\end{equation*}
$$

The retailer maximizes its expected profit, leading to the following proposition.
Proposition 8. In scenario $R R$, for a given premium $m$, the optimal price is
(i) $p_{R R}^{*}=\frac{c+\lambda_{h}+m+\lambda_{h} s-s}{2 \lambda_{h}}$ if $m \leq m_{4}$. The retailer restricts uninformed consumers.
(ii) $p_{R R}^{*}=\frac{\beta \theta \lambda_{h}+(\beta-1) c \theta+c+h\left(-\theta \lambda_{l}+\theta+\lambda_{l}-1\right)-\theta \lambda_{l}+\lambda_{l}+\beta \theta m+\beta \theta \lambda_{h} s-\beta \theta s-\theta \lambda_{l} s+\theta s+\lambda_{l} s-s}{2\left(\beta \theta \lambda_{h}-\theta \lambda_{l}+\lambda_{l}\right)}$ if $m_{4}<m \leq$ $\max \left(m_{2}, m_{4}\right)$. The retailer does not restrict uninformed consumers and adopts the low-price strategy.
(iii) $p_{R R}^{*}=p_{1}$ if $\max \left(m_{2}, m_{4}\right)<m \leq \max \left(m_{2}, m_{3}\right)$. The retailer does not restrict uninformed consumers and adopts the high-price strategy.
(iv) $p_{R R}^{*}=\frac{c+\lambda_{h}+m+\lambda_{h} s-s}{2 \lambda_{h}}$ if $m>\max \left(m_{2}, m_{3}\right)$. The retailer does not restrict uninformed consumers and adopts the high-price strategy.

Proposition 8 shows that even if the insurer restricts uninformed consumers from obtaining RSI, the retailer may still restrict them from buying the product. This happens when the premium is sufficiently low $\left(m \leq m_{4}\right)$ as Figure 6 illustrates. The low premium incentivizes the retailer to set a lower price to increase informed consumers' purchases, whereas this would encourage more uninformed consumers to buy the product, leading to more return loss. To avoid return loss from uninformed consumers' purchases, the retailer further restricts uninformed consumers from buying the product, even though the insurer
restricts uninformed consumers from obtaining RSI. When the premium is high ( $m>m_{4}$ ), the results are the same as scenario RN .


Figure 6. Seller's pricing strategy in scenario $R R\left(c=0.4, \theta=0.6, \lambda_{l}=0.4, \lambda_{h}=0.6, \beta=2\right.$, $h=0.1, k=0.0002$ ).

Proposition 8 implies that when the premium is sufficiently low or high, the retailer should not allow uninformed consumers to buy the product. However, the retailer should adopt different methods according to the premium. When the premium is sufficiently low, the retailer should restrict uninformed consumers from buying the product and set a low price. When the premium is sufficiently high, the retailer sets a high price and does not need to restrict uninformed consumers from buying the product.

The insurer's expected profit is $\pi_{I}^{R R}=\pi_{I}^{R N}=(1-p) \theta\left(m-h\left(1-\lambda_{h}\right)\right)$. The insurer maximizes its expected profit, leading to the following proposition.

Proposition 9. In scenario $R R$, the optimal premium is
$m_{R R}^{*}= \begin{cases}\frac{1}{2}\left(-c-h \lambda_{h}+h+\lambda_{h}-s \lambda_{h}+s\right) & \text { if } s \leq s_{4} \\ m_{3} & \text { if } s_{4}<s \leq \min \left(s_{5}, s_{6}\right) \\ m_{5} & \text { if } \min \left(s_{5}, s_{6}\right)<s \leq s_{5} \\ \bar{m}_{3} & \text { if } s_{5}<s \leq \max \left(s_{4}, s_{5}\right) \\ m_{R N}^{1} & \text { if } s>\max \left(s_{4}, s_{5}\right)\end{cases}$
Compared with Proposition 7, we derive that the insurer sets a new optimal premium $m_{5}$ when the salvage $s$ is medium $\left(\min \left(s_{5}, s_{6}\right)<s \leq s_{5}\right)$. In this case, the retailer restricts uninformed consumers from buying the product, and premium $m_{5}$ is the lowest premium. When the retailer restricts uninformed consumers, the retailer cannot buy the product, which does not lead to losses to the retailer. This incentivizes the retailer to set a lower price to increase informed consumers' purchases. In this case, a lower premium can increase more sales. Thus, the insurer sets the lowest premium $m_{5}$. Otherwise, the optimal premium is the same as scenario RN.

## 5. Conclusions

Return shipping insurance is an effective tool to solve consumer returns. However, RSI results in a high return rate, which can also increase the retailer's cost. In practice, platforms may give insurers and retailers the right to restrict consumers. In this paper, we examine the impact of RSI under different restricting rights and derive some interesting results.

First, whether the retailer restricts uninformed consumers from buying the product depends on the salvage and whether the insurer restricts uninformed consumers from obtain-
ing RSI. When the insurer does not restrict uninformed consumers, the retailer will restrict uninformed consumers if the salvage is sufficiently low. When the insurer restricts uninformed consumers, the retailer will restrict uninformed consumers if the salvage is medium. The insurer restricting uninformed consumers makes fewer uninformed consumers buy the product. Thus, the retailer does not need to restrict uninformed consumers if the salvage is low. This helps explain why many retailers on Taobao.com (https:/ /www.douban. com/group/topic/262036714/?_i=7639899j6Ys52s, accessed on 20 February 2023) or Pingduoduo (https:/ /baijiahao.baidu.com/s?id=1747351930076458925\&wfr=spider\&for=pc, accessed on 20 February 2023) still further restrict uninformed consumers from buying products, even though insurers have restricted them from obtaining RSI.

Second, when the salvage is low, the retailer should adopt the high-price strategy; otherwise, the retailer should adopt the low-price strategy. When the salvage is low, any return means high return loss. The high-price strategy can prevent uninformed consumers from buying the product, which reduces the return loss. This implies that when RSI is offered, retailers should adopt corresponding pricing strategies according to the salvage of returned products.

Third, RSI may hurt the retailer and the retail price is higher when consumers are not restricted. When the salvage is low, purchasing RSI increases returns, which hurts the retailer. This suggests that retailers should only buy RSI if the salvage is high. The practice that high-quality retailers, such as Adidas (https://adidas.tmall.com, accessed on 20 February 2023) and FILA (https:/ /fila.tmall.com, accessed on 20 February 2023), purchase RSI and small brands, such as Warrior (https:/ /huili.tmall.com, accessed on 20 February 2023) and Zulun (https:/ /zulun.tmall.com, accessed on 20 February 2023), do not purchase RSI, verifies this result. Purchasing RSI also increases consumers' utility, which allows the retailer to set a higher price. Thus, when retailers buy RSI, they should increase the retail price.

Fourth, the insurer may set a higher premium when the salvage is low. When the salvage is low, the retailer may restrict uninformed consumers from buying the product or adopt the high-price strategy. This reduces the demand of RSI, which makes the insurer set a higher premium. This implies that if the product salvage is low, such as fresh products and customized products, the insurer should charge a higher premium.

This study has some limitations. First, we consider one online retailer and one insurer. In practice, there are multiple online retailers and insurers on e-commerce platforms, and they compete with each other. Second, we focus on the online return. In reality, some retailers not only allow consumers to return products by express but also allow them to return products through brick-and-mortar stores. Finally, future research could consider more complex valuation distribution.

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## Appendix A

Proof of Proposition 1. To focus on the interesting cases, we assume that $h<\min \left(\lambda_{l}, h_{1}, h_{2}\right)$; otherwise, only informed consumers buy the product or consumers
would not return the product when there is no RSI. $h_{1}=\frac{\lambda_{l}\left(-c-\theta \lambda_{l}+\theta \lambda_{h}+\lambda_{l}+s\left(\theta \lambda_{l}-\theta \lambda_{h}-\lambda_{l}+1\right)\right)}{(\theta-1) \lambda_{l}^{2}-\theta \lambda_{l}\left(\lambda_{h}+2\right)+2 \theta \lambda_{h}+\lambda_{l}}$, $h_{2}=\frac{c-\theta \lambda_{l}+\theta \lambda_{h}+\lambda_{l}+s\left(-\theta \lambda_{l}+\theta \lambda_{h}+\lambda_{l}-1\right)}{-\theta \lambda_{l}+\theta \lambda_{h}+\lambda_{l}+1}$.
(1) From Section 4.1, in scenario N, in which there is no insurance, we know that given the retail price $p$, informed consumers buy the product if $v>v_{n h}$, and uninformed consumers buy the product if $v>v_{n l}$. By Equation (3), we know that the retailer's profit is $\pi_{R}^{N}$. To maximize the retailer's profit, we derive the second derivative of $\pi_{R}^{N}$ with respect to $p$ which is $2(\theta-1) \lambda_{l}-2 \theta \lambda_{h}<0$. Solving the first-order condition yields the optimal price $p_{N}^{*}=\frac{-c+h \theta \lambda_{l}-h \theta \lambda_{h}-h \lambda_{l}+\theta \lambda_{l}-\theta \lambda_{h}+h-\lambda_{l}+\theta s \lambda_{l}-\theta s \lambda_{h}-s \lambda_{l}+s}{2\left(\theta \lambda_{l}-\theta \lambda_{h}-\lambda_{l}\right)}$.
(2) The first derivative of $\pi_{R}^{N}$ with respect to $s$ is $\frac{(\theta-1) \lambda_{l}-\theta \lambda_{h}+1}{2(\theta-1) \lambda_{l}-2 \theta \lambda_{h}}<0$. Thus, $\pi_{R}^{N}$ decreases in $s$.
(3) The first derivative of $\pi_{R}^{N}$ with respect to $h$ is $\frac{(\theta-1) \lambda_{l}-\theta \lambda_{h}+1}{2(\theta-1) \lambda_{l}-2 \theta \lambda_{h}}<0$. Thus, $\pi_{R}^{N}$ decreases in $h$.

Proof of Proposition 2. (1) We derive the optimal price given the premium $m$. From Section 4.2, in the scenario NN, we know that the retailer's profit is $\pi_{R}^{N N}$ from Equation (6). To maximize the retailer's profit, we derive the second derivative of $\pi_{R}^{N N}$ with respect to $p$ which is $2 \beta\left((\theta-1) \lambda_{l}-\theta \lambda_{h}\right)<0$. Solving the first-order condition yields the optimal price $p_{N N}^{1}=\frac{c-\theta \lambda_{l}+\theta \lambda_{h}+\lambda_{l}+m-\theta \lambda_{l} s+\theta \lambda_{h} s+\lambda_{l} s-s}{-2 \theta \lambda_{l}+2 \theta \lambda_{h}+2 \lambda_{l}}$. Thus, we derive the optimal price given the premium $m$.
(2) Then, we derive the optimal premium by maximizing the insurer's profit. By Equation (7), we know that the insurer's profit is $\pi_{I}^{N N}$. Substituting the optimal price $p_{N N}^{1}$ into $\pi_{I}^{N N}$ gives

$$
\pi_{I}^{N N}=\frac{\left((\theta-1) h \lambda_{l}-\theta h \lambda_{h}+h-m\right)\left(c+\theta \lambda_{l}-\theta \lambda_{h}-\lambda_{l}+m-\theta \lambda_{l} s+\theta \lambda_{h} s+\lambda_{l} s-s\right)}{2(1-\theta) \lambda_{l}+2 \theta \lambda_{h}} .
$$

The second derivative of $\pi_{I}^{N N}$ with respect to $m$ is $\frac{1}{(\theta-1) \lambda_{l}-\theta \lambda_{h}}<0$. Solving the firstorder condition yields the optimal premium $m_{N N}^{*}=\frac{1}{2}\left(-c+h\left(\theta \lambda_{l}-\theta \lambda_{h}-\lambda_{l}+1\right)-\theta \lambda_{l}+\right.$ $\left.\theta \lambda_{h}+\lambda_{l}+\theta \lambda_{l} s-\theta \lambda_{h} s-\lambda_{l} s+s\right)$.
(3) Substituting $m=m_{N N}^{*}$ into $p_{N N}^{1}$ gives

$$
p_{N N}^{*}=\frac{c+h \theta \lambda_{l}-h \theta \lambda_{h}-h \lambda_{l}-3 \theta \lambda_{l}+3 \theta \lambda_{h}+h+3 \lambda_{l}-\theta \lambda_{l} s+\theta \lambda_{h} s+\lambda_{l} s-s}{-4 \theta \lambda_{l}+4 \theta \lambda_{h}+4 \lambda_{l}} .
$$

(4) The first derivative of $p_{N N}^{*}$ with respect to $h$ is $\frac{-\theta \lambda_{l}+\theta \lambda_{h}+\lambda_{l}-1}{4(\theta-1) \lambda_{l}-4 \theta \lambda_{h}}>0$. Thus, $p_{N N}^{*}$ increases in $h$.
(5) The first derivative of $m_{N N}^{*}$ with respect to $h$ is $\frac{1}{2}\left((\theta-1) \lambda_{l}-\theta \lambda_{h}+1\right)>0$. Thus, $m_{N N}^{*}$ increases in $h$.

Proof of Proposition 3. (1) $p_{N N}^{*}-p_{N}^{*}=\frac{c+3 h\left(-\theta \lambda_{l}+\theta \lambda_{h}+\lambda_{l}-1\right)+\theta \lambda_{l}-\theta \lambda_{h}-\lambda_{l}-\theta \lambda_{l} s+\theta \lambda_{h} s+\lambda_{l} s-s}{4(\theta-1) \lambda_{l}-4 \theta \lambda_{h}}$. Because $m_{N N}^{*}<h$, we derive that $p_{N N}^{*}>p_{N}^{*}$.
(2) Let $\pi_{R}^{N N}-\pi_{R}^{N}=0$, we can obtain two roots. Define $s_{1}$ as the larger non-negative real root of the equation. It is easy to find that $\pi_{R}^{N N}<\pi_{R}^{N}$ if $s<s_{1}$, and $\pi_{R}^{N N} \geq \pi_{R}^{N}$ if $s \geq s_{1}$.

Proof of Proposition 4. (1) We derive the optimal price if the retailer restricts uninformed consumers. From Section 4.3, in scenario NR, the retailer has the right to restrict uninformed consumers. We know that the retailer's profit is $\pi_{R}^{N R}$ from Equation (8) if the retailer restricts uninformed consumers. To maximize the retailer's profit, we derive the second derivative of $\pi_{R}^{N R}$ with respect to $p$, which is $-2 \beta \theta \lambda_{h}<0$. Solving the first-order condition yields the optimal price $p_{N R}^{1}=\frac{c+\lambda_{h}+m+\lambda_{h} s-s}{2 \lambda_{h}}$.
(2) Compare the profit $\pi_{R}^{N R}\left(m, p_{N R}^{1}\right)$ when restricting uninformed consumers with the profit $\pi_{R}^{N N}\left(m, p_{N N}^{1}\right)$ when not restricting consumers. Let $\pi_{R}^{N R}\left(m, p_{N R}^{1}\right)-\pi_{R}^{N N}\left(m, p_{N N}^{1}\right)=0$; we can obtain two roots. Define $m_{1}$ as the smaller non-negative real root of the equation. It is easy
to find that $\pi_{R}^{N R}\left(m, p_{N R}^{1}\right) \leq \pi_{R}^{N N}\left(m, p_{N N}^{1}\right)$ if $m<m_{1}$, and $\pi_{R}^{N R}\left(m, p_{N R}^{1}\right)>\pi_{R}^{N N}\left(m, p_{N N}^{1}\right)$ if $m>m_{1}$.
(3) $p_{N R}^{1}-p_{N N}^{1}=\frac{(\theta-1)\left(\lambda_{l}-\lambda_{h}\right)(c+m-s)}{2 \lambda_{h}\left(-\theta \lambda_{l}+\theta \lambda_{h}+\lambda_{l}\right)}$. It is easy to find that $p_{N R}^{1}-p_{N N}^{1}<0$. Thus, as the premium $m$ changes such that the retailer's optimal strategy switches from not restricting uninformed consumers to restricting uninformed consumers, $p_{N R}^{*}$ decreases discontinuously.

Proof of Proposition 5. From Section 4.3, in scenario NR, we know that the insurer's profit is $\pi_{I}^{N R}$ from Equation (9). From Proposition 4, we know that the retailer has two strategies according to the premium. Thus, we derive the optimal premium in the following two cases. Then, we compare the insurer's profits of the two cases to derive the optimal premium.
(1) Consider the case in which $m \leq m_{1}$. In this case, the retailer does not restrict uninformed consumers. From Proposition 2, we know that $\pi_{I}^{N N}\left(p_{N N}^{*}, m\right)$ is a concave function in $m$, and the optimal premium is $m_{N N}^{*}$. Let $m_{N N}^{*}-m_{1}=0$, we can obtain two roots. Define $s_{3}$ as the larger non-negative real root of the equation. It is easy to find that $m_{N N}^{*} \geq m_{1}$ if $s \leq s_{3}$, and $m_{N N}^{*}<m_{1}$ if $s>s_{3}$. So, in this case $m \leq m_{1}$, we derive that $m_{N R}^{1}=m_{1}$ if $s \leq s_{3}$, and $m_{N R}^{1}=m_{N N}^{*}$ if $s>s_{3}$.
(2) Consider the case in which $m>m_{1}$. In this case, the retailer restricts uninformed consumers. Substituting $p=p_{N R}^{1}$ into $\pi_{I}^{N R}$ gives $\frac{\left.\theta\left(h \lambda_{h}-1\right)+m\right)\left(c-\lambda_{h}+m+\lambda_{h} s-s\right)}{-2 \lambda_{h}}$. The second derivative of $\pi_{I}^{N R}\left(m, p_{N R}^{1}\right)$ with respect to $m$ is $-\frac{\theta}{\lambda_{h}}<0$. Solving the first-order condition yields the optimal premium $\bar{m}_{1}=\frac{1}{2}\left(-c-h \lambda_{h}+h+\lambda_{h}-s \lambda_{h}+s\right)$.
(3) Compare the insurer's profit under $m \leq m_{1}$ with that under $m>m_{1}$. Define $s_{2}$ as the smaller non-negative real root of the equation $\pi_{I}^{N R}\left(p_{N R}^{1}, \bar{m}_{1}\right)=\pi_{I}^{N N}\left(p_{N N}^{*}, m_{1}\right)$. It is easy to find that $\pi_{I}^{N R}\left(p_{N R}^{1}, \bar{m}_{1}\right) \geq \pi_{I}^{N N}\left(p_{N N}^{*}, m_{1}\right)$ if $s \leq s_{2}$, and $\pi_{I}^{N R}\left(p_{N R}^{1}, \bar{m}_{1}\right)<$ $\pi_{I}^{N N}\left(p_{N N}^{*}, m_{1}\right)$ if $s>s_{2}$. Because $\pi_{I}^{N N}\left(p_{N N}^{*}, m_{1}\right)<\pi_{I}^{N N}\left(p_{N N}^{*}, m_{N N}^{*}\right), \pi_{I}^{N R}\left(p_{N R}^{1}, \bar{m}_{1}\right)<$ $\pi_{I}^{N N}\left(p_{N N}^{*}, m_{N N}^{*}\right)$ if $s>s_{3}$.

Proof of Proposition 6. In scenario RN, from Equations (10) and (11), we know that the two pricing strategies lead to two different retailers' profits. Thus, we derive the optimal price in the following two cases. Then, we compare the retailer's profits of the two cases to derive the optimal price.
(1) Consider the low-price strategy $p<p_{1}$. In this case, uninformed consumers and informed consumers would buy the product. From Equation (10), the retailer's profit is $\pi_{R}^{R N-L}$. The second derivative of $\pi_{R}^{R N-L}$ with respect to $p$ is $-2\left(\beta \theta \lambda_{h}-\theta \lambda_{l}+\lambda_{l}\right)<0$. Solving the first-order condition yields the optimal price

$$
p_{R N-L}^{*}=\frac{\beta \theta \lambda_{h}+(\beta-1) c \theta+c+h\left(-\theta \lambda_{l}+\theta+\lambda_{l}-1\right)-\theta \lambda_{l}+\lambda_{l}+\beta \theta m+\beta \theta \lambda_{h} s-\beta \theta s-\theta \lambda_{l} s+\theta s+\lambda_{l} s-s}{2\left(\beta \theta \lambda_{h}-\theta \lambda_{l}+\lambda_{l}\right)}
$$

Then, compare $p_{R N-L}^{*}$ with $p_{1}$.

$$
p_{R N-L}^{*}-p_{1}=\frac{c\left((\beta-1) \theta \lambda_{l}+\lambda_{l}\right)+h\left(\lambda_{l}-1\right)\left((\theta-1) \lambda_{l}-2 \beta \theta \lambda_{h}\right)-\lambda_{l}\left(\beta \theta \lambda_{h}-\theta \lambda_{l}+\lambda_{l}-\beta \theta m+\theta s\left(-\beta \lambda_{h}+\beta+\lambda_{l}-1\right)-\lambda_{l} s+s\right)}{2 \lambda_{l}\left((1-\theta) \lambda_{l}+\beta \theta \lambda_{h}\right)} .
$$

The first derivative of $p_{R N-L}^{*}-p_{1}$ with respect to $m$ is $\frac{\beta \theta}{2 \beta \theta \lambda_{h}-2 \theta \lambda_{l}+2 \lambda_{l}}>0$. Let $p_{R N-L}^{*}-p_{1}=$ 0 , we obtain
$\bar{m}_{2}=\frac{-c\left((\beta-1) \theta \lambda_{l}+\lambda_{l}\right)-h\left(\lambda_{l}-1\right)\left((\theta-1) \lambda_{l}-2 \beta \theta \lambda_{h}\right)+\lambda_{l}\left(\beta \theta \lambda_{h}-\theta \lambda_{l}+\lambda_{l}+\theta s\left(-\beta \lambda_{h}+\beta+\lambda_{l}-1\right)-\lambda_{l} s+s\right)}{\beta \theta \lambda_{l}}$. So in this case, the optimal price is $p_{R N-L}^{*}$ if $m \leq \bar{m}_{2}$, and $p_{1}$ otherwise.
(2) Consider the high-price strategy $p \geq p_{1}$. In this case, only informed consumers would buy the product. From Equation (11), the retailer's profit is $\pi_{R}^{R N-H}$. The second derivative of $\pi_{R}^{R N-H}$ with respect to $p$ is $-2 \beta \theta \lambda_{h}<0$. Solving the first-order condition yields the optimal price $p_{R N-H}^{*}=\frac{c+\lambda_{h}+m+\lambda_{h} s-s}{2 \lambda_{h}}$.

Then, compare $p_{R N-H}^{*}$ with $p_{1} \cdot p_{R N-H}^{*}-p_{1}=-1+h\left(\frac{1}{\lambda_{l}}-1\right)+\frac{c+\lambda_{h}+m+\lambda_{h} s-s}{2 \lambda_{h}}$. The first derivative of $p_{R N-L}^{*}-p_{1}$ with respect to $m$ is $\frac{1}{2 \lambda_{h}}>0$. Let $p_{R N-H}^{*}-p_{1}=0$; we obtain
$m_{3}=-c-\frac{2 h \lambda_{h}}{\lambda_{l}}+2 h \lambda_{h}+\lambda_{h}-\lambda_{h} s+s$. So in this case, the optimal price is $p_{1}$ if $m \leq m_{3}$, and $p_{R N-L}^{*}$ otherwise.
(3) Compare the low-price strategy with the high-price strategy.
$\bar{m}_{2}-m_{3}=\frac{(\theta-1)\left(c-h \lambda_{l}+h-\lambda_{l}+\lambda_{l} s-s\right)}{\beta \theta}$. Let $\bar{m}_{2}-m_{3}=0$, we obtain $\overline{s_{1}}=\frac{-c+h\left(\lambda_{l}-1\right)+\lambda_{l}}{\lambda_{l}-1}$. It is easy to find that $\bar{m}_{2} \leq m_{3}$ if $s \leq \overline{s_{1}}, \bar{m}_{2}>m_{3}$ otherwise. When $s \leq \overline{s_{1}}$, it is easy to find that

$$
p_{R N}= \begin{cases}p_{R N-L}^{*} & \text { if } m \leq \bar{m}_{2} \\ p_{1} & \text { if } \bar{m}_{2}<m \leq m_{3} \\ p_{R N-H}^{*} & \text { if } m>m_{3}\end{cases}
$$

When $s>\overline{s_{1}}$, compare $\pi_{R}^{R N-L}\left(p_{R N-L}^{*}\right)$ with $\pi_{R}^{R N-H}\left(p_{R N-H}^{*}\right)$. Let $\pi_{R}^{R N-L}\left(p_{R N-L}^{*}\right)-$ $\pi_{R}^{R N-H}\left(p_{R N-H}^{*}\right)=0$, we obtain two roots. Define $\bar{m}_{3}$ as the larger non-negative real root of the equation. It is easy to find that $\pi_{R}^{R N-L}\left(p_{R N-L}\right) \geq \pi_{R}^{R N-H}\left(p_{R N-H}^{*}\right)$ if $m \leq \bar{m}_{3}$, $\pi_{R}^{R N-L}\left(p_{R N-L}^{*}\right)<\pi_{R}^{R N-H}\left(p_{R N-H}^{*}\right)$ otherwise. Finally, we define

$$
m_{2}= \begin{cases}\overline{m_{2}} & \text { if } s \leq \overline{s_{1}} \\ \overline{m_{3}} & \text { if } s>\overline{s_{1}}\end{cases}
$$

Proof of Proposition 7. From Section 4.4, in scenario RN, we know that the insurer's profit is $\pi_{I}^{R N}$ from Equation (12). From Proposition 6, we know that the retailer has three strategies according to the premium. Thus, we derive the optimal premium in the following three cases. Then, we compare the insurer's profits of the three cases to derive the optimal premium.
(1) We derive the optimal premium in each case. Consider the case in which $m \leq m_{2}$ and the retailer adopts the low-price strategy. Substituting $p_{R N-L}^{*}$ into $\pi_{I}^{R N}$ gives

$$
\pi_{I}^{R N}\left(p_{R N-L}^{*}\right)=\frac{\theta\left(h\left(\lambda_{h}-1\right)+m\right)\left(\beta \theta \lambda_{h}+c(-\beta \theta+\theta-1)+h(\theta-1)\left(\lambda_{l}-1\right)-\theta \lambda_{l}+\lambda_{l}-\beta \theta m-\beta \theta \lambda_{h} s+\beta \theta s+\theta \lambda_{l} s-\theta s-\lambda_{l} s+s\right)}{2\left(\beta \theta \lambda_{h}-\theta \lambda_{l}+\lambda_{l}\right)}
$$

The second derivative of $\pi_{I}^{R N}\left(p_{R N-L}^{*}\right)$ with respect to $m$ is $-\frac{\beta \theta^{2}}{\beta \theta \lambda_{h}-\theta \lambda_{l}+\lambda_{l}}<0$. Solving the first-order condition yields the optimal premium
$m_{R N}^{1}=\frac{\beta \theta \lambda_{h}+c(-\beta \theta+\theta-1)+\theta h\left(-\beta \lambda_{h}+\beta+\lambda_{l}-1\right)-h \lambda_{l}-\theta \lambda_{l}+h+\lambda_{l}-\beta \theta \lambda_{h} s+\beta \theta s+\theta \lambda_{l} s-\theta s-\lambda_{l} s+s}{2 \beta \theta}$.
Consider the case in which $m>\max \left(m_{2}, m_{3}\right)$ and the retailer adopts the high-price strategy. Substituting $p_{R N-H}^{*}$ into $\pi_{I}^{R N}$ gives $\pi_{I}^{R N}\left(p_{R N-H}^{*}\right)=-\frac{\theta\left(h\left(\lambda_{h}-1\right)+m\right)\left(c-\lambda_{h}+m+\lambda_{h} s-s\right)}{2 \lambda_{h}}$. The second derivative of $\pi_{I}^{R N}\left(p_{R N-H}^{*}\right)$ with respect to $m$ is $-\frac{\theta}{\lambda_{h}}<0$. Solving the first-order condition yields the optimal premium $m_{R N}^{2}=\frac{1}{2}\left(-c-h \lambda_{h}+h+\lambda_{h}-\lambda_{h} s+s\right)$. Compare $m_{R N}^{2}$ with $m_{3}$. Solving $m_{R N}^{2}=m_{3}$ yields $\overline{s_{2}}=\frac{c \lambda_{l}+h\left(-5 \lambda_{l} \lambda_{l}+\lambda_{l}+4 \lambda_{h}\right)-\lambda_{l} \lambda_{h}}{\lambda_{l}-\lambda_{l} \lambda_{h}}$. It is easy to find that $m_{R N}^{2} \geq m_{3}$ if $s \leq \overline{s_{2}}, m_{R N}^{2}<m_{3}$ otherwise. We compare $m_{R N}^{1}, m_{R N^{\prime}}^{2}, \bar{m}_{2}$ and $\bar{m}_{3}$ using the same method. We define

$$
\overline{s_{3}}=\frac{\lambda_{l}\left((\theta-1) \lambda_{l}-\beta \theta \lambda_{h}\right)+c\left((\beta-1) \theta \lambda_{l}+\lambda_{l}\right)+h\left(\lambda_{l}\left(\theta\left(-5 \beta \lambda_{h}+\beta-3\right)+3\right)+4 \beta \theta \lambda_{h}+3(\theta-1) \lambda_{l}^{2}\right)}{\lambda_{l}\left(\theta\left(\beta\left(-\lambda_{h}\right)+\beta+\lambda_{l}-1\right)-\lambda_{l}+1\right)}, \overline{s_{4}} \text { as the non- }
$$

negative real root of the equation $m_{R N}^{2}=\bar{m}_{3}, s_{6}$ as the non-negative real root of the equation $m_{R N}^{1}=\bar{m}_{3}$.
(2) Compare $\pi_{I}^{R N}\left(p_{1}, m_{3}\right), \pi_{I}^{R N}\left(p_{R N-L}^{*}, m_{2}\right), \pi_{I}^{R N}\left(p_{R N-H}^{*}, m_{3}\right), \pi_{I}^{R N}\left(p_{R N-L}^{*}, m_{R N}^{1}\right)$ and $\pi_{I}^{R N}\left(p_{R N-H}^{*}, m_{R N}^{2}\right)$. We derive that $\pi_{I}^{R N}\left(p_{R N-H}^{*}, m_{R N}^{2}\right)>\pi_{I}^{R N}\left(p_{1}, m_{3}\right)$ if $s<\overline{s_{2}}$, $\pi_{I}^{R N}\left(p_{R N-H}^{*}, m_{3}\right)=\pi_{I}^{R N}\left(p_{1}, m_{3}\right)$ if $\overline{s_{2}}<s<\overline{s_{3}}, \pi_{I}^{R N}\left(p_{R N-L}^{*}, m_{R N}^{1}\right)>\pi_{I}^{R N}\left(p_{R N-H}^{*}, \overline{m_{3}}\right)$ if $s>\overline{s_{1}}, \pi_{I}^{R N}\left(p_{R N-H}^{*}, m_{R N}^{2}\right)>\pi_{I}^{R N}\left(p_{1}, m_{3}\right)$ if $s<\overline{s_{2}}$, and $\pi_{I}^{R N}\left(p_{R N-L}^{*}, \bar{m}_{3}\right)>\pi_{I}^{R N}\left(\left(p_{R N-H}^{*}, \bar{m}_{3}\right)\right.$ if $s>\overline{s_{1}}$. We derive some threshold values. Define $\overline{s_{5}}$ as the non-negative real root of the equation $\pi_{I}^{R N}\left(p_{R N-L}^{*}, m_{R N}^{1}\right)=\pi_{I}^{R N}\left(p_{1}, m_{3}\right)$ and $\overline{s_{6}}$ as the non-negative real root of the equation $\pi_{I}^{R N}\left(p_{R N-L}^{*}, \bar{m}_{3}\right)=\pi_{I}^{R N}\left(p_{R N-H}^{*}, m_{R N}^{2}\right)$. Define

$$
\begin{aligned}
& s_{4}= \begin{cases}\overline{s_{2}} & \text { if } \overline{s_{2}} \leq \overline{s_{1}} \\
\overline{s_{6}} & \text { if } \overline{s_{2}}>\overline{s_{1}}\end{cases} \\
& s_{5}= \begin{cases}\overline{s_{5}} & \text { if } \overline{s_{2}}<\overline{s_{3}} \leq \overline{s_{1}} \\
\overline{s_{1}} & \text { if } \overline{s_{2}}<\overline{s_{1}} \leq \overline{s_{3}} \\
\overline{s_{6}} & \text { if } \overline{s_{1}}<\overline{s_{2}}<\leq \overline{s_{3}}\end{cases}
\end{aligned}
$$

Proof of Proposition 8. From Section 4.5, in scenario RR, we know that the retailer's profit is $\pi_{R}^{R R}$ from Equation (13) if the retailer restricts uninformed consumers. First, we derive the optimal price when the retailer restricts uninformed consumers. Then, to derive the retailer's decisions, we compare the retailer's profits with and without consumer restrictions.
(1) The second derivative of $\pi_{R}^{R R}$ with respect to $p$ is $-2 \beta \theta \lambda_{h}<0$. Solving the firstorder condition yields the optimal price $p_{R R}^{1}=\frac{c+\lambda_{h}+m+\lambda_{h} s-s}{2 \lambda_{h}}$.
(2) Compare $\pi_{R}^{R R}\left(p_{R R}^{1}\right)$ with $\pi_{R}^{R N}\left(p_{R N-H}^{*}\right)$ of the high-price strategy. $\pi_{R}^{R R}\left(p_{R R}^{1}\right)-$ $\pi_{R}^{R N-H}\left(p_{R N-H}^{*}\right)=k \theta-k<0$. Compare $\pi_{R}^{R R}\left(p_{R R}^{1}\right)$ with $\pi_{R}^{R N-L}\left(p_{R N-L}^{*}\right)$ of the low-price strategy. Solving $\pi_{R}^{R R}\left(p_{R R}^{1}\right)=\pi_{R}^{R N-L}\left(p_{R N-L}^{*}\right)$ yields two roots. Define $\bar{m}_{4}$ as the smaller non-negative real root of the equation. It is easy to find that $\pi_{R}^{R R}\left(p_{R R}^{1}\right) \geq \pi_{R}^{R N-L}\left(p_{R N-L}^{*}\right)$ if $m \leq \bar{m}_{4}, \pi_{R}^{R R}\left(p_{R R}^{1}\right)<\pi_{R}^{R N-L}\left(p_{R N-L}^{*}\right)$ otherwise. Compare $\pi_{R}^{R R}\left(p_{R R}^{1}\right)$ with $\pi_{R}^{R N-L}\left(p_{1}\right)$ of the low-price strategy. Solving $\pi_{R}^{R R}\left(p_{R R}^{1}\right)=\pi_{R}^{R N-L}\left(p_{1}\right)$ yields two roots. Define $\bar{m}_{5}$ as the smaller non-negative real root of the equation. It is easy to find that $\pi_{R}^{R R}\left(p_{R R}^{1}\right) \geq$ $\pi_{R}^{R N-L}\left(p_{1}\right)$ if $m \leq \bar{m}_{5}, \pi_{R}^{R R}\left(p_{R R}^{1}\right)<\pi_{R}^{R N-L}\left(p_{1}\right)$ otherwise. Finally, Solving $\bar{m}_{4}=\bar{m}_{5}$ yields $s=\frac{c(-\theta)+c+h(\theta-1)\left(\lambda_{l}-1\right)+\theta \lambda_{l}-2 \sqrt{\lambda_{h}} \sqrt{\beta(\theta-1) \theta(-k)}-\lambda_{l}}{(\theta-1)\left(\lambda_{l}-1\right)}$. Finally, define

$$
m_{4}= \begin{cases}\bar{m}_{4} & \text { if } s \leq \overline{s_{7}} \\ \overline{s_{5}} & \text { if } s>\overline{s_{7}}\end{cases}
$$

Proof of Proposition 9. In the scenario RR, to derive the optimal premium, we compare the insurer's profits with and without consumer restrictions. If the retailer has no right to restrict consumers, the results converge to scenario RN . When the retailer has the right to restrict consumers, the retailer restricts when the premium $m \leq m_{4}$. We only need to compare $\pi_{I}^{R R}$ under the condition $m \leq m_{4}$ with $\pi_{I}^{R N}$ in scenario RN. There are cases: $s \leq s_{7}$ and $s>$ $s_{7}$. Consider the case in which $s \leq s_{7}$. In this case, the retailer does not restrict uninformed consumers. Substituting $p_{R R}^{1}$ into $\pi_{I}^{R R}$ gives $\pi_{I}^{R R}\left(p_{R R}^{1}\right)=-\frac{\theta\left(h\left(\lambda_{h}-1\right)+m\right)\left(c-\lambda_{h}+m+\lambda_{h} s-s\right)}{2 \lambda_{h}}$. $\pi_{I}^{R R}\left(p_{R R}^{1}\right)$ is concave in premium $m$, and the first-order condition yields the optimal premium $m_{R R}^{1}=\frac{1}{2}\left(-c-h \lambda_{h}+h+\lambda_{h}-\lambda_{h} s+s\right)$. For each region in Proposition 7 , we compare $\pi_{I}^{R R}\left(p_{R R}^{1}, m_{4}\right)$ and $\pi_{I}^{R R}\left(p_{R R}^{1}, m_{R R}^{1}\right)$ with the insurer's profits in scenario $R N$, which can yield threshold values. It is easy to find that $\pi_{I}^{R R}\left(p_{R R}^{1}, m_{4}\right)$ or $\pi_{I}^{R R}\left(p_{R R}^{1}, m_{R R}^{1}\right)$ is larger when $s$ is higher or lower than the threshold values. For example, when $s_{4}<s \leq s_{5}$ where $m_{N R}^{*}=m_{3}, p_{R N}^{*}=p_{1}$, solving $\pi_{I}^{R R}\left(p_{R R}^{1}, m_{5}\right)-\pi_{I}^{R N}\left(p_{1}, m_{3}\right)=0$ yields $\overline{s_{8}}$. It is easy to find that $\pi_{I}^{R R}\left(p_{R R}^{1}, m_{5}\right) \leq \pi_{I}^{R N}\left(p_{1}, m_{3}\right)$ if $s \leq \overline{s_{8}}, \pi_{I}^{R R}\left(p_{R R}^{1}, m_{5}\right)>\pi_{I}^{R N}\left(p_{1}, m_{3}\right)$ otherwise. $\pi_{I}^{R R}\left(p_{R R}^{1}, m_{R R}^{1}\right)-\pi_{I}^{R N}\left(p_{1}, m_{3}\right)=\frac{\theta\left(c \lambda_{l}+h\left(-5 \lambda_{l} \lambda_{h}+\lambda_{l}+4 \lambda_{h}\right)-\lambda_{l} \lambda_{h}+\lambda_{l}\left(\lambda_{h}-1\right) s\right)^{2}}{8 \lambda_{l}^{2} \lambda_{h}}>0$. Define $s_{6}$ as the threshold value. When $s>s_{6}, \pi_{I}^{R R}>\pi_{I}^{R N}$ in each region in Proposition 7. Define $s_{7}$ as the threshold value. When $s<s_{7}, \pi_{I}^{R R}<\pi_{I}^{R N}$ in each region in Proposition 7. Let $m_{5}=\min \left(m_{4}, m_{R R}^{1}\right)$.

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